8.03 at ESG **Fall 2002**

Supplemental Notes

Consideration of Equation (4-23), Page 98 in French, is a large part of the early stages of 8.03. Plots such as Figure 4-12 are useful, and used often (and are easily duplicated). Following Equation (4-23), French describes the determination of the width of the function $\overline{P}(\omega)$ for large Q by making an approximation that puts $\overline{P}(\omega)$ in a form that is symmetric about $\omega = \omega_0$, yielding $2\,\Delta\omega \approx \omega_0/Q$ (Equation (4-27)) quite readily.

As it turns out, the result that the width of the peak is exactly ω_0/Q is valid for any Q. Showing this is more math than physics. Apart from the entertaining math aspects, it might come up (I've seen physics instructors argue about this, to little advantage).

Equation (4-23) shows that $\overline{P}(\omega)$ is an *even* function of ω, in that $\overline{P}(\omega) = \overline{P}(-\omega)$. There is some question as to whether or not it makes physical sense to talk about negative frequencies, but taking this as a math question, it makes all sorts of sense.

The math question reduces to finding ω such that

$$\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2} = \frac{2}{Q^2}, \qquad \text{or}$$

$$\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} = \pm\frac{1}{Q}.$$

The $\pm$ is crucial. The last relation can be solved by multiplying by ω and using the quadratic formula, yielding

$$\omega = \frac{\omega_0}{2}\left[\pm\frac{1}{Q} \pm \sqrt{\frac{1}{Q^2} + 4}\right],$$

a total of *four* roots. We want only two roots, and this is where we have to be careful. All four roots are real, two postive and two negative. To find the width, we want the difference between the positive roots (which would also be the difference between the negative roots as well). So, the roots we want, call them ω_1 and ω_2, with $\omega_1 < \omega_2$, are

$$\omega_1 = \frac{\omega_0}{2}\left[-\frac{1}{Q} + \sqrt{\frac{1}{Q^2} + 4}\right], \qquad \omega_1 = \frac{\omega_0}{2}\left[+\frac{1}{Q} + \sqrt{\frac{1}{Q^2} + 4}\right],$$

so that

$$\omega_2 - \omega_1 = \frac{\omega_0}{Q}.$$