Quiz 2b

\[ \frac{q}{E} d\alpha = \frac{Q_m}{\varepsilon_0} \]

\[ E - 2A = \frac{qA}{\varepsilon_0} \]

\[ E = \frac{\varepsilon_0}{2\varepsilon_0} \]

+ \( \frac{q}{A} \)

\[ tE, \downarrow E \Rightarrow \vec{E} = 0 \]

- \( \frac{q}{A} \)

\[ tE, \downarrow E \Rightarrow \vec{E} = \frac{E}{\varepsilon} \]

\[ \downarrow E, \uparrow E \Rightarrow \vec{E} = 0 \]

\[ V_{at} = - \int_0^t E \cdot ds = - \int_0^t \frac{E}{\varepsilon_0} \cdot t \cdot 3d = \frac{Q}{A} \frac{d}{\varepsilon_0} \]

\[ Q = CV \Rightarrow C = \frac{AE_0}{d} \]

a) \( d \rightarrow 2d \)

\[ C \rightarrow C_0/2 \]

\[ Q \rightarrow Q \]

\[ V \rightarrow 2AV \]

b) \( P = 2V = \frac{dQ}{dt} \)

\[ P = \int_0^t \frac{dQ}{d\alpha} \cdot \frac{d\alpha}{dt} \]

\[ P = 2 \]

\[ C \rightarrow C/2 \]

\[ U \rightarrow 2U = \left( \frac{Q^2}{\varepsilon_0} \right) \]

d) \[ U = \frac{1}{2} \frac{Q^2}{\varepsilon} = \frac{1}{2} CV^2 \]

\[ V \rightarrow V_0 \]

\[ C \rightarrow C_0/2 \]

\[ U = U_0/2 = \frac{Q^2}{4\varepsilon_0} \]

Notice: \( E \) remains the same in parts a-c but the volume it is affecting is doubled. In part d, \( E \) reduces to half its value.

c) Energy was added to the system by doing work against the electrical force to move the plates.
Problem 2 (20 points)

Shown below is the cross-section of a parallel plate capacitor with distance 2*d between the plates. The capacitor is given a charge Q using a power supply and then disconnected from the power supply. Then a dielectric with thickness d and dielectric constant K=2 is inserted between the plates.

(a) Does the stored energy increase, decrease or stay the same when the dielectric is inserted?

\[ U = \frac{1}{2} \frac{Q^2}{C} \]

\[ C = \frac{\varepsilon_0}{2} + K \frac{\varepsilon_0}{2} \]

The energy decreases

(b) On the graph below, draw a qualitative sketch of the electric potential between the capacitor plates as a function of x between x=0 and x=2d. At which value of x did you choose to set V=0?

[Sketch of capacitor with labels for x=0, x=2d, and potential V vs. x graph]

I chose x=2d as V=0
A different choice of V=0 would also give a correct result
(By moving this sketch up or down)
I will draw the multimeters as:

before: $S_2 = 0$ (circuit is open), $V_a = V_b$

a) $V = 400V$

b) $ΔV = V_a - V_b = 0$

c) $V_b = 0$

d) $ΔV = V_a - V_b = 3.9V$

e) The HVPS produces more current so there is a higher potential drop caused by the internal resistance of the HVPS

f) $V = F\cdot d = \frac{F}{m}d$

\[
\begin{align*}
F &\rightarrow F \\
m &\rightarrow m \\
V &\rightarrow 2V \\
d &\rightarrow 2d
\end{align*}
\]

MM would read 800V
a) \[ P = IV = \frac{V^2}{R} \]

\[ R_1 = \frac{V^2}{P} = \frac{144}{36} = 4 \, \Omega \]

b) \[ P = IV = I^2R \]

Since they have the same resistance and the same current flows through them, they would show the same brightness.

c) \[ R_2 = \frac{V^2}{P} = \frac{144}{72} = 2 \, \Omega \]

\[ P = I^2R \]

\[ I_1 = I_2 \]

\[ R_1 > R_2 \]

Bulb 1 would burn brighter.
Practice (a)

1.
(a) \( I_1 = I_2 + I_3 \)
\[ I_1 > I_2 \quad \text{and} \quad I_1 > I_3 \]
\[ P_1 = I_1^2 R_1 \]
So bulb 1 is **brightest**
(b) bulb 1 is brighter, bulb 3 is less brighter.

If the resistance of bulb 2 is reduced to \( \frac{1}{2} \),
Then \( I_1 \) increases, \( U_1 \) increases, \( U_3 \) decreases,
\[ P_1 = I_1^2 R_1 \quad \text{and} \quad P_3 = \frac{U_3^2}{R_3} \]
So bulb 1 is brighter, bulb 3 is less brighter.

2.
(a) \[ C = \frac{2\varepsilon_0 A}{d_0} \]
\[ E = \frac{1}{2} \frac{Q^2}{C} = \frac{d_0}{\varepsilon_0 A} Q^2 \]
(b) \[ U = \frac{Q^2}{C} = \frac{d_0 Q}{2\varepsilon_0 A} \] is unchanged.

After separating,
\[ \frac{1}{C'} = \frac{1}{C_{\text{glass}}} + \frac{2}{C_{\text{air}}} = \frac{d_0}{2\varepsilon_0 A} + \frac{2}{\varepsilon_0 A} \]
\[ = \frac{3d_0}{2\varepsilon_0 A} \]
\[ U_{\text{stored}} = \frac{1}{2} CV^2 \]
\[ = \frac{1}{2} \frac{2\varepsilon_0 A}{3\varepsilon_0} \left( \frac{d\varepsilon Q}{d\varepsilon} \right)^{-2} \]
\[ = \frac{\varepsilon_0}{12\varepsilon_0 A} Q^2 \]

3.
(a) \[ \Delta V = U \]
\[ \frac{\Delta V}{\Delta t} = \frac{dU}{d\varepsilon} \]
\[ \frac{\Delta V}{\Delta t} \rightarrow \Delta V = \frac{dU}{d\varepsilon} \]

\[ U = \int \Delta V \]
\[ U = \int \frac{dU}{d\varepsilon} \]
\[ \int \Delta V = U \]
\[ \frac{1}{\varepsilon} \int \Delta V = U \]

\[ U = \int \Delta V \]

\[ \Delta V + \int \varepsilon d\varepsilon = U_c \]
\[ \Delta V + \frac{\varepsilon_0}{\varepsilon} \frac{dU_c}{dt} = U_c \]
\[ \Rightarrow U_c = \Delta V (1 - e^{-\frac{t}{\varepsilon r}}) \]

\[ Q = C U_c = C \Delta V (1 - e^{-\frac{t}{\varepsilon r}}) \]
(b) \[ P = \Delta V I_c \]
\[ = \Delta V (1 - e^{-\frac{t}{cr}}) \frac{\Delta V}{\sqrt{r}} e^{-\frac{t}{cr}} \]

So when \( 1 - e^{-\frac{t}{cr}} = e^{-\frac{t}{cr}} \)

\( P \) gets maximum.

\[ \Rightarrow t = cr \ln 2. \]
\[ = 100 \times 10^{-6} \times 10 \times 10^3 \ln 2 \]
\[ = \ln 2 \times 5 \]

(c) \[ P_{\text{max}} = \frac{1}{4} \frac{\Delta V^2}{r} = \frac{1}{4} \frac{4000^2}{10 \times 10^3} = 400 \text{ W} \]

4.

(a) \[ V_{\text{mmi}} = 150 \text{ V} \quad V_{\text{mii}} \]
\[ V_{\text{miii}} = 150 \text{ V} \]

(b) \[ V_{\text{mmi}} = 300 \text{ V} \]
\[ V_{\text{mii}} = 0 \text{ V} \]
(C) before foil jumps

\[ E = \frac{\Delta V}{\alpha} \]

\[ F = QE = \frac{Q \Delta V}{\alpha} \]