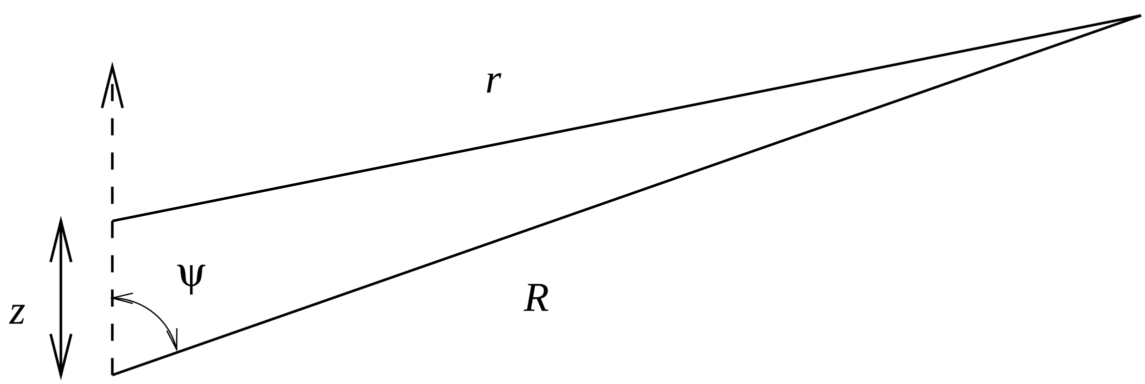
An After-the-Fact Dicsussion of Part of Problem 4.5
and
a Glimpse into the Future

Equations 4.16, 4.22 and 4.23, as well as Problem 4.4, all involve terms of the form

$$\frac{\sin(\omega t - kr)}{r}.$$

In Eq. 4.16, it was asumed that Δl is "short", so that no integral was needed. What "short" meant was (a) $\Delta l \ll r$, which has been sort of assumed from the beginning of Chapter 4, and (b) $k\Delta l \ll 1$, or $\Delta l \ll \lambda$. The other uses still assume $l \ll r$ (l as opposed to Δl, now), but l might be comparable to λ.

In any case, with $|z| \leq l$, we have



$$r^2 = R^2 + z^2 - 2zR\cos\psi.$$

Note that in Figure 4.11(b), $\psi = \theta$, but in Problem 4.4(e), $\psi = \phi$, and $z \to x$, but in Problem 4.4(c), $\psi = \frac{\pi}{2}$. We don't want to be confused by angle labels, so I'll stick to ψ as our generic angle.

Anyhow, the above form for $r = r(R, z, \phi)$ is rarely useful without some approximations. For practical purposes, we will usually have $l \ll R$, hence $z \ll R$, and so

$$\begin{aligned} r &= R\left(1 - \frac{2z}{R}\cos\psi + \frac{z^2}{R^2}\right)^{1/2} \\ &\sim R\left(1 - \frac{z}{R}\cos\psi + \frac{z^2}{2R^2} + \frac{z^2}{2R^2}\cos\psi\right) \\ &= R\left(1 - \frac{z}{R}\cos\psi + \frac{z^2}{2R^2}(1+\cos\psi)\right) \\ &= R\left(1 - \frac{z}{R}\cos\psi + \left(\frac{z}{R}\cos\frac{\psi}{2}\right)^2\right), \end{aligned}$$

keeping terms to order $(z/R)^2$.

Well, here's the deal; for $z \ll R$, we can ignore the second-order $(z/R)^2$ term, and $r \sim R - z\cos\psi$. But what, you may ask, if $\psi = \pm\frac{\pi}{2}$, so $\cos\psi = 0$? Then, $\cos^2\left(\pm\frac{\psi}{2}\right) = \frac{1}{2}$, so $r \sim R + \frac{z^2}{2R^2}$ (which we could have obtained directly from $r^2 = R^2 + z^2$).

The question is; when $\cos\psi = 0$, why don't we include the $(z/R)^2$ term? The answer is, because we don't want to! This may seem like a cheap shot, but it's really not. What's happening is that including the $(z/R)^2$ correction is of the same order as accounting for the variation of r in the denominator of $(\sin(\omega t - kr)/r)$. Yes, the correction is there, but we are usually justified in ignoring it.

Now, to be a real stinker, I introduce the reason for this explanation; in Chapter 8, Equations 8.64-8.67, we *do* include this correction, and it's essential. In fact, it's the only way Eq. 8.65 can be integrated (please note that here, $R \to z$, but $z \to \rho$). However, sneak a peek at Figure 8.22, and note that the oscillatory part is $R \sim l$ if $\lambda \sim l$, so the previously used approximation is *not* valid.

Well, then, what happens if we're off-axis in Figure 8.21? Then, we can't do the integral unless $z \gg a$. Then, things get interesting. Stay tuned.

A good reference for the criteria to determine if an observer is in the "local" (or "near") field or the "far" (or "distant") field, see Rojansky, *Electromagnetic Fields and Waves*, Chapter 24, but watch out for unorthodox notation.