Massachusetts Institute of Technology *Spring Term 2005*

8.02X Electricity and Magnetism

Quiz #1 Solutions

Tuesday, Feb 22 10:05-10:55am Room 26-100

The quiz has four questions. It is a closed book quiz. No calculators are allowed. A letter-size formula sheet can be used, but has to be signed and submitted together with the quiz.

LAST NAME

FIRST NAME

MIT ID#

RECITATION SECTION

			MARK YOURS
Rec01	MW11	B. Zeng	
Rec02	MW12	B. Zeng	
Rec03	MW1	D. Nagaj	
Rec04	TR11	G. Benedek	
Rec05	TR1	G. Benedek	
Rec06	TR2	G. Benedek	
Rec07	TR3	Daniel Nagaj	

Problem #1	
Problem #2	
Problem #3	
Problem #4	
TOTAL	

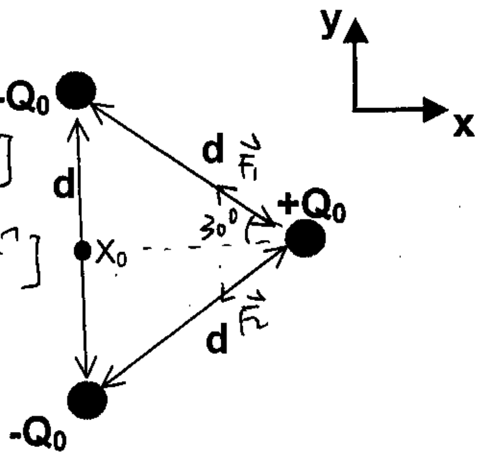
Problem 1 (25 points)

Consider the configuration of point charges shown below, with two negative charges $-Q_0$ and a positive charge $+Q_0$ forming a equilateral triangle (all sides have length d) in the x-y plane.

(a) What is the direction and magnitude of the force on the positive charge $+Q_0$ in terms of the given quantities?

(b) What is the direction and magnitude of the electric field at point x_0 halfway in between the two negative charges?

(c) Now, assume that the two negative charges are fixed in space and that $+Q_0$ is freely movable. Describe the motion $+Q_0$ would undergo if released from rest from the original position shown below (2-3 sentences)



a) $\vec F = \vec F_1 + \vec F_2$

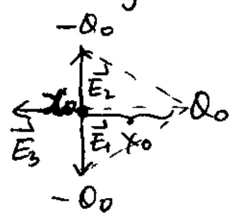
$$= \frac{Q_0^2}{4\pi\varepsilon_0 d^2}\left[-\cos30^\circ \hat i + \sin30^\circ \hat j\right]$$

$$+ \frac{Q_0^2}{4\pi\varepsilon_0 d^2}\left[-\cos30^\circ \hat i - \sin30^\circ \hat j\right]$$

$$= -\frac{2Q_0^2}{4\pi\varepsilon_0 d^2}\cos30^\circ \hat i$$

$$= -\frac{\sqrt{3}\,Q_0^2}{4\pi\varepsilon_0 d^2}\hat i$$

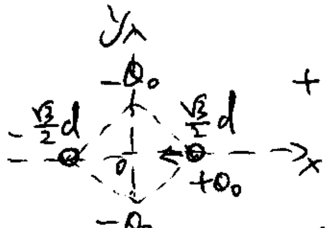
b) Apparently $\vec F_1$ and $\vec F_2$ cancel each other



$$\vec E = \vec E_3 = \frac{Q_0}{4\pi\varepsilon_0 x_0^2}(-\hat i)$$

$$= \frac{Q_0}{4\pi\varepsilon_0\left(\frac{\sqrt3}{2}d\right)^2}(-\hat i) = \frac{Q_0}{3\pi\varepsilon_0 d^2}(-\hat i)$$

c)



$+Q_0$ will oscillate back and forth between $-\frac{\sqrt3}{2}d$ and $\frac{\sqrt3}{2}d$. Since energy in conserved, $+Q_0$ will keep oscillating forever.

Problem 2 (25 points)

In lecture, you saw that an electrically charged plexiglass rod could be used to attract electrically neutral objects like a balloon made out of conducting foil.

(a) In a few sentences, explain the origin of the force between a charged object like the rod and an electrically neutral conducting object.

(b) Attraction can also be seen between a charged object and electrically neutral insulators. For example, the rod can be used to pick up pieces of confetti. How does this differ from the process described in (a)?

(a) The E.S. Force between the charges on the charged rod and the mobile charge carriers on the conductor causes charges with the same sign as those on the rod to move to the far side of the conductor, whereas unlike charges on the conductor are attracted and move to the side of the conductor close to the rod. i.e. we get an <u>induced dipole</u>. B/c the field created by the rod drops with increasing distance, the attractive force on the near unlike-charges wins over the repulsive force on the far-away like charges, giving a net attractive force between rod and conductor.

(b) This is a similar effect, however the dipole is not induced by a global charge separation across the object, but by _polarizing_ the +, - negative charges within the molecules making up the insulator

Problem 3 (25 points)

Shown below is the cross-section of a conducting sphere of radius R/2, surrounded by a very thin conducting spherical shell of radius R. The inner sphere carries a charge $+Q_0$ and the outer shell carries a charge $-Q_0$.

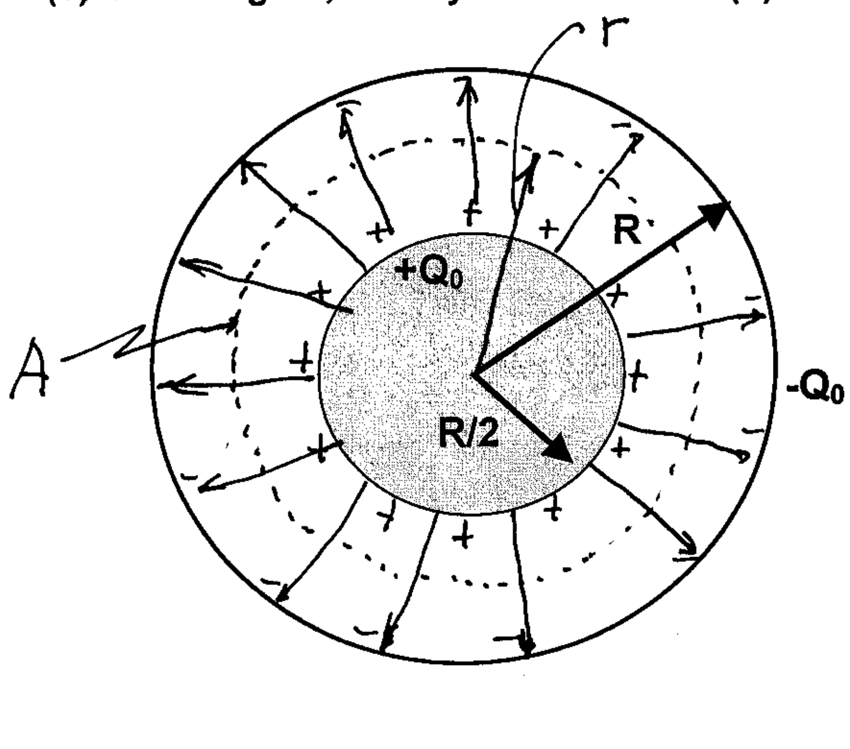
(a) On the figure, indicate the distribution of charge on the inner sphere. $+Q$ uniformly distributed on surface of inner sphere

(b) Using Gauss's Law, find the strength of the electric field E(r) as a function of r from r=0 to r > R, where r is the distance from the center of the sphere. Results without work will not receive credit. $0 < r < R/2$; $E = 0$

$\frac{R}{2} < r < R$; $E = Q_0 / 4\pi\epsilon_0 r^2$

$r > R/2$: $E = 0$

(c) On the figure, show your solution to (b) using field lines



$$\oint E \cdot dA = \frac{Q_{encl.}}{\epsilon_0}$$

Problem 4 (25 points)

Shown below is the cross-section of two large parallel plates carrying charges +Q (top) and -Q (bottom). Each plate has area A. Vertically between the plates, a small charged particle with charge q and mass m is suspended at y=d/2, i.e. the force of gravity F_G = -m*g and the electrostatic force on the particle cancel.

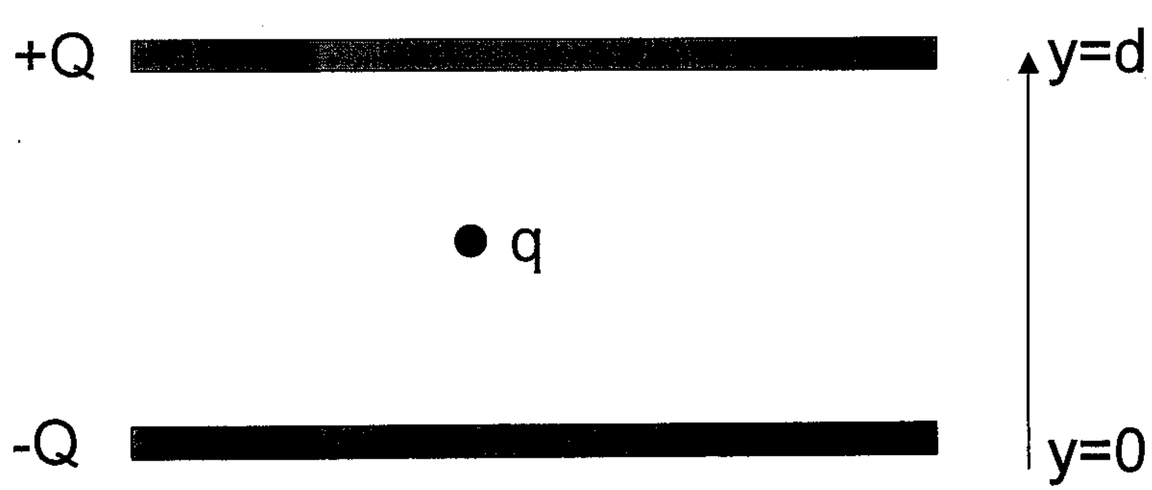
(a) What is the sign of the small particles charge q?

(b) Determine q in terms of the other quantities given. Neglect fringe effects for the electric field created by the two plates.

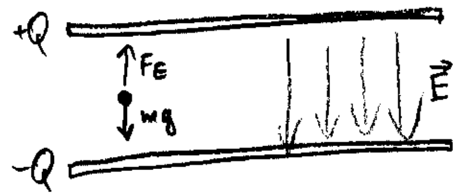
(c) Sketch the electric potential energy U_E of the charged particle as a function of y from y=0 to y=d, assuming U_E = 0 at y=0.

(d) Sketch the total potential energy U_T of the particle as a function of y from y=0 to y=d.

(e) Sketch the electric potential V between the plates (ignore the charge q) from y=0 to y=d.



PROBLEM 4



the field is constant between the plates, with $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

you can get it as a superposition of fields from two infinite plates

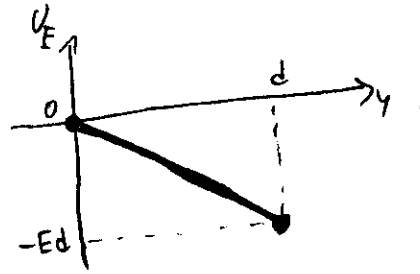
$$E = E_+ + E_- = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

a) the charge must be attracted to the top plate, so it is <u>negative</u> (2 points)

b) $F_E = F_G$ ... $Eq = \frac{Q}{A\varepsilon_0} q = -mg$ ... $\boxed{q = -\frac{mg\,A\varepsilon_0}{Q}}$ (8 points)

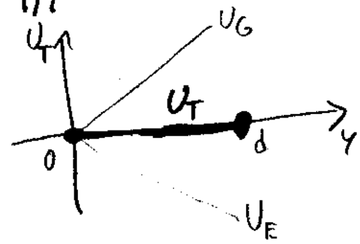
c) the potential energy in a constant field is <u>linear</u>. $U(y) - U(0) = \int_y^0 \vec{E}q \cdot d\vec{r} = qEy$

now q is negative, so we have



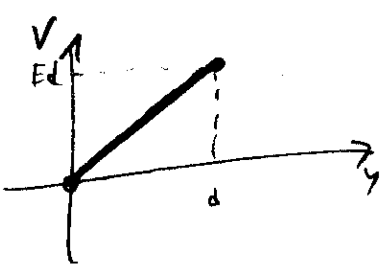
(5 points)

d) the potential energy due to gravity is $U_G = mgy = (-Eq)y$ ... it is exactly opposite to the potential energy in the electric field. So the total potential energy is <u>constant</u>



(5 points)

e) the potential between two plates is $V(0) - V(y) = \int_0^y \vec{E} \cdot d\vec{r} = -Ey$ ... <u>$V(y) = V(0) + Ey$</u>



(5 points)

You can get this also from $V = \frac{U_E}{q}$ (now q was negative, so V has opposite slope)