Problem 1. Aliens have established a base on the moon (distance from earth \( d \approx 4 \times 10^8 \) m) and are preparing to attack the earth. They are using giant laser guns about 30 m long, emitting a beam of intense radiation through a circular aperture of radius \( r \approx 1 \) m. Their first target is the MIT campus, where they proceed to destroy one building at a time, starting with Building 20 (and demonstrating that, whatever their other failings, the aliens have a keen sense of the aesthetic value of proper campus planning). Estimate the wavelength of the radiation used by the aliens. What part of the electromagnetic spectrum does this wavelength correspond to? Neglect any effect of the Earth’s atmosphere and take \( l \approx 100 \) m to be the characteristic size of Building 20.

Problem 2. Sketch the main sound radiation patterns of a 10-inch diameter loudspeaker cone for middle C (256 Hz) and successively higher octaves. Assume that the vibration amplitude is approximately constant everywhere inside the circular opening. Why is it better to use a narrower cone for higher frequency sound?

Problem 3. A diffraction grating consists of a large number \( N \) of parallel narrow slits in an opaque screen. Light of wavelength \( \lambda \) normally incident on the grating will be strongly diffracted in directions making an angle \( \psi \) to the incident direction if \( \delta \equiv kd \sin \psi = 2\pi m \), where \( m \) is an integer called the spectral order. Here \( k = 2\pi/\lambda \) and \( d > \lambda \) is the separation between slits.

(a) Show that if a second component of the light with wavelength \( \lambda' = \lambda + \Delta \lambda \), with \( |\Delta \lambda/\lambda \ll 1 \), is diffracted through the same grating, the smallest wavelength difference \( \Delta \lambda \) that can just barely be resolved at order \( m \) is given simply by

\[
\frac{\Delta \lambda}{\lambda} = \frac{1}{mN}.
\]

Here ”barely resolved” means that the main intensity peaks corresponding to \( \lambda \) and \( \lambda' \) do not overlap by more than half their width (this is called the “Rayleigh criterion” in optics).

(b) The yellow doublet of sodium consists of two wavelengths \( \lambda = 589.0 \) nm and \( \lambda' = 589.6 \) nm. What is the minimum value of \( N \) needed to resolve this doublet in the third spectral order?

(c) When the slits are extremely narrow, with their width \( D \ll \lambda \), higher spectral orders are in principle visible (allowing for better spectral resolution), but very little light can go through the grating. As one increases \( D \) to allow more light to go through, higher spectral orders become suppressed compared to low spectral orders. For what critical value \( D_{\text{max}} \) of the slit width does the fourth spectral order first disappear completely?
Problem 4. A plane electromagnetic wave is incident on the interface between two perfect (nonmagnetic) dielectrics. The index of refraction is $n_1$ on the side of the incident wave and $n_2$ on the other side (recall that the speed of propagation inside a dielectric material of permittivity $\varepsilon$ is $v = \frac{1}{\sqrt{\varepsilon \mu_0}} \equiv \frac{c}{n}$). The angle of incidence is $\theta_i$ and the incident wave is polarized with its electric field in the plane of incidence. Assume from the start that there is no reflected wave. Write down the boundary conditions for the tangential components of the electric field and magnetic field at the interface and show that these boundary conditions can be satisfied only if $\tan \theta_i = n_2/n_1$, i.e., $\theta_i = \theta_B$, the Brewster angle.