An Alternative Solution to Problem 4.2(b)

As an alternative to the solution using

$$y(x,\, t) = f(x - vt) + g(x + vt),$$

the following, using French's notation of Equation (8-1), Page 256, should work just as well:

$$y(x,\, t) = f\left(t - \frac{x}{v}\right) + g\left(t + \frac{x}{v}\right).$$

As before, $f(t - x/v)$ represents the incident pulse, and $g(t + x/v)$ repsents the reflected pulse. (Yes indeed, they can't possibly be the same "f" and "g", but that's not important to the solution of this problem.) Proceding as before,

$$\frac{\partial y}{\partial x} = -\frac{1}{v}\, f'\left(t - \frac{x}{v}\right) + \frac{1}{v}\, g'\left(t + \frac{x}{v}\right),$$
$$\frac{\partial y}{\partial t} = f'\left(t - \frac{x}{v}\right) + g'\left(t + \frac{x}{v}\right).$$

At the end (still $x = 0$), then,

$$-\frac{1}{v}\, f'(t) + \frac{1}{v}\, g'(t) = -\frac{b}{T}\, \left(f'(t) + g'(t)\right),$$

$$g'(t) = \frac{\frac{1}{v} - \frac{b}{T}}{\frac{1}{v} + \frac{b}{T}}\, f'(t) = \frac{T - v\,b}{T + v\,b}\, f'(t).$$

Integrating (and ignoring constants of integration, as is our right in this case) gives

$$g(t) = \frac{T - v\,b}{T + v\,b}\, f(t).$$

Following the discussion in French, then, we have

$$g\left(t + \frac{x}{v}\right) = \frac{T - v\,b}{T + v\,b}\, f\left(t - \frac{x}{v}\right),$$

as obtained by the other method; the methods are equivalent.