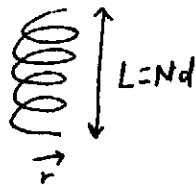


Problem 1 (15 points) Mutual Inductance

Consider a long solenoid that has N turns and is tightly wound using wire of diameter d . The radius of the solenoid is r . Neglect fringe fields.

- (a) For a given current I , how big will the magnetic field in the solenoid be?



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$BL = \mu_0 NI$$

$$BNd = \mu_0 NI$$

$$B = \frac{\mu_0 I}{d}$$

- (b) A second solenoid is wound around the first one. The second solenoid has the same length as the first one, but uses wire that is twice as thick. What is the mutual inductance of the two solenoids? Neglect fringe fields.

oops: $R=r$

$$\Phi = \int \vec{B} \cdot d\vec{a} = \left(\frac{\mu_0 I}{d} \right) (\pi R^2) (N_2)$$

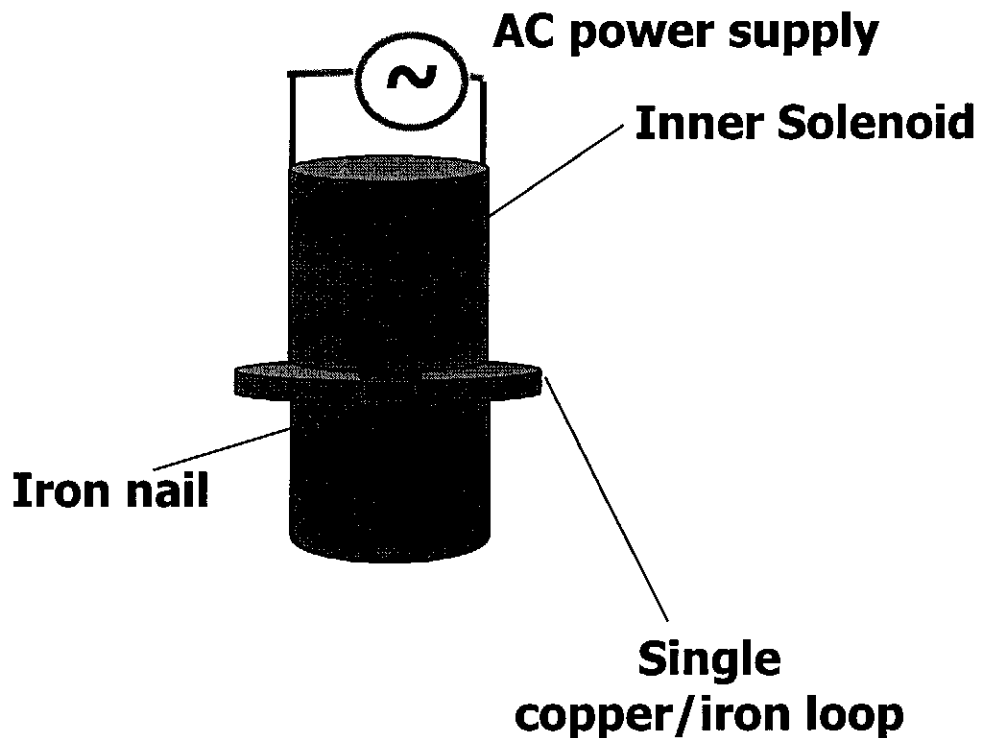
$$L = N_2 2d = Nd \Rightarrow N_2 = \frac{N}{2}$$

$$\Phi = \frac{\mu_0 I}{d} \pi R^2 \frac{N}{2} = \left(\frac{\mu_0 N}{2d} \pi R^2 \right) I$$

$$M = \frac{\mu_0 N}{2d} \pi R^2$$

Problem 2 (20 points) Transformer action

In one of the lecture demos, a long solenoid with 100 turns was connected to an AC power supply providing a varying voltage of $V_{rms} = 100V$ on average. A single loop was put around the solenoid. The loop consists of a thick copper wire and a thin iron nail. When the AC was powered up, the iron nail heated up and eventually melted.



(a) How are power, voltage and current in the primary solenoid related to those in the iron/copper loop?

$$P_{\text{primary}} = P_{\text{loop}}$$

$$V_{\text{primary}} = N_{\text{primary}} V_{\text{loop}} = 100 V_{\text{loop}}$$

$$I_{\text{primary}} V_{\text{primary}} = I_{\text{loop}} V_{\text{loop}} \Rightarrow I_{\text{primary}} 100 V_{\text{loop}} = I_{\text{loop}} V_{\text{loop}}$$

$$\Rightarrow I_{\text{loop}} = 100 I_{\text{primary}}$$

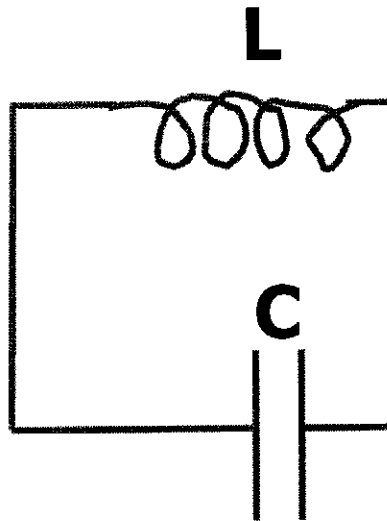
(b) Assume that the resistance of the nail is 0.1 Ohm. Estimate the power dissipated in the nail.

$$V_{\text{loop}} = 1V$$

$$P = \frac{V^2}{R} = \frac{1}{0.1} = 10 \text{ W}$$

Problem 3 (20 points) LC circuit

Shown below is an ideal LC circuit, consisting of an inductor with $L=0.1\text{H}$ and a capacitor with $C=0.1\text{F}$. At time $t=0$, the current in the circuit is zero. The total energy stored in the circuit is 0.5 Joule.



(a) The charge Q on the capacitor will vary as a function of time as $Q(t) = Q_0 \sin(\omega t + \phi)$. Based on the information given above, find the numerical values for Q_0 , ω and ϕ .

$$\frac{Q}{C} + L \frac{dI}{dt} = 0$$

\Downarrow

$$\ddot{Q} + \frac{1}{LC} Q = 0$$

$$\Rightarrow \omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1\text{H} \cdot 0.1\text{F}}} = 10 \text{ Hz}$$

$$I(t) = Q_0 \omega \cos(\omega t + \phi)$$

$$\Rightarrow 0 = Q_0 \omega \cos \phi$$

$$\Rightarrow \phi = \pm \frac{\pi}{2}$$

At $t=0$, capacitor has all energy

$$E = \frac{1}{2C} Q_0^2 \sin^2 \phi = \frac{1}{2C} Q_0^2$$

$$\Rightarrow Q_0 = \sqrt{2CE} = \sqrt{2 \cdot 0.1\text{F} \cdot 0.5\text{J}} = \sqrt{0.1} \text{ C}$$

$$= 0.32 \text{ C}$$

(b) If the maximum field the inductor reaches is 0.1 T, what is the volume of the inductor? You can leave your answer in terms of μ_0 and the values given in the problem.

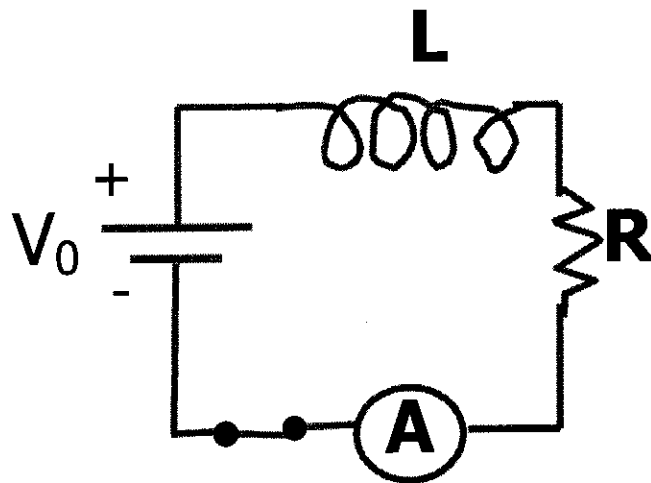
$$E = \frac{B^2}{2\mu_0} V$$

$$\Rightarrow V = \frac{2\mu_0 E}{B^2} = \mu_0 \cdot \frac{2 \cdot 0.5}{0.1^2} = 100 \mu_0$$

$$V = 100 \mu_0$$

Problem 4 (15 points) LR circuit

Shown below is a an ideal LR circuit, connected to a DC power supply with a switch. The resistance of the resistor is 10 Ohm. The output voltage of the power supply is 10V. The self-inductance of the inductor is 0.1H. The switch has been closed for a long time and is then opened at time $t=0$. The ammeter is connected such that it gives positive readings for a current flowing clockwise.



- (a) What is the magnitude and sign of the current I in measured by the ammeter at $t=0.01$ second, after the switch has been opened? For full credit, your answer needs to be correct to within 10%.

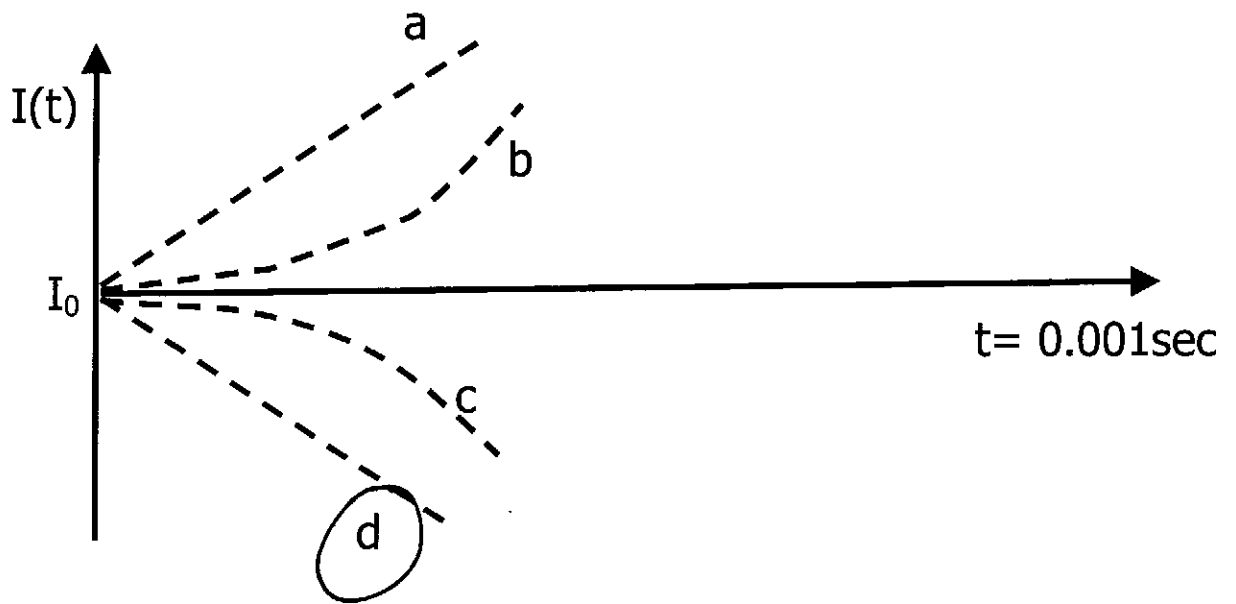
Current should drop Exponentially

$$I = I_0 e^{-\frac{R}{L}t} \quad \text{with steady state current being } \frac{V_0}{R}$$

$$I = \frac{V_0}{R} e^{-\frac{R}{L}t}, \quad I(0.01s) = \frac{10V}{10\Omega} e^{-\frac{10\Omega}{0.1H} \cdot 0.01} = \frac{1}{e} \text{ Amps}$$

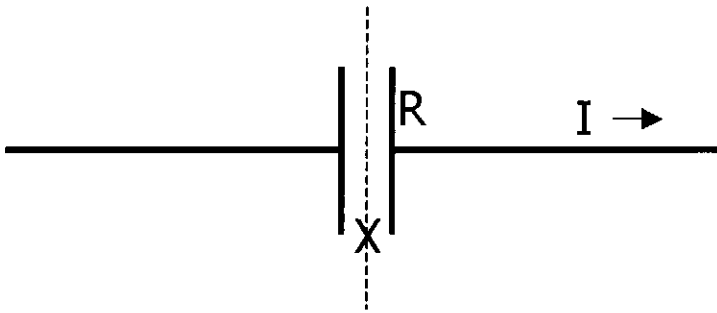
$$= \boxed{0.368 \text{ A}}$$

(b) Which of the following curves best represents the reading of the ammeter as a function of time between $t=0\text{sec}$ and $t \ll 0.001\text{ sec}$?



Problem 5 (15 points) Maxwells equations/Displacement current

Shown below is a circular parallel plate capacitor of radius R and distance d between the plates. The capacitor is being charged with a current I flowing through the wire. Neglect fringe fields.



- (a) How big is the rate of change of the electric flux through a plane in the middle of the capacitor (dashed line)?

$$Q = CV$$

$$C = \frac{\epsilon A}{d}$$

$$I = C \frac{dV}{dt} = \epsilon d \frac{dE}{dt}$$

$$\frac{d\Phi}{dt} = \frac{I}{\epsilon_0}$$

$$IA = \epsilon d \frac{d\Phi}{dt}$$

$$\frac{d\Phi}{dt} = \frac{IA}{\epsilon d} = \frac{IA}{\epsilon_0 A} = \frac{I}{\epsilon_0}$$

- (b) How big is the magnetic field at a point X on this plane at a distance R from the center line of the capacitor?

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi R}$$

Problem 6 (16 points) Waves

Consider a plane wave with an amplitude that is described by the following equations:

$$A_x = 0$$

$$A_y = 0$$

$$A_z = A_0 \cos(1000/\text{sec} * t - 0.3/\text{m} * x)$$

(c) Which direction is the wave traveling in?

$$\vec{k} \cdot \vec{r} - \omega t = N(\omega t - \alpha x) \Rightarrow \text{traveling in } x \text{ direction}$$

(d) What is the propagation speed of this wave?

$$x - vt = -\frac{\omega}{k}(x - \frac{\omega}{v}t)$$

$$v = \frac{1000}{\text{s}} \cdot \frac{\text{m}}{0.3} = \frac{10000}{3} \text{ m/s} = 3333\frac{1}{3} \text{ m/s}$$