Problem 1. A mechanical system initially at rest is set into vibrations by a harmonic driving force whose frequency is 1% greater than the resonant frequency of the system. Estimate (approximately) the \( Q \)-value the system must have if its amplitude during the build-up is not to exceed the steady-state amplitude by more than 10%. Is it a lower limit or an upper limit?

Problem 2. Two identical simple pendulums are connected by a light spring attached to the bobs. Each bob has a mass of 1.0 kg and the coupling spring constant is 0.8 N m\(^{-1}\). When one pendulum is clamped, the period of the other is found to be 1.25 s. Find the periods of the two modes of the system with the clamp removed. \textit{Hint:} you can easily guess what the two normal mode frequencies are; there is no need to solve the equations of motion (or even to write them down).

Problem 3. A mass \( M \) is moving horizontally on a frictionless surface. It is connected to a rigid wall on the left via a spring of constant \( k_M \) and to another mass \( m \) on the right, also moving horizontally on the same frictionless surface, via a spring of constant \( k_m \). The equations of motion are

\[
M \ddot{x}_M = -(k_M + k_m)x_M + k_m x_m,
\]

\[
m \ddot{x}_m = +k_m x_M - k_m x_m,
\]

where \( x_M \) and \( x_m \) are the displacements of \( M \) and \( m \) from their equilibrium positions.

(a) Derive the general expressions for the two normal mode frequencies.

(b) For the particular case where \( M = 2m \) and \( k_M = 2k_m \), find the normal mode frequencies in terms of \( \omega_0^2 \equiv k_m/m \) and determine the corresponding amplitude ratios. Write down the general solution for \( x_M(t) \) and \( x_m(t) \) in this case in terms of the two normal modes. Identify clearly which parameters in your general solution can be used to satisfy the initial conditions.

(c) Now add a horizontal periodic force \( F(t) = F \cos \omega t \) acting on \( M \) alone. The equations of motion become

\[
M \ddot{x}_M = -(k_M + k_m)x_M + k_m x_m + F \cos \omega t
\]

\[
m \ddot{x}_m = +k_m x_M - k_m x_m.
\]

Show that for any given value of \( \omega \) it is always possible to adjust the values of \( k_m \) and \( m \) in such a way that \( x_M(t) = 0 \) at all times! This is the basic principle used in all dynamic vibration absorbers. \textit{Hint:} set \( x_M = \ddot{x}_M = 0 \) in the equations of motion, derive the result in about 3 lines of algebra, then read pp. 132–134 in French and add a few comments to your solution.

Problem 4. Consider two identical series \( RLC \) circuits coupled by their mutual inductance, i.e., the two circuits are coupled because the current oscillating in one circuit sets up an oscillating magnetic flux in the other, thereby inducing an oscillating e.m.f. in that
circuit. This is an example of “dynamic coupling,” i.e., coupling through inertia rather than stiffness. The equations of the two coupled circuits can be written

\[
L \frac{di_1}{dt} + M \frac{di_2}{dt} + R i_1 + \frac{1}{C} q_1 = 0,
\]
\[
L \frac{di_2}{dt} + M \frac{di_1}{dt} + R i_2 + \frac{1}{C} q_2 = 0,
\]

where \(M\) is the mutual inductance, \(i_1 = dq_1/dt\) and \(i_2 = dq_2/dt\). Assume that \(M > 0\) and \(M < L\).

(a) Show that the two normal mode frequencies and corresponding damping factors are

\[
\omega_1^2 = \frac{1}{(L + M)C}, \quad \gamma_1 = R/(L + M),
\]
\[
\omega_2^2 = \frac{1}{(L - M)C}, \quad \gamma_2 = R/(L - M).
\]

Hint: define normal coordinates \(\psi_1 = q_1 + q_2\) and \(\psi_2 = q_1 - q_2\).

(b) Sketch how \(\omega_1, \omega_2, \gamma_1,\) and \(\gamma_2\) vary as a function of the dimensionless coupling strength \(M/L\). Describe qualitatively what happens at late times after we start an arbitrary vibration?