Electromagnetic Fields in Fields in Matter

You have likely seen the expressions for electric and magnetic fields in matter in some guise or another. The purpose of these notes is to reconcile B&B's notation with more common usages.

Start by combining the expressions in Eq. 6.27 with the expressions for the fields $\vec{\boldsymbol{D}}$ and $\vec{\boldsymbol{H}}$ as given on the next page (414). Maxwell's equations then become

$$\nabla\cdot\vec{\boldsymbol{D}} = \rho \qquad \nabla\times\vec{\boldsymbol{E}} + \frac{\partial}{\partial t}\vec{\boldsymbol{B}} = 0$$
$$\nabla\cdot\vec{\boldsymbol{B}} = 0 \qquad \nabla\times\vec{\boldsymbol{H}} = \left[\frac{\partial}{\partial t}\vec{\boldsymbol{D}} + \vec{\boldsymbol{J}}\right].$$

Note that the homogeneous equations (those without charges densities or currents) are unchanged. Note also that taking the divergence of the equation involving $\vec{\boldsymbol{H}}$, and recognizing that the divergence of a curl vanishes, the continuity equation $\nabla\cdot\vec{\boldsymbol{J}} = -\frac{\partial\rho}{\partial t}$, is recovered. This is mentioned partly as a reminder that the ρ and $\vec{\boldsymbol{J}}$ that appear in the above equations are the *free* charge density and the *conduction* current.

So, these are fine and admirable. For now, note that the divergence of $\vec{\boldsymbol{H}}$ need not vanish, and that it is possible for $\vec{\boldsymbol{B}}$ to have a non-zero curl even if $\vec{\boldsymbol{J}}$ is absent and $\vec{\boldsymbol{D}}$ is constant in time.

To proceed, let's make two wild assumptions. In fact, everything else in this chapter will presume that $\vec{\boldsymbol{E}} \parallel \vec{\boldsymbol{D}}$ and $\vec{\boldsymbol{B}} \parallel \vec{\boldsymbol{H}}$. We have discussed briefly the situations where this is *not* the case. Including the more general situation can be done, but at the expense of introducing relative permittivity as a tensor and relative permeability as an invertible tensor. Such considerations are necessary when dealing with materials such as birefringent crystals or LCDs.

So, we forge ahead; let

$$\vec{\boldsymbol{D}} = \epsilon_0\vec{\boldsymbol{E}} + \vec{\boldsymbol{P}} = \epsilon\vec{\boldsymbol{E}}, \qquad \vec{\boldsymbol{H}} = \frac{\vec{\boldsymbol{B}}}{\mu_0} - \vec{\boldsymbol{M}} = \frac{\vec{\boldsymbol{B}}}{\mu}.$$

We can solve these for

$$\vec{\boldsymbol{P}} = (\epsilon - \epsilon_0)\,\vec{\boldsymbol{E}}, \qquad \vec{\boldsymbol{M}} = \left(\frac{1}{\mu_0} - \frac{1}{\mu}\right)\vec{\boldsymbol{B}}.$$

Now, introduce the notation on page 420, Eq. 6.34b. This leads to

$$\vec{\boldsymbol{P}} = \epsilon_0 \chi_E \vec{\boldsymbol{E}}, \qquad \vec{\boldsymbol{M}} = \frac{\chi_M}{\mu_0 (1 + \chi_M)} \vec{\boldsymbol{B}}.$$

To get Eq. 6.34a, μ_0 is eliminated in favor of μ, and this is what accounts for the seeming assymetry in the two expressions that comprise Eq. 6.34a. It may seem to be a typo, but it's not.

A few further points: We still have (as of February 24, 1997) $\boldsymbol{\nabla} \cdot \vec{\boldsymbol{B}} = 0$, but $\boldsymbol{\nabla} \cdot \vec{\boldsymbol{M}}$ and $\boldsymbol{\nabla} \cdot \vec{\boldsymbol{H}}$ need not be zero. Indeed, this is the situation for a permanent magnet. If ϵ or μ varies spatially (*not* unusual), we need to account for $\boldsymbol{\nabla}\epsilon$ or $\boldsymbol{\nabla}\frac{1}{\mu}$, quantities that are not pleasant but shouldn't be frightening.

If you're gonna be a physics major, or EE, or something that will involve radiation from antennae, you'll be using the fields $\vec{\boldsymbol{D}}$ and $\vec{\boldsymbol{H}}$. If you're interested, I can recommend the text by Lorrain & Corson, a venerable copy of which I have and keep in my bookshelf, and which you are welcome to borrow. One of the neat things about the book is that it's well-illustrated.

`/mit/esg/pers/watko/8.03/notes/mats.tex`