Physics 8.05 Oct 14, 2007

SUPPLEMENTARY NOTES ON NEUTRINO OSCILLATIONS AND KAON PHYSICS

1 Introduction

We have spent the first part of 8.05 setting up the kinematic and dynamical framework of quantum physics. We have developed a structure that is much more general than wave mechanics. In this third section of the course, in order to solidify our understanding, we are working through several examples of operator quantum mechanics. Fortunately many beautiful and relatively modern applications of quantum mechanics involve the time development of systems where only *two quantum states* are important. A two-dimensional Hilbert space is so simple that all of the properties of these systems can be displayed analytically without approximation. Such systems are not only of pedagogical interest. Many interesting modern developments in quantum theory and applications to the real world involve systems where the restriction to two quantum states is natural.

In lecture so far we have seen several examples of this formalism; we worked out the time evolution of the ammonia molecule; we then considered the precession of a spin-1/2 particle in a static magnetic field, using this to illustrate the analysis of the time evolution of the most general physical observable in any two-state system with a time-independent Hamiltonian. We then analyzed one particular example of a time-dependent Hamiltonian, namely that which allowed us to describe the physics of NMR.

In these notes, we will pursue applications to the physics of neutrino oscillations and kaon propagation, decay, and regeneration. A complete treatment of either of these topics would require physics beyond what we will cover in 8.05. In both cases, I will try in lecture to give a presentation which both introduces you to the physics and serves to reinforce some previous quantum mechanics lessons, and teach you some new ones. These notes are at the level I will follow in lecture, and are also at a level such that they could be the source of problems on the midterm or final. You are responsible for this material. However, for both the midterm and final you are not responsible for *memorizing* physics special to neutrinos or kaons. If I pose a problem in this area, I will tell you whatever you need to know about neutrinos or kaons, and ask you to solve a problem which demonstrates your understanding of quantum mechanical two-state systems. Although memorization of the special features of these systems is not called for, if you do face a problem of this nature on an exam you will be in much better shape if you have read through these notes.

Neutrino oscillations have been much in the news over the last few years. For example, in 2002 the Nobel prize was awarded to Ray Davis and Masatoshi Koshiba for detecting neutrinos from the sun. The fact that they detected fewer neutrinos than expected was the earliest indication that neutrinos may oscillate. Many of you may wish to learn more about neutrino oscillations than I can provide in lecture and in these notes. Resources for you to begin to do that are summarized at the end of these notes, and have been posted on the course web page. This material is marked as optional reading, and is only for those of you that wish to go beyond these notes.

You might seek out Prof. Formaggio, as he is the MIT faculty member studying neutrino oscillations. He is a member of the SNO and KATRIN collaborations.

2 Neutrino Oscillations

These notes have grown over time. Prof. Jaffe started them in 1996. Prof. Rajagopal and I extended and updated them in several subsequent years, and I have just done so again. Prof. Formaggio has also given helpful suggestions. The notes are still evolving, as is the subject.

Neutrinos are very light, perhaps massless, particles produced by radioactive decays that involve the *weak interactions*. Neutrinos are examples of relatively simple particles known as "leptons", simple because they do not experience the strong interactions that bind quarks into hadrons (protons, neutrons, pions, kaons, *etc.*). Neutrinos are intimately associated with equally simple, negatively charged particles — the charged leptons. The most familiar charged lepton is the electron (e), which has a mass $0.511\,\mathrm{MeV}/c^2$. Two others, the *muon* (μ), with mass $105.66\,\mathrm{MeV}/c^2$ and the *tau* (τ), with mass $m_\tau = 1777.0\,\mathrm{MeV}/c^2$ are known. (For some reason, even though in speech we refer to the electr*on* and mu*on*, the τ is referred to as the tau, not the tauon.) All three have the same (negative) charge as the electron. Each lepton has its own associated neutrino, the electron (ν_e), muon (ν_μ), and tau (ν_τ) neutrino respectively. Neutrinos are neutral. As far as we can tell from experiments which attempt to measure the neutrino masses directly[1], neutrinos appear to be massless. (The present limits from direct mass measurements are $m_{\nu_e} < 2.2\,\mathrm{eV}/c^2$, $m_{\nu_\mu} < 0.19\,\mathrm{MeV}/c^2$ and $m_{\nu_\tau} < 18.2\,\mathrm{MeV}/c^2$.)[2]

Neutrinos and their associated leptons appear to be linked in the following way: If we assign a "lepton number" $+1$ to the lepton and its associated neutrino, and -1 to the associated antilepton and antineutrino, then Nature appears to conserve electron number, muon number and tau number separately. Note that every particle has an associated *antiparticle* with opposite electric charge and lepton number and the same mass (for example, $\bar{e}$ is an antielectron, or *positron* and has the opposite electric charge and electron number, but otherwise same properties as the electron). The conservation of lepton number implies that a high energy photon passing through matter can disassociate into an $e\bar{e}$, $\mu\bar{\mu}$ or $\tau\bar{\tau}$ pair, but not into a $e\bar{\mu}$ or $\mu\bar{\tau}$ pair. Likewise a weak decay like $\pi \to \mu\bar{\nu}_\mu$, or $\pi \to e\bar{\nu}_e$ is allowed, but $\pi \to \mu\bar{\nu}_e$ has never been observed.[3] Let's check lepton number conservation explicitly in the

[1] As opposed to via oscillations, as we describe below.

[2] As we shall see below, neutrino oscillation experiments are only sensitive to the *differences* among neutrino masses; they cannot teach us the absolute scale of the neutrino masses in the way that a direct mass measurment can. In addition to being a member of the SNO collaboration — more on that below — Prof. Formaggio is helping to construct a new experiment in Germany (called KATRIN) which will make the most precise direct measurement of m_{ν_e} ever, which will either determine its mass for the first time or set a new upper limit on it that could be as low as $0.2\,\mathrm{eV}/c^2$.

[3] The π-meson is a particle of mass about 1/7 that of the proton and lifetime about 26 nanoseconds. It plays an important role in the dynamics of nuclei, but other than its propensity to decay into leptons, its properties need not concern us here.

latter example: In the decay $\pi \to e\bar{\nu}_e$, the electron number, muon number and tau number are all zero both before the decay and after. In the decay $\pi \to \mu\bar{\nu}_e$, all lepton numbers are zero before the decay, while after the decay the muon number is +1 and the electron number is -1. This decay violates lepton number conservation, and has never been observed.

Lepton number and its conservation are simple examples of the phenomenological regularities on which our theory of electroweak interactions is based. In this theory, which is one of the pillars of the "standard model of particle physics", the different interactions involving the three lepton groups $(e,\ \nu_e)$, $(\mu,\ \nu_\mu)$ and $(\tau,\ \nu_\tau)$ are such that the number of leptons present from each of the groups is conserved. We say that there are three conserved lepton numbers in the standard model.

In the standard model, neutrinos are precisely massless. Our present understanding of the electroweak interactions gives no acceptable explanation *why* neutrinos should be massless, given that all other fermions in the standard model have nonzero masses. This ranks as one of the principal mysteries of the standard model of modern particle physics. Furthermore, it is a relatively simple matter to extend the standard model to include nonzero neutrino masses in ways which do not upset the very many other precise predictions which the model makes.[4] So, are neutrinos precisely massless? Or are their masses merely too small for our experiments to detect? The phenomenon of "neutrino oscillations" is a method to search indirectly for possible neutrino masses. The mass is not observed directly but instead by a change in neutrino identity caused by quantum propagation. Neutrino oscillations, for example in which a ν_e changes to a ν_μ as it propagates, require neutrino masses to be nonzero, and also require that the lepton numbers are not conserved. Their discovery would therefore demonstrate that the standard model of particle physics is incomplete.

Lets assume the neutrinos have a mass and see where this leads us. For simplicity we ignore tau neutrinos for the present, and treat the $\nu_e - \nu_\mu$ system in isolation. The extension of our treatment to three neutrino species is a relatively easy exercise in the dynamics of a three-state system. There is no reason *a priori* for the (at present unknown) dynamics that gives rise to neutrino masses to respect the conservation of lepton number.[5] The physics of neutrinos makes use of two bases:

[4] For example, neutrino masses are nonzero in many "grand unified theories". These are theories in which the strong interactions (which bind quarks into protons and neutrons, for example), the weak interactions (responsible for beta decay, for example, and for the processes like pion decay which produce neutrinos which we discuss further below) and the electromagnetic interactions, are different manifestations of the same underlying force law. At very short distances and very high energies in such theories, the photon (carrier of the electromagnetic interactions), the W- and Z-bosons (carriers of the weak interactions) and the gluons (carriers of the strong interactions) are indistinguishable. At these very short distances and very high energies, quarks also become indistinguishable from leptons. Grand unified theories are, however, not the only context in which the standard model can be extended to include nonzero neutrino masses. Discovery of neutrino oscillation certainly does not imply the existence of a grand unified theory.

[5] A similar situation occurs with quarks: the eigenstates of the weak interactions are not the same as the eigenstates of the mass generating dynamics. The mixing among quark species is determined by a transformation matrix known as the "Cabbibo, Kobayashi, Maskawa matrix" whose entries are fundamental parameters of the Standard Model — but that is the subject of a course in particle physics. Discovery of mixing among neutrinos and nonzero masses for neutrinos would make the lepton sector more similar to the quark sector. This is an incomplete glimpse of why neutrino masses and mixing arise in the grand unified theories mentioned in the previous footnote.

1) the weak interaction eigenstates produced in weak decay processes, denoted $|\nu_e\rangle$ and $|\nu_\mu\rangle$, and

2) the mass (or Hamiltonian) eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$ with masses m_1 and m_2 respectively.

The basis sets are related by a change of basis,

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle\,, \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle\,, \end{aligned} \tag{1}$$

where we have chosen the phases of the states so the (unitary) matrix is real. The angle θ is presumably determined by the same dynamics that determines the neutrino masses, and it is just as mysterious at the present time.

To summarize the "ground rules" — the eigenstates of the Hamiltonian (of Nature) are $|\nu_1\rangle$ and $|\nu_2\rangle$ with eigenvalues m_1c^2 and m_2c^2 for neutrinos *at rest*. A neutrino of type j with momentum p is an energy eigenstate with energy $E = \sqrt{p^2c^2 + m_j^2c^4}$. Neutrinos are produced by weak interactions in eigenstates of lepton number ($|\nu_e\rangle$, $|\nu_\mu\rangle$ and $|\nu_\tau\rangle$) that are not energy eigenstates. The two sets of states are related by a unitary transformation as given in the two state case by (1).

The first step in looking for neutrino oscillations is to produce a beam of neutrinos of a specific weak interaction type and definite (or nearly definite) energy. One mechanism makes use of π^+ mesons. π^+ mesons decay overwhelmingly to $\mu^+\nu_\mu$.[6] Furthermore, π^+ mesons are produced copiously in collisions of high energy protons with matter. They can be selected from the refuse of these collisions by a sequence of analysing magnets and collimators. It is not too difficult to produce a well collimated, nearly monoenergetic beam of π^+'s that can then be allowed to decay in flight. It would seem we now have a beam of pure ν_μ's, but we'd better be careful. What happens to the muons, also produced in the decay of the π^+'s? The muons will themselves decay a few microseconds (an eternity) later, producing ν_e's, among other things. Our goal is a beam of pure ν_μ's, so we must avoid having the neutrinos produced by muon decay in our beam. One solution (which looks simple to a theorist, but which is not used, and so is presumably impractical) might be to use magnets to sweep the muons out of the beam, before they decay. Two strategies are actually used: Either the muons are stopped in a big pile of dirt, so that when they decay they make low energy neutrinos which are completely uncollimated. The effects of these uncollimated and feeble neutrinos are negligible relative to the collimated, energetic, ν_μ's. Alternatively, the pion beam is produced in short pulses and the detector is then set up to only look at neutrinos produced "promptly" and to ignore those neutrinos produced "an eternity" later.

The bottom line is that we may assume that a beam of muon neutrinos, $|\nu_\mu\rangle$, all with the same (or approximately the same) momentum p, emerges from our π^+ decay apparatus at $t = z = 0$. Our initial beam is a superposition of mass eigenstates according to (1),

$$\begin{aligned} |\nu_\mu(0)\rangle &= \cos\theta|\nu_2\rangle - \sin\theta|\nu_1\rangle\,, \\ &= \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \end{aligned} \tag{2}$$

[6]The most common π^+ decay that yields a neutrino of a different weak interaction type is $\pi^+ \to e^+\nu_e$, which occurs at a level of one part in 10^4.

where the second line is in a two component vector notation in the $\{1, 2\}$ basis. The neutrino is produced by the decay $\pi^+ \to \mu^+ + \nu_\mu$, and so is in the state $|\nu_\mu\rangle$ when it is produced. The mass eigenstates propagate through space with momentum E_j determined by relativistic kinematics, $E_j = \sqrt{p^2c^2 + m_j^2c^4}$. We will assume that each mass eigenstate propagates as a plane wave, $\langle z|\nu_j\rangle = e^{i(pz - E_j t)/\hbar}$. For small mass (the case of interest) the energy can be expanded, $E_j \approx (pc) + (m_j^2c^3/2p)$. At a time t and a distance z the wavefunction of the initial ν_μ beam has become

$$\psi(z,t) = e^{-i\frac{pc}{\hbar}(t - z/c)} \begin{pmatrix} -e^{-i\frac{m_1^2c^3}{2\hbar p}t} \sin\theta \\ e^{-i\frac{m_2^2c^3}{2\hbar p}t} \cos\theta \end{pmatrix} . \tag{3}$$

This equation is the starting point for several aspects of neutrino oscillation phenomenology. Above I used infinite plane wave wavefunctions. The analysis should really be done using more realistic wave packets. This is done in the notes by Stelitano and also in the article by Boris Kayser, both of which you can find in the "extra reading" packet.[7] We are interested in circumstances in which the wave function describing the neutrino of interest is (i) a wave packet which is much wider than the wavelength $2\pi\hbar/p$, and yet (ii) is much narrower than the distance scale which we will see below is actually of interest to us, namely the distance over which the neutrino will travel and oscillate. (ii) means that when we discuss a neutrino travelling over many metres or many km, we will think of a travelling wave packet, and not a plane wave delocalized over many m or km. (i) means that we can nevertheless get away with using plane waves as an approximation. The result of the wave packet analysis agrees with (3) up to corrections which are negligible for any practical purposes. We can rewrite (3) as

$$\psi(z,t) = e^{-i\phi(z,t)} \begin{pmatrix} -e^{-i\frac{m_1^2c^3}{2\hbar E}L} \sin\theta \\ e^{-i\frac{m_2^2c^3}{2\hbar E}L} \cos\theta \end{pmatrix} , \tag{4}$$

where $\phi(z,t)$ is a phase common to both neutrinos that depends on z and t. Here we have replaced t by L/c, where L is the distance travelled by the wave packet, and have defined $E \equiv pc$, because the wave packet is travelling with a group velocity which is c to very high accuracy. This is the form which we shall use.

2.1 Neutrino Oscillation in Laboratory Experiments

In this subsection, I describe one type of laboratory scale experiment to look for neutrino masses. Let us suppose that a beam of initially pure muon neutrinos is produced in the manner described above. At some distance from the source, the mixing described by (4) will result in a component of electron neutrinos in the beam, and an associated reduction in the number of muon neutrinos. The electron neutrinos can be detected by looking for a characteristic reaction like $\nu_e + n \to e^- + p$. In the case of the LSND detector, located

[7] You may well be concerned that we have applied 8.05 methods developed for non-relativistic quantum mechanics to highly relativistic neutrinos. It would go beyond the scope of 8.05 for me to present a relativistic treatment leading to (3). I have posted notes prepared by D. Stelitano, who was the TA for 8.05 in 1996, which make an attempt at doing this. Although these notes may not answer all your questions, they should clarify the derivation of the result (3).

at the Los Alamos National Laboratory in New Mexico, the detector consisted of a large volume of mineral oil, which contains many carbon atoms. The "n" in the above reaction is a neutron within a carbon nucleus. The detector detected the electrons, kicked out of the carbon nucleus by the neutrino. Note that only a ν_e can make an electron when it strikes a nucleus; ν_μ's can make muons but not electrons. The LSND experiment was an "appearance experiment", in that it looked for the appearance of ν_e's in what was originally a beam of pure ν_μ's. Another possibility would be to look for a depletion in the number of μ neutrinos in the beam — a "disappearance experiment". In the case of LSND, the reduction in the number of μ neutrinos is too small to detect; we shall see examples of disappearance experiments below, however.

It is hard to overstate how difficult these experiments are. The fundamental difficulty is that neutrinos interact with matter only incredibly weakly. For example, an electron neutrino with energy 100 MeV/c^2 would need to travel through a tank of mineral oil which is about 10^7 earth radii in size in order to have a 50% chance of interacting with a carbon nucleus and being detected. As the LSND detector is not quite this big (!!), the experimenters must use a beam containing very many neutrinos and must have a detector which can detect the very few electrons which are produced.

The change in the composition of the neutrino beam depends on L and E. An experiment aiming to be sensitive to the smallest possible masses and mixing angles would seek to maximize L and minimize E (see Eq. (4)). However, practicalities limit L and E. The neutrino beam diverges so the surface area of a detector placed at a distance L must grow like L^2 and so must the cost. Neutrinos are less reactive the lower their energy, so detector costs limit how low an energy one can make use of; also, lower energy neutrino beams are less well collimated and less monochromatic raising other disadvantages. For some fixed L and E an experiment is sensitive to neutrino oscillation provided $|m_1^2 - m_2^2|$ and/or $\sin 2\theta$ is not too small.

As an explicit example, let us compute the probability that an electron neutrino will be found in an initially pure ν_μ beam. This is determined by the square of the overlap of (4) with the electron neutrino state, $(\cos\theta, \sin\theta)$. A little bit of algebra (which you will do on problem set 7) yields,

$$P_{\nu_e}(L) = \sin^2(2\theta)\sin^2\left(\frac{c^3}{4\hbar}\frac{\Delta m^2 L}{E}\right), \tag{5}$$

where $\Delta m^2 = m_1^2 - m_2^2$. We see that the neutrino, initially a $|\nu_\mu\rangle$, oscillates until P_{ν_e} reaches the maximum possible value, namely $\sin^2 2\theta$, and then oscillates back to being purely $|\nu_\mu\rangle$ as it travels an oscillation length

$$L_{osc} = \frac{4\pi\hbar E}{c^3\Delta m^2} \ .$$

From (5) we now see explicitly what was already evident in our earlier discussion: in order for neutrino oscillation to occur, we must have a nonzero mixing angle θ and must have a nonzero neutrino mass difference, and therefore at least one nonzero neutrino mass. Substituting values for the fundamental constants we find

$$P_{\nu_e}(L) = \sin^2(2\theta)\sin^2\left(\frac{1.27\,\Delta m^2 c^4 L}{E}\right). \tag{6}$$

In (6), $\Delta m^2 c^4$ is the neutrino mass-squared difference measured in eV^2, the neutrino energy E is measured in GeV, and L is measured in km. (Equivalently, one could measure E in MeV and L in m.)[8] Note that neutrino oscillation experiments are sensitive to the mass *differences* between neutrino species (actually mass^2 differences) and cannot be used to measure the absolute values of the neutrino masses. Of course, if the difference between two neutrino masses is nonzero, at least one mass must be nonzero.

The results of a search for $\nu_\mu \to \nu_e$ is a bound on the probability P_{ν_e} at some reasonably sharp values of E and L. Eq (6) allows us to translate the bound into a contour in the Δm^2 — $\sin^2 2\theta$ plane. The shape of the contour is quite complicated. For example, no matter how good the bound, there are values of Δm for which m_1 and m_2 are not zero, but $\sin^2 \frac{1.27\Delta m^2 c^4 L}{E} = 0$ and therefore any value of $\sin^2 2\theta$ is allowed. Clearly, ruling out a wide range of masses and mixing angles requires several precise experiments, using different values of L/E.

In 1996, there was a tantalizing report of a positive result in a $\bar{\nu}_\mu \to \bar{\nu}_e$ experiment done at Los Alamos using the LSND detector. The experiment used neutrinos with energies between 20 and 60 MeV which fly about 30 m between the point at which they are produced and the detector. The experimenters report seeing about 50 $\bar{\nu}_e$ events corresponding to an oscillation probability of about $P_{\nu_e} = 0.003 \pm .001 \pm .0005$. The first error estimate is the statistical error; the second is an estimate of the systematic error. In your problem set, you will discover what values of $(m_1^2 - m_2^2)$ and $\sin^2(2\theta)$ are consistent with this observation.[9] Between 1996 and the present, questions have been raised about possible systematic effects in this experiment not taken into account in the above error estimate. The detection is just at the edge of the limits of sensitivity which this detector can achieve, and many experimenters have been skeptical of the claim that P_{ν_e} has been demonstrated to be nonzero.

A new experiment called MiniBooNe was designed to explore the same range of $\sin^2(2\theta)$ and Δm^2 with much greater sensitivity than in the Los Alamos experiment. It was constructed at the Fermi National Accelerator Laboratory, and began taking data in 2003. The goal of this experiment was to either convincingly contradict or convincingly confirm the claim from LSND for $\nu_\mu \to \nu_e$ oscillations.[10] The ν_μ neutrinos were produced by an initial

[8] Some of you may have wondered "Why not measure the momenta of the pion which decayed and the muon which was created along with the neutrino? Knowing these, one could learn the precise energy of the neutrino, and thus learn whether one had a $|\nu_2\rangle$ or a $|\nu_1\rangle$. Then, there would be no oscillations. This is correct. However, it turns out that if one measures the momenta of the μ and π to this accuracy, their positions are very poorly known. The uncertainty in the position from which the neutrino originates is then so large that the neutrino wave packet is broader than the oscillation length, and it is in fact true that no oscillation occurs!

[9] If you wish to read about the experiment, and to compare your results to those of the people who analyzed the data in full, you can read the paper. It is published as Physical Review Letters **77** (1996) 3082-3085 and is available on the web at http://xxx.lanl.gov/abs/nucl-ex/9605003. This experiment looked for positrons produced via the appearance of $\bar{\nu}_e$ in a beam which was initially $\bar{\nu}_\mu$; the experiment we described above, namely looking for electrons produced via the apperance of ν_e in a beam which was initially ν_μ has also been done by LSND. See Physical Review Letters **81** (1998) 1774-1777 and http://xxx.lanl.gov/abs/nucl-ex/9706006.

[10] In a two-state system the observation of oscillations for antineutrinos, $\bar{\nu}_\mu \to \bar{\nu}_e$, implies the same oscillation pattern for the corresponding neutrinos, $\nu_\mu \to \nu_e$. This follows due to a fundamental symmetry of nature known as "CPT", where the C stands for charge-conjugation, which switches particles and antiparticles, the P stands for parity, and the T stands for time-reversal. CPT means we perform all three of these.

beam of 8 GeV protons on a beryllium target, resulting in a mean energy $E_{\nu_\mu} \sim 500\,\text{MeV}$, which is much larger than LSND. To compensate, the distance was chosen as $L \sim 500\,m$, thus making L/E for Eq. (6) similar to LSND. In LSND, the ν_μ neutrinos were not energetic enough to produce a μ^- which has $m_\mu = 106\,\text{MeV}$. MiniBooNE's more energetic neutrinos mean that both conversions $\nu_\mu \to \mu^-$ and $\nu_e \to e^-$ occur in their detector, so that it is no longer a simple appearance experiment. Instead their detector (800 tons of mineral oil) distinguishes between electrons and muons directly (they do this using Chrenekov radiation, which I will hold off on explaining until the next section). Their data was analyzed using a "blind analysis" to avoid any experimenter bias and improve confidence in the final result.[11] In every prior year of 8.05 this is where the story ended; without the MiniBooNE data we had no confirmation or satisfactory contradiction with the LSND oscillation result. In April of 2007 MiniBooNE reported their first results. After a long and careful analysis they concluded that the data in their signal region, $E_\nu = 475$–$3000\,\text{MeV}$ was inconsistent with a $\nu_\mu \to \nu_e$ interpretation of LSND at a 98% confidence level. Hence, it is best that we search elsewhere for neutrino oscillations.[12]

The figure of merit for a neutrino oscillation experiment are the quantities Δm^2 and $\sin^2 2\theta$ — these are the quantities bounded by the experiment; and the sensitivity of an experiment is determined by the quantity L/E which the experimenters typically want to maximize. As bounds on neutrino masses and mixing have improved, proposals for experiments with larger and larger "baselines" (L) have appeared. We will return to these longer baseline laboratory experiments after first discussing two different kinds of experiments, in which the neutrino sources are "natural" rather than man-made.

2.2 The Solar Neutrino Problem

There is a more than thirty year old problem in particle astrophysics known as the "Solar Neutrino Problem" that seems now to be solved via neutrino oscillations. The nuclear processes that fuel the sun produce *electron* neutrinos as a by-product. The energies of the neutrinos range from keV up to about 10 MeV. Experiments to detect "solar neutrinos" have been undertaken for decades, pioneered by Ray Davis at the University of Pennsylvania, who won the Nobel prize in 2002. This first experiment employed the nuclear reaction $\nu_e + {}^{37}Cl \to e^- + {}^{37}Ar$ to look for the ν_e's produced by the sun. The detector consists of a large tank of perchloroethylene cleaning fluid (C_2Cl_4) located deep underground in the Homestake Gold Mine in South Dakota. Every few months for about 25 years, the experimenters extracted the small sample of Argon atoms which had accumulated in the tank. Once again,

[11] The idea of a blind analysis is that an offset is introduced into the data, with the offset not known to any of the scientists who are doing the analysis. The scientists then analyze the data, focussing in particular on determining their estimate of the systematic error in their result. All this time they do not know whether their result is zero or nonzero, since they do not know the offset. Finally, after all the analysis is completed and the systematic errors are all understood and quantified, the collaboration decides to "open the black box", and learn what their result is. In any experiment in which understanding subtle systematic (as opposed to statistical) errors is crucial, and where there is a prior result to be confirmed (or refuted), a blind analysis provides added confidence in the final result.

[12] As is often the case in science, resolving one question brings up a new one. MiniBooNE sees an excess number of ν_e events at lower energies, and while this excess was outside the signal region for their analysis, it is still a fairly large fluctuation (3.7σ) that is not yet understood.

the fact that neutrinos interact extremely weakly makes the experiment fiendishly difficult: after each few months Davis and collaborators typically extracted about 15 Argon atoms from a tank containing more than 10^{30} atoms of Carbon and Chlorine. (That's not a typo; I really do mean fifteen Argon atoms. The 15 ^{37}Ar atoms are extracted chemically, stored, and counted as they decay radioactively, one by one, over several months. Calibrations are done by introducing a known number of Argon atoms, and then removing and counting them: these calibration studies show that about 90% of the ^{37}Ar atoms produced in the tank are extracted and counted.) To the surprise and consternation of all, Davis' experiments detect electron neutrinos from the sun at a level about 1/3 that predicted by standard models of the sun. Solar neutrino experiments like this one are examples of disappearance experiments: fewer ν_e's are seen then expected, but if they have oscillated in ν_μ or ν_τ, this cannot be checked because the detector is only sensitive to ν_e.

The nuclear reaction employed to detect the neutrinos in the Davis experiment has an energy threshold which is high enough that this experiment only detects the neutrinos among those produced by the sun which have rather high energies. In recent years new generations of solar neutrino detectors (based on tanks of Gallium instead of tanks of Chlorine) which are sensitive to lower energy neutrinos have been developed and brought on-line. Some have low enough energy thresholds that they are sensitive to the "bulk" of the neutrinos coming from the sun.[13] These see about 55% of the electron neutrinos which they should. It is very important that these gallium experiments were done, as they clearly demonstrate that the solar neutrino deficit is energy-dependent: fewer of the lower energy neutrinos are missing.

The deficit of solar neutrinos is also seen in a third type of experiment, in which the neutrinos are detected in quite a different way.[14] SuperKamiokande is a 50,000 ton tank of water, surrounded by 11,146 light detectors called photo-multiplier tubes. It is located deep underground in the Kamioka zinc mine in Japan. The water serves both as the target for the neutrinos and as the detection medium for the by-products of the neutrino interactions. When a neutrino from the sun interacts with an electron in the water, it gives the electron such a kick that the electron ends up travelling faster than the speed of light in water, which

[13] The sun shines by virtue of a cycle of many nuclear reactions whose net effect is to turn 4 protons into one ^{4}He nucleus, two positrons, and two electron neutrinos. Since we know how brightly the sun shines, we know the total number of neutrinos coming from the sun with fairly good accuracy. The Davis experiment is sensitive only to neutrinos produced in reactions involving ^{8}B and ^{7}Be. In the cycle of nuclear reactions, these reactions are "side-branches" which are only responsible for a tiny fraction of the total luminosity of the sun. Furthermore, these rare reactions have rates which are very sensitive to the temperature at the center of the sun. The theoretical prediction ("standard solar model") for how many neutrinos should be seen in the Davis experiment is therefore more uncertain than the prediction for the "bulk" of the neutrinos: we know the total luminosity of the sun very well but until recently our knowledge of the conditions at the center of the sun was less precise. This was one motivation for the gallium experiments. By now, though, astrophysicists have used "helioseismology" observations to test the standard solar model quite stringently, making its predictions reliable even quite close to the center of the sun. This is an impressive piece of physics, but the analysis of the vibration modes of the sun is not a subject in which quantum physics plays a role, so we will not explore it.

[14] Although presented third in these notes, an experiment of this type was the second experiment to detect neutrinos from the sun and discover that there is a deficit. This experiment, called Kamiokande, was lead by Masatoshi Koshiba who shared the 2002 Nobel prize with Ray Davis. In these notes, I will focus on the current generation of water-Cherenkov experiments: SuperKamiokande (a larger successor to Kamiokande) and SNO.

is about 3/4 of the speed of light in vacuum. Just as a supersonic plane makes a sound shockwave, a charged particle travelling in a medium faster than light can travel in that medium makes a "light shock wave". This light, called Cherenkov light after its discoverer, radiates from the track of the charged particle in a distinctive conical pattern, and is detected by the photomultiplier tubes. The orientation of the cone tells us which direction the muon or electron was moving, and this in turn tells us from which direction the neutrino came. Thus, the SuperKamiokande detector can almost literally "see" neutrinos, and in fact data from this detector has been used to create a picture of the sun as seen by its neutrino-shine. In understanding the implications of SuperKamiokande's data on the solar neutrino problem, it is important to note that it is sensitive only to the very highest energy neutrinos from reactions involving ^{8}B: the SuperKamiokande energy threshold is even higher than that of the Chlorine experiment. In this experiment, the number of electron neutrinos which is observed is 0.46 that which is expected.

One proposed explanation for the lack of electron neutrinos is that they may have oscillated into, say, muon neutrinos in the way that we now know how to describe using "8.05-physics". It is even possible to choose parameters such that the distance from the sun to the earth is "just so" that 40% of the lower energy ν_e's have oscillated into muon neutrinos, while 50% of the very highest energy neutrinos have oscillated into muon neutrinos, and almost all the "intermediate energy" neutrinos detectable by the Chlorine experiment but not by the SuperKamiokande experiment have oscillated into muon neutrinos. Although it is an interesting exercise to work out the values of $\sin^2(2\theta)$ and Δm^2 which can explain the data in this way — it turns out that the Δm^2 required is about 10^{-10}eV^2 — this solution has never appealed widely in the particle/astrophysics community because it attributes the experimental results to an accident of the layout of our solar system (the earth-sun distance).

Unfortunately for us in 8.05, the neutrino oscillation scenario which is much more plausible, and which is presently considered the most likely explanation of the solar neutrino data, including its energy dependence, requires some 8.06 physics to explain. It involves enhanced neutrino oscillation as the neutrinos are propagating through the sun. By "enhanced" I mean enhanced relative to the degree of oscillation which occurs in vacuum as we have described in 8.05. We will return to this next semester. For the present, suffice to say that these "matter enhanced oscillations" can explain the data with $\sin^2(2\theta) \sim 1$ and $\Delta m^2 c^4 \sim 10^{-5}$ (up to date results for $\sin^2(2\theta)$ and $\Delta m^2 c^4$ are given at the end of this section).

In April 2002, an experiment called the Sudbury Neutrino Observatory reported crucial confirmation of the hypothesis that the neutrinos from the sun are oscillating. This detector consists of a large tank of "heavy water" deep in the INCO nickel mine near Sudbury, Canada. Heavy water is chemically the same as ordinary water, except that the nuclei of the two hydrogen atoms are deuterons, instead of protons. (A deuteron is a nucleus made of one proton and one neutron. It has charge +1, so the resulting hydrogen atom has the same chemistry as ordinary hydrogen. It is just heavier.) The most important property of a deuteron for our purposes is that it is a very fragile nucleus: it only takes about 2 MeV/c^2 of energy to break apart a deuteron into a proton and a neutron. Solar neutrinos have energies up to about 10 MeV/c^2. This means that solar neutrinos, *whether they are* ν_e*,* ν_μ *or* ν_τ, can break up deuterons into neutrons and protons. And, the SNO detector can detect the resulting neutrons. Therefore, if the electron neutrinos from the sun are oscillating into muon neutrinos or tau neutrinos, and becoming invisible to all detectors except SNO, they are *visible* in Sudbury.

The SNO detector can detect neutrinos via three different reactions. First, electron scattering (ES):

$$\nu + e^- \rightarrow \nu + e^- \ .$$

This reaction involves electrons only, and so it does not matter whether the water is "ordinary" or "heavy". With respect to this reaction, the SNO detector can be thought of as a somewhat smaller version of the SuperKamiokande detector. Although I did not mention it before, according to the standard model of particle physics the ES reaction has some sensitivity to muon and tau neutrinos. You should think of this reaction as measuring $\phi(\nu_e) + \phi(\nu_\mu)/6.6 + \phi(\nu_\tau)/6.6$, where by $\phi(\nu_x)$ I mean the flux of x-neutrinos passing through the detector. Because the SuperKamiokande detector can only use this single reaction, it has no way of separately measuring $\phi(\nu_e)$ or $\phi(\nu_\mu) + \phi(\nu_\tau)$. Now we come to the second and third reactions, both of which require the presence of deuterons and thus can only be used at SNO, In the second reaction,

$$\nu_e + d \rightarrow p + p + e^-$$

a deuteron is converted into two protons and an electron. This reaction, which is called the "charged current" (CC) reaction, can only be initiated by electron neutrinos. It therefore measures $\phi(\nu_e)$. Finally, and most crucially, we come to the "neutral current" (NC) reaction,

$$\nu + d \rightarrow p + n + \nu$$

which is equally sensitive to ν_e, ν_μ and ν_τ and thus measures $\phi_{\rm total} \equiv \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)$. The experimental challenge here is to detect the neutron. SNO will ultimately do this in three different ways. The first method they used relied on the fact that some of the neutrons react with other deuterons in the heavy water, in a reaction which produces a high energy photon (a gamma ray). This gamma then in turn scatters on electrons, which are seen via their Cherenkov emission. Thus, SNO has to disentangle the electrons ultimately produced after a neutrino initiates an ES, CC or NC reaction. It does so via the different energies of the electrons produced from the three reactions, and via the fact that only the ES-electrons point away from the sun, whereas the CC and NC reactions produce electrons going in all directions.[15]

[15] Remember that NC electrons don't directly come from the NC reaction. These electrons come from the shower induced by a gamma that came from the interaction between a neutron made in the NC reaction and a deuteron in a different water molecule. The neutron from the NC reaction diffuses around in the water for a little while before making the gamma, meaning that the electron which is eventually detected has no memory of the direction in which the neutron was going when it was originally produced. Thus, the NC electrons are isotropic. The CC electrons are not isotropic, but are also not all going in the direction away from the sun. For reasons that I will not go into, they are distributed in angle like $1 - (1/3)\cos\theta$ where θ is the angle between the emitted electron and the direction of the incident neutrino. So, the angular dependences of the electrons produced in the three reactions are distinct: electrons from NC are isotropic; electrons from CC go like $1 - (1/3)\cos\theta$ and electrons from ES are strongly peaked at $\theta = 0$. Also, the energy distributions of the electrons are different. As you might guess, given their indirect origin, the electrons that result from the NC reaction have lower energies than those from the other two reactions. The combination of the three distinct angular distributions and the three distinct energy distributions make it possible to separately measure the number of electrons coming from ES, CC and NC reactions.

In April 2002, SNO reported its measurement of all three reactions. They saw 264 ± 26 neutrinos that induced ES reactions, 1968 ± 61 neutrinos that induced CC reactions, and 577 ± 49 neutrinos that induced NC reactions in their giant tank of heavy water over a period of 306.4 days. Knowing the "efficiency" with which their detector sees each of these reactions, they can convert these measurements into measurements of neutrino fluxes,[16] in units of 10^6 cm^{-2}s^{-1}:

$$\begin{aligned}
\phi_{\rm CC} \equiv& \quad \phi(\nu_e) &= 1.76 \pm .06({\rm stat.}) \pm .09({\rm syst.}) \\
\phi_{\rm ES} \equiv& \quad \phi(\nu_e) + \tfrac{(\phi(\nu_\mu)+\phi(\nu_\tau))}{6.67} &= 2.39 \pm .24({\rm stat.}) \pm .12({\rm syst.}) \\
\phi_{\rm NC} \equiv& \quad \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau) &= 5.09 \pm .44({\rm stat.}) \pm .46({\rm syst.})\ .
\end{aligned} \tag{7}$$

Lets first pause to appreciate the magnitude of these fluxes. These experiments tell us that several million neutrinos from the sun with an energy greater than 5 MeV pass through every square centimeter of the detector (and of us) every second. And, these energetic ^{8}B neutrinos constitute only 1 in 10^4 of all the neutrinos from the sun. In total, more than 10^{10} solar neutrinos per second pass through every square centimeter. Second, lets appreciate once more how the feebleness of neutrino interactions makes the experiments difficult. Even with millions of neutrinos with sufficient energy to be detected passing through each square centimeter each second, and even with an enormous detector, SNO detected fewer than 3000 neutrinos in about one year. And, third, the punch line. By measuring $\phi_{\rm NC}$ and $\phi_{\rm CC}$, SNO reveals that the total flux of electron muon and tau neutrinos from the sun is about three times larger than than the flux of electron neutrinos alone. Since we *know* that the neutrinos are electron neutrinos when they are produced, this convinces us that the neutrinos must oscillate (to either muon or tau neutrinos or both) en route from production in the sun to detection in Sudbury.[17]

Finally, we can ask whether the observed oscillation fully resolves what used to be called the "solar neutrino problem". The standard solar model (SSM) predicts that the flux of neutrinos with energy greater than 2.2 MeV is

$$\phi_{\rm SSM} = 5.05^{+1.01}_{-0.81}\ {\rm cm}^{-2}{\rm s}^{-1}\ .$$

[16] The SNO and SuperKamiokande measurements of $\phi_{\rm ES}$ are in good agreement. The SuperK measurement of $\phi_{\rm ES}$ has much smaller error bars than that from SNO, and was made earlier, but SuperK cannot measure the other two fluxes. Since the point I want to make requires comparing fluxes, I will give you only the SNO data.

[17] In Sept. 2003 SNO reported results from a second method where they dissolved salt in their heavy water. Because the interaction between neutrons and chlorine nuclei is much stronger than that between neutrons and deuterons, the salt provides a more efficient way of detecting the neutrons which are produced by the neutral current reaction, with the result that the error bar on $\phi_{\rm NC}$ has been reduced significantly since 2002. I will show you the current SNO results in lecture, and they are also incorporated in the final results for the mass difference and mixing angle quoted later in these notes. Prof. Formaggio and his colleagues are now completing the third phase of the SNO experiment, in which they have inserted an array of neutron detectors hanging within the SNO tank of heavy water. In all three phases, the CC and ES processes are measured in the same way. What has changed in the three phases is the method by which the neutrons from the NC reaction are detected. This NC reaction is the linch pin of SNO's success, as it is what allows them to see *all* the neutrinos the sun is producing. So, it is a credit to the designers of this experiment that they have come up with three different ways of measuring the crucial quantity, with three quite different sets of systematic errors.

If there were no oscillations, these should all be electron neutrinos. The detected flux of electron neutrinos is about one third this. (That was the solar neutrino problem.) Now, we see from the SNO measurement that the detected flux of *all three* neutrinos is in very good agreement with the SSM prediction! So, the sun is burning and emitting neutrinos as theory predicts. But, neutrino oscillation turns two thirds of them from electron neutrinos into muon or tau neutrinos en route. The Sudbury experiment has therefore clinched the fact that neutrinos from the sun are oscillating, and therefore have mass, unlike in the standard model of particle physics.

In December 2002, an experiment called KamLAND reported results that constituted a measurement of $\Delta m^2 c^4$ using "manmade" neutrinos rather than those from the sun. KamLAND was designed to confirm the solar neutrino result (namely that the electron neutrino oscillates into some other species, with a mass difference $\Delta m^2 c^4 \geq 10^{-5} eV^2$ and a mixing angle $\sin^2(2\theta)$ near 1). It is an ingenious experiment, located in the Kamioka mine, that relies on the fact that there are many nuclear reactors (producing 20% of the world's nuclear power) within about 200 kilometers of Kamioka. Each nuclear reactor is pumping out a flux of electron antineutrinos that is proportional to the power output, and thus known. Furthermore, different nuclear reactors are turned off for maintenance and then turned back on at known times. The mean antineutrino energy is about 3 MeV, and the mean baseline between reactor and detector is about 180 km. You can check that with this E/L, the vacuum neutrino oscillations described by Eq. (6) will be significant as long as electron antineutrinos oscillate with $\Delta m^2 \sim 10^{-5} eV^2$ or larger. In KamLAND's reported results they saw clear evidence for neutrino oscillations, and thus definitively ruled out solutions with $\Delta m^2 c^4 \sim 10^{-10} eV^2$ that were still consistent with previous data. Their results instead indicate that $\Delta m^2 c^4 \sim 10^{-5} eV^2$, which is compatible with the solution to the solar neutrino problem that was favored by SNO and SuperKamiokande.

The year 2002 was a triumphant moment in solar neutrino physics. After decades of experimental effort, for the first time we had agreement between all of the following:

- The theoretical astrophysics and nuclear physics that goes into the standard solar model calculation of how the sun burns, how many neutrinos it emits, and with what distribution of energies, supported by helioseismological observations having nothing to do with neutrino physics.

- The long-standing experiments (Ray Davis' chlorine experiment, the gallium experiments and SuperK) that reveal a deficit in the number of electron neutrinos from the sun.

- The data from the Sudbury experiment which clinched the fact that neutrinos from the sun are oscillating into muon and tau neutrinos, with the total number of neutrinos of all three flavors being consistent with the predictions of the standard solar model even though the number of electron neutrinos is reduced by about a factor of 3.

- And, the decisive determination by the KamLAND experiment that $\Delta m^2 c^4 \sim 10^{-5} eV^2$, in quantitative agreement with one of the possible explanations of the SNO results.

In addition, the experimental work that laid the foundation for all this, namely the detection of neutrinos from the sun by Davis et al and Koshiba et al, was recognized with the 2002 Nobel prize.

As of summer 2007 a global analysis of the solar-neutrino data, including the latest data from SNO, SuperKamiokande and the KamLAND data gives

$$\sin^2(2\theta) = 0.86^{+0.03}_{-0.04}\,, \quad \Delta m^2 c^4 = (8.0 \pm 0.3) \times 10^{-5}\, eV^2\,. \tag{8}$$

Interestingly, this rules out a so-called maximal mixing solution which would have had $\theta = 45^\circ$ degrees and $\sin^2(2\theta) = 1$. This solution is called maximal mixing, because for this angle the weak $|\nu_e\rangle$ eigenstate would have an equal amount of the two mass eigenstates, see equation (1). From Eq. (5) we see that it is only for the maximal mixing solution that a weak neutrino eigenstate has 100% probability of fully converting to the other state in the two-state system at some distance. There are several current and planned experiments that will give even more precise results for these neutrino parameters. To improve the precision on $\Delta m^2 c^4$ both further results from KamLAND and more precise NC measurements from SNO will be important. There are also more experiments coming online that are designed to study the lower energy solar neutrinos.

2.3 Atmospheric Neutrinos

Nature provides us with another source of neutrinos, in addition to those coming from the sun. When cosmic rays strike nuclei in air in the upper atmosphere, they produce showers of particles, including charged pions. Just as in the laboratory experiments, these pions decay via $\pi^+ \to \mu^+ + \nu_\mu$. Nature does nothing, however, to get rid of the muons from the "beam", i.e. from the shower of particles produced in the upper atmosphere. These muons later (a little lower, but still before crashing into the earth) decay via $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$. (Note that these decays conserve all lepton numbers.) The result is that for every π^+ created by the cosmic ray, one gets one muon neutrino, one muon anti-neutrino and one electron neutrino. Similarly, for every π^- created by the cosmic ray, one gets one muon anti-neutrino, one muon neutrino and one electron anti-neutrino. The detectors used to see these neutrinos cannot distinguish neutrinos from anti-neutrinos, so what matters is the 2:1 ratio. Each cosmic ray (and there are many of them) yields a small neutrino beam with twice as many muon neutrinos as electron neutrinos.

These so-called "atmospheric neutrinos" have been seen in a variety of detectors, but the experiment which really clinched the discovery of neutrino oscillations was the SuperKamiokande experiment, which has also played such an important role in studying solar neutrinos as described above. The clinching experiments that demonstrated that atmospheric neutrinos oscillate were reported in 1998. This was the first completely convincing demonstration of neutrino oscillation.

Recall that the SuperKamiokande detector is a giant tank of ordinary water, deep underground in Japan. The water serves both as the target for the neutrinos and as the detection medium for the by-products of the neutrino interactions. The atmospheric neutrinos have energy of order one or a few GeV. They are therefore about 1000 times more energetic than neutrinos from the sun. This means that atmospheric ν_μ's have enough energy that in a collision with an electron or nucleus in the water, they can create a charged muon: μ^-.[18] (Recall that in the SNO CC reaction, solar neutrinos scatter off electrons and nuclei and produce electrons. Muons have a mass $m_\mu = 106\,\text{MeV}$, and so muon neutrinos from the sun do not have enough energy to make muons via the CC reaction. That's why SNO needed the NC reaction to see the muon neutrinos. Atmospheric muon neutrinos, however, have more than enough energy to produce muons via the CC reaction.) The experimental challenge is to differentiate between the Cherenkov light emitted when either a muon or an electron travels through water faster than the speed of light in water. Because muons are heavier, they travel farther in water. This means that muons make long tracks whereas electrons make showers of shorter tracks, and the Cherenkov light patterns produced by an electron is clearly distinguishable from that produced by a muon. This allows one to know whether the incident neutrino was either a $\nu_e/\bar{\nu}_e$ or a $\nu_\mu/\bar{\nu}_\mu$. The charge of the charged particle is not measured, and so neutrinos and anti-neutrinos are not distinguished. The orientation of the cone tells us which direction the muon or electron was moving, and this in turn tells us from which direction the neutrino came.[19]

The results of the first year of data taking were reported at a conference in Takayama, Japan in June 1998, and made it onto the front page of the New York Times, and into President Clinton's speech at that year's MIT commencement. Because the detector can distinguish the direction from which the neutrinos come, it can distinguish those neutrinos produced by cosmic rays in the upper atmosphere above the detector, say 10 km away, from those neutrinos produced by cosmic rays hitting the upper atmosphere on the other side of the earth, 13,000 km away, which then travel through the earth to Japan to be detected. Thus, we have an experiment with a baseline ranging from 10 km to 13,000 km! The experimenters find that the neutrinos coming from above, which have only 10 km over which to oscillate before being detected, are as numerous as expected. They see the expected number of muon neutrinos, the expected number of electron neutrinos, and the expected 2:1 ratio. However, for the neutrinos entering the detector from below, which have had thousands of kilometers over which to oscillate, the story is different. The expected number of electron neutrinos are seen, but only about 60% of the expected number of muon neutrinos are seen. This suggests that the muon neutrinos have oscillated, but not into electron neutrinos. The paper reporting these results is in your "extra reading" packet, so that you can read more about the error bars on these results. By studying the dependence of the reduction in the number of ν_μ's on the neutrino energy, and on the distance over which the oscillation has occurred, the authors concluded that $0.82 < \sin^2 2\theta \leq 1$ and $5 \times 10^{-4} < \Delta m^2 c^4 < 6 \times 10^{-3}\ eV^2$. Atmospheric ν_μ's are oscillating into something other than ν_e, since the number of ν_e's does *not* go up as the number of ν_μ's goes down. These experimental results were the first convincing evidence that neutrinos oscillate, and therefore have mass, and that therefore the standard model of particle physics is incomplete as it stands.

To bring things up to date, with several more years of data now analyzed the current

[18] Similarly, the ν_e's make negatively charged electrons: e^-. The antineutrinos make positively charged positrons and antimuons.

[19] I should mention that other experiments studied atmospheric neutrinos before Super-K, and found the first hints of neutrino oscillations therein. Super-K is much bigger, and so it has obtained results which are much more statistically significant.

Super-Kamiokande result is that atmospheric neutrino oscillations are described by

$$0.93 < \sin^2(2\theta) \leq 1\,, \qquad \Delta m^2 c^4 = (2.4 \pm 0.4) \times 10^{-3}\ eV^2\,. \tag{9}$$

The data still allows for a maximal mixing angle of $\theta = 45°$.

Finally, there are two experiments whose goals are to confirm the Super-Kamiokande atmospheric neutrino results using man-made neutrinos. Both these experiments have the necessary baselines of several hundred kilometers. In the first, a beam of ν_μ's with a mean energy of about 1 GeV is being created at the KEK accelerator in Japan and aimed at the Super-Kamiokande detector 230 km away. This "K2K" experiment has reported data which are fully consistent with the expected oscillation based on the atmospheric neutrino results, but the error bars on $\Delta m^2 c^4$ and $\sin 2\theta$ are larger than those in (9). In the second experiment, called MINOS, the ν_μ beam with mean energy about 3-5 GeV is created at Fermilab, outside Chicago, and aimed at a neutrino detector in a mine in northern Minnesota, 730 km from Fermilab. (The idea is that the neutrinos will be aimed slightly downward, so that they travel in a straight line path from Illinois, under Wisconsin, and arrive at the detector in Minnesota.) As you can confirm using eq. (6), both these experiments are designed to have an L/E such that they are sensitive to the same region of the Δm^2 - $\sin^2 2\theta$ plane which the Super-Kamiokande data suggests can explain the disappearance of the atmospheric muon neutrinos. Minos reported its first results in March 2006. You can find the plot showing their results for Δm^2 and $\sin^2 2\theta$ linked from their home page, http://www-numi.fnal.gov/. The result is actually a nontrivial contour in the $(\Delta m^2, \sin^2 2\theta)$ plane, but it can be caricatured as showing that

$$0.7 < \sin^2(2\theta) \leq 1\,, \qquad \Delta m^2 c^4 = (2.7 \pm 0.5) \times 10^{-3}\ eV^2\,, \tag{10}$$

in excellent agreement with the results (9) obtained from the SuperKamiokande analysis of the oscillation of atmospheric neutrinos. So, just as the KamLAND experiment has confirmed the neutrino oscillations first discerned in solar neutrinos, as of this year the atmospheric neutrino oscillations have now also been confirmed using manmade neutrinos.

2.4 Putting all the experiments together

Let us try to put the results from the three kinds of experiments we have described into a single picture. The atmospheric neutrino results show that ν_μ's oscillate into some species of neutrino which is not ν_e, since the number of atmospheric ν_e's observed does not go up when the number of ν_μ's goes down. Let us assume that Super-Kamiokande has detected $\nu_\mu - \nu_\tau$ oscillations, so that $\Delta m^2 c^4 = (240 \pm 40) \times 10^{-5} eV^2$ in the two-state system consisting of oscillating muon and tau neutrinos. The solar neutrino deficit could then be explained if the mass difference in the two state system consisting of ν_e and one of the ν_μ/ν_τ mass eigenstates is $\Delta m^2 c^4 = (8.2 \pm 0.6) \times 10^{-5} eV^2$. Explaining the solar and atmospheric neutrino oscillations in this way makes it impossible to have the larger $\Delta m^2 c^4 \sim$ (0.1 to a few) eV^2 needed to explain the LSND reactor experiment results! Thus it is somewhat gratifying that the new data from MiniBooNE is inconsistent with the old LSND result.[20]

[20]Its only somewhat gratifying because the alternative would have been quite remarkable. If the LSND result of $\nu_\mu - \nu_e$ oscillations with this larger $\Delta m^2 c^4$ had been confirmed by MiniBooNE, and if the atmospheric

Although we did not analyze the three-state problem describing oscillation among three neutrino species, this has been done and the conclusions I am sketching can be phrased precisely in terms of the three mass eigenvalues therein. These three mass eigenvalues define *two* mass-squared-differences. One of them has to be $\Delta m^2 c^4 \simeq 240 \times 10^{-5} eV^2$ and the other has to be $\Delta m^2 c^4 \simeq 8.2 \times 10^{-5} eV^2$. Its also worth mentioning that if we assume that there are three neutrinos and that the pattern of mixing described here is correct, then so far there is no conclusive experimental evidence for the θ that would indicate mixing between $\nu_e - \nu_\tau$. There is however an upper bound of $\sin^2(2\theta) < 0.19$ from the CHOZ reactor experiment.

The solar and atmospheric neutrino results by now look quite solid, having been cross-checked by many experiments and furthermore having been confirmed using neutrinos made on earth. The situation with the LSND experiment has also apparently been resolved by MiniBooNE (future data from MiniBooNE on antineutrino oscillations has the potential to further collaborate their initial $\nu_\mu \to \nu_e$ results). Continuing experiments will further reduce these error bars, and provide more information on the unmeasured mixing angle.

I hope you have enjoyed this somewhat extended description of how the quantum mechanics of a two (or three) state system is being used to learn whether neutrinos have a mass, and thus whether the (old) standard model of particle physics is incomplete. By now, as of 2007, neutrino oscillation and hence neutrino mass have been convincingly demonstrated by sufficiently many techniques that we now have a new standard model, with masses and mixing angles in the neutrino sector added as fundamental parameters, which the experiments I have described are measuring. There are many fundamental questions posed by this new addition to the standard model. Here is a sampling, but as these do not involve neutrino oscillations we shall not discuss them in 8.05:

- Neutrino oscillation experiments measure neutrino mass differences. But, what is the absolute mass scale? For example, is the lightest neutrino massless or not? This question will be probed by Prof. Formaggio's KATRIN experiment, which will make a direct measurement of the mass of the electron neutrinos emitted in the decay of tritium nuclei.

- Now that we know that (some) neutrinos have a mass, the cosmic neutrino background contributes a certain fraction to the mass density of the universe. What fraction? Is it a large enough fraction that the neutrinos could be the long sought dark matter? Here, cosmological observations (for example made by the WMAP satellite) having nothing to do with neutrino oscillations are providing us with an answer to the last question: no; the neutrinos cannot be so heavy as to be the major component of the dark matter. This yields an indirect constraint on the sum of neutrino masses can be derived from cosmology, $\sum_i m_i \leq 0.7\, eV$, which uses the cosmic microwave background anisotropy

neutrino oscillation results are assumed to be correct, then explaining the solar neutrino deficit by neutrino oscillations would have required the introduction of a hitherto never seen fourth neutrino! The three presently known neutrinos are all produced in decays of the Z-boson. The decays of the Z have been analyzed with sufficient precision to determine that *only* three different neutrinos are produced. Thus, a fourth neutrino would have to have different interactions from the other three, as it must not interact with the Z. Such a neutrino is called "sterile". Whereas the standard model can easily be extended to give mass to the three presently known neutrinos, there is no "empty slot" in the model for a fourth neutrino with the necessary properties.

data, galaxy redshift surveys, other data, and also has some cosmological assumptions. Unfortunately, describing the physics behind this limit would take us too far afield.

- Is there a natural theoretical explanation of how it is that neutrinos turn out to be so many orders of magnitude lighter than all other known elementary particles? A class of explanations (called "see-saw models", but I won't explain the name) have been proposed which are interesting on two fronts. First, they tie the lightness of neutrino masses to the heaviness of masses of certain new particles predicted by grand unified theories. Second, they predict that neutrinos are their own antiparticles. The antiparticle of an electron is a positron, with positive charge. Since neutrinos are neutral, once we know that in the new standard model lepton number is not conserved — after all neutrinos oscillate from one lepton type to another — it is natural to ask whether neutrinos can be their own antiparticle. In see-saw models, the answer is yes. There are a number of experiments on the drawing board that will test this hypothesis by looking for nuclear decays in which two protons within a nucleus turn into neutrons, emitting two positrons. In the standard model, these two positrons must be accompanied by two neutrinos. But, if and only if the neutrino is its own antiparticle, it is possible for "neutrinoless double beta decay reactions" to occur. This is what this new class of experiments will be looking for.

If you would like to learn more about neutrino oscillations and neutrino physics, one option is to seek out Prof. Formaggio. On the 8.05 webpage I am also providing you with links to *optional* extra reading from the following 4 items:

- Notes on the space-time evolution of a neutrino beam, prepared by D. Stelitano who was the TA for 8.05 in 1996.

- An article by Boris Kayser entitled "On the quantum mechanics of Neutrino Oscillations."

- The paper published in Physical Review Letters which describes the 1998 discovery of evidence for the oscillation of atmospheric neutrinos obtained using the SuperKamiokande detector.

- The paper published in Physical Review Letters which describes the 2002 discovery of direct evidence for neutrino oscillations via the SNO measurement of the NC flux.

- The paper published in Physical Review Letters which describes the 2007 MiniBooNE results.

The last four items are given as links to journal articles to which MIT has an electronic subscription. If you want to go further still, you can consult http://www.hep.anl.gov/ndk/hypertext/ for links to the web pages of all the ongoing, completed, and proposed experiments studying neutrino oscillations, including many that I did not have time to mention in these notes. You might also try reading Hitoshi Murayama's 2002 Physics World article, which you can find near the top of http://hitoshi.berkeley.edu/neutrino/.

3 The Physics of Neutral Kaons

Kaons are short lived elementary particles discovered in the 1950's. They provide a wonderful laboratory for exploring the exotic world of quantum mechanics. No discussion of two state systems would be complete without an exploration of some of the phenomena that arise in the world of neutral kaons. In this section I first (very) briefly introduce the kaons and summarize some of their properties, trying not to confuse you with too much extra information. Then I describe three unusual phenomena that follow from the elementary properties of kaons and the fundamental structure of quantum mechanics. To quote Richard Feynman,

> If there is any place where we have the chance to test the main principles of quantum mechanics in the purest way — does the superposition of amplitudes work or doesn't it? — this is it.

3.1 Properties of neutral kaons

The strong interactions bind quarks together to form a panoply of particles known as *hadrons*. Hadrons are small — the characteristic size is the *femtometer*$= 10^{-15}$m known also as the Fermi (abbreviated fm). It takes light about 3×10^{-23} seconds to cross a hadron. This is the natural time scale for strong interaction physics. Excited states of the proton, for example, live only about 10^{-23} sec. before decaying. Eigenstates of the stong interaction Hamiltonian may still decay due to electromagnetic or weak interactions. Because these interactions are weaker, the instability takes a while to develop, leading to a much longer lifetime. Hadrons that are unstable against the weak interactions characteristically have lifetimes of order a nanosecond. Although this seems inconceivably short to us, it represents of order 10^{14} light-crossings of the hadron — the instability requires about 10^{14} chances to finally make it. For this reason, particle physicists refer to hadrons that can only decay by weak or electromagnetic interactions as "*stable*" particles!

Among the more common stable hadrons are four quark-antiquark bound states where one of the quarks or antiquarks is the "strange" quark. Their names and quark contents are as follows,

$$\begin{aligned} K^+ &\Leftrightarrow u\bar{s} \\ K^- &\Leftrightarrow s\bar{u} \\ K^0 &\Leftrightarrow d\bar{s} \\ \overline{K}^0 &\Leftrightarrow s\bar{d} \end{aligned} \tag{11}$$

The superscript denotes the electric charge. K^0 and $\overline{K}^0$ are electrically neutral. They are eigenstates of the strong interaction Hamiltonian, $\hat{H}$. (The total Hamiltonian is the sum of $\hat{H}$ and a smaller term due to the weak interactions which we will call $\hat{H}_W$ and will discuss below.) For a particle at rest, the eigenvalue of $\hat{H}$ is simply its rest energy (mc^2). The masses of the K^0 and the $\overline{K}^0$ are equal (on account of a deep symmetry of Nature),

$$\begin{aligned} \hat{H}|K^0\rangle &= Mc^2|K^0\rangle \\ \hat{H}|\overline{K}^0\rangle &= Mc^2|\overline{K}^0\rangle. \end{aligned} \tag{12}$$

Mc^2 is about 497.7 MeV (where the proton mass ($m_p c^2$) is about 938 MeV). The neutral kaons are unstable on account of the weak interactions. They decay away, usually to π-mesons, with lifetimes of the order of nanoseconds.

When hadrons interact, neutral kaons are often produced. K^0 is produced more copiously than $\overline{K}^0$ when protons or neutrons are scattered off matter. Kaons live long enough that we can select them out of the refuse of collisions, make "secondary" beams of them and study their scattering in matter. It turns out that the $\overline{K}^0$'s are more reactive than the K^0's. In particular, matter is quite a bit more effective at absorbing $\overline{K}^0$'s than K^0's.

Things get interesting when we consider the principal decays of the neutral kaons. The $\overline{K}^0$ and K^0 are not eigenstates of the weak interaction Hamiltonian, $\hat{H}_W$, which has matrix elements connecting the kaons to states of two and three π mesons. In other words, if there were no weak interactions and $\hat{H}_W$ were zero, then the $\overline{K}^0$ and K^0 would not decay. The decay of the kaons is determined by the matrix element of $\hat{H}_W$ between the initial state, K^0 or $\overline{K}^0$ and the final state 2π or 3π,[21] $\langle 2\pi|\hat{H}_W|K^0\rangle$, for example. Each of these matrix elements is a complex number (depending on the particular types of pions, their momenta, *etc.*. One of the great triumphs of particle physics in the 1950's was to understand that the four matrix elements for the weak decay of neutral kaons form the following pattern,

$$\begin{aligned} \langle 2\pi|\hat{H}_W|K^0\rangle &= A \\ \langle 2\pi|\hat{H}_W|\overline{K}^0\rangle &= A \\ \langle 3\pi|\hat{H}_W|K^0\rangle &= B \\ \langle 3\pi|\hat{H}_W|\overline{K}^0\rangle &= -B \end{aligned} \tag{13}$$

on account of a deep symmetry of Nature.[22]

Eqs. (13) may not seem very exotic. They seem to say that the K^0 and $\overline{K}^0$ have equal probabilities to decay to (say) two pions. Not quite: the matrix elements in (13) are *probability amplitudes* and their absolute squares give probabilities. This requires us to consider the implications of superposition. Consider the *symmetric and antisymmetric* linear combinations of K^0 and $\overline{K}^0$,

$$|K_S\rangle \equiv \frac{1}{\sqrt{2}}\{|K^0\rangle + |\overline{K}^0\rangle\}$$

[21]This is a result of the study of *perturbation theory* — a subject for the next term. However, it is reasonable that $\langle f|\hat{O}|i\rangle$ should be a measure of the amplitude for the initial state i to transition to the final state f by the action of the operator $\hat{O}$.

[22]Symmetries and their violation have been the premier subject of late 20^{th} century physics. The symmetry responsible for (13) is time reversal invariance. Even more careful study of the neutral kaons showed that the consequences of (13) are not exactly obeyed in Nature, so time-reversal invariance is not a perfect symmetry.

$$|K_L\rangle \equiv \frac{1}{\sqrt{2}}\{|K^0\rangle - |\overline{K}^0\rangle\}. \tag{14}$$

Looking back to (13) we see that K_L *has no amplitude to decay to* 2π and K_S *has no amplitude to decay to* 3π. To put it somewhat differently, K_S and K_L are eigenstates of the weak interaction Hamiltonian with the decays $K_S \to \pi\pi$ and $K_L \to \pi\pi\pi$. Since the decay amplitudes A and B are quite different, the lifetimes of the K_L and K_S are quite different — $\tau_S = 0.8926 \pm 0.0012 \times 10^{-10}$ sec and $\tau_L = 5.17 \pm 0.04 \times 10^{-8}$ sec.[23] Hence the names: K_S for K-short and K_L for K-long.

Here then are the few facts about neutral kaons that are needed to continue our discussion:

- The K^0 and $\overline{K}^0$ are eigenstates of the strong interactions.
- K^0 is more copiously produced when protons hit matter; $\overline{K}^0$ is more strongly absorbed when propagating through matter.
- K_S and K_L are eigenstates of the weak interactions, with very different lifetimes.

In light of this situation, it is clearly better to think of the neutral kaons as a *two-state system* conveniently viewed in one basis (K^0 and $\overline{K}^0$) for production and interaction and in another basis (K_L and K_S) for decay.

3.2 Evolution of a Beam of Neutral Kaons

To warm up, let's consider a beam of neutral kaons propagating through empty space. The only thing they can do is decay, so it is best to use the $K_S - K_L$ basis. If we denote $K_S \Rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $K_L \Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, then the time evolution operator $\hat{U}(t)$ is given by

$$\hat{U}(t) = \begin{pmatrix} e^{-iEt/\hbar - t/2\tau_S} & 0 \\ 0 & e^{-iEt/\hbar - t/2\tau_L} \end{pmatrix} \tag{15}$$

This requires some explanation. $\hat{U}$ is not unitary. The K_S amplitude decays away exponentially with a lifetime τ_S, and similarly for the K_L. This is not a violation of some sacred principle of quantum mechanics. The probability to observe a K_S or K_L really does decay to zero with time because the particles decay to 2π and 3π respectively.[24] Eq. (15) has been written assuming that the K_S and K_L masses are the same (hence the same energy eigenvalue, E, appears in both entries of $\hat{U}$). This is not quite right. Although the mass of the K^0 and $\overline{K}^0$ are the same, the weak interactions that allow the kaons to decay also generate a small mass difference between K_S and K_L. We ignore the $K_L - K_S$ mass difference here, but you will study it on your problem set.

[23] This gives us a glimpse of the puzzle that confronted physicists in the 1950's when neutral kaons were first encountered. Experimenters looking at cloud chambers saw two different particles (they called them the τ and the θ) one decaying to two pions, the other to three. Their lifetimes differed by a factor of about 500, but their masses seemed to be the same.

[24] If one insisted to write down a Hamiltonian corresponding to $\hat{U}$, it would not be Hermitian. The imaginary part of $\hat{H}$ — $-i/2\tau$ is there on account of particle decay. The compensating appearance of pions is left out of our discussion and our Hamiltonian because we don't care what happens to them.

Since K^0 are most copiously produced, let's suppose that a pure K^0 beam is produced at $t = 0$

$$|\psi(0)\rangle = |K^0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{16}$$

As it propagates through space it evolves in time according to $\hat{U}(t)$,

$$\begin{aligned} |\psi(t)\rangle &= \hat{U}(t)|\psi(0)\rangle \\ &= \frac{e^{-iEt/\hbar}}{\sqrt{2}}\begin{pmatrix} e^{-t/2\tau_S} \\ e^{-t/2\tau_L} \end{pmatrix} \end{aligned} \tag{17}$$

The square of the wavefunction tells us how the beam is decaying,

$$\langle\psi(t)|\psi(t)\rangle = \frac{1}{2}\{e^{-t/\tau_S} + e^{-t/\tau_L}\} \tag{18}$$

To interpret this result remember that the K_S decays to 2π and the K_L decays to 3π. Eq. (18) tells us that the K^0 beam rapidly decays away (over a time scale τ_S) by two pion emission to half its original intensity. Travelling at speeds close to the speed of light, the beam traverses about 1 cm in a time τ_S. Allowing for relativistic time dilation, the burst of two pion emission occurs over distances of a few centimeters. Later, over a time scale τ_L that is about 500 times longer the rest of the beam decays away by three pion emission. In short the beam behaves like a 50/50 mixture of two different particles with two different lifetimes and two different decays. No wonder early particle physicists were confused! At long times, when the K_S component is gone, the wavefunction is pure $K_L = \frac{1}{\sqrt{2}}\{|K^0\rangle - |\overline{K}^0\rangle\}$.

3.3 Regeneration

The different interactions of K^0 and $\overline{K}^0$ in matter offer a new twist on the propagation we have just discussed. Suppose a neutral kaon beam is allowed to propagate many K_S decay lifetimes. The beam will then be essentially pure K_L and the number of K_L's will be about $\frac{1}{2}$ the original number of K^0's. Now suppose this K_L beam is directed at a sheet of material thick compared to the $\overline{K}^0$ attenuation length, but thin compared to the K^0 attenuation length. The sheet will have the effect of quenching the $\overline{K}^0$ component of the wavefunction. Thus the beam will emerge as a fairly pure K^0 beam, reduced again by a factor of two in intensity. This beam is again a (50/50) coherent superposition of K_L and K_S. Once again the K_S decay away leaving a pure K_L beam. K_S's seem to have been "regenerated" in the beam. A beam which was originally completely depleted in K_S's, is reduced in total intensity by 50%, but becomes a 25%:25% mixture of K_L and K_S.

3.4 $\phi \to K\overline{K}$

The neutral kaon system is a convenient place to confront one of the more alarming issues in the interpretation of quantum mechanics. First I will present the example, then make a few comments on its implications for how we think about Nature and measurement in a quantum world.

The ϕ meson is an *unstable* hadron. It is rather long-lived — $\tau_\phi \approx 2 \times 10^{-22}$ sec — compared to the natural time scale of the strong interactions. When it decays, about 35% of the time it decays to neutral kaons, $\phi \to K^0\overline{K}^0$. Notice that the decay is specified in terms of K^0 and $\overline{K}^0$ because they are the strong interaction eigenstates. When a ϕ at rest decays, the kaons come out back to back. At the moment of decay the wavefunction of the $K\overline{K}$ system is

$$|K\overline{K}\rangle = \frac{1}{\sqrt{2}}\Big\{|K^0(\vec{p})\ \overline{K}^0(-\vec{p})\rangle - |\overline{K}^0(\vec{p})\ K^0(-\vec{p})\rangle\Big\} \tag{19}$$

The superposition and the relative minus sign between the two terms are dictated by the particle $\leftrightarrow$ antiparticle symmetry of the strong interactions. The neutral kaons will be observed by their decays. To understand what will be seen, we change to the K_S/K_L basis, where the state (19) is described as

$$|K\overline{K}\rangle = \frac{1}{\sqrt{2}}\{|K_L(\vec{p})\ K_S(-\vec{p})\rangle - |K_S(\vec{p})\ K_L(-\vec{p})\rangle\}\ . \tag{20}$$

When we watch a given ϕ decay from a particular location, we have equal probability to observe a K_S or K_L according to (20). Things get more interesting when we consider *correlations*. Suppose we (observer I) sit on one side of an apparatus that creates ϕ's at rest,[25] and suppose another observer (observer II) sits on the other side. In each decay each of us has a 50% likelihood of seeing a K_S and a 50% likelihood of seeing a K_L. However, for any specific decay, if I see a K_S then II must see a K_L and *vice versa*.[26] According to (20) the wavefunction of the *two kaon system* is a superposition of two terms — one describing a K_S heading toward observer II, the other describing at K_L heading toward observer II. When observer I records a K_S, the term in the wavefunction so selected requires a K_L heading toward observer II. Apparently the act of observation by I has affected the observation by II even though the observers may be separated by macroscopic distances.

Although this result may seem surprising, by itself it could be explained by purely a classical model. To wit: suppose ϕ mesons are little spring-loaded devices consisting of a K_L and a K_S held together by a spring that decays. When the device snaps apart the K_L shoots one way and the K_S shoots the other way. Either observer has a 50% chance to see a K_S and a 50% chance to see a K_L. However, if I sees a K_S, then II must see a K_L. So the quantum result is not so unusual after all.

But this is not the end. Suppose we put a filter very close to the location where the ϕ is made. The filter sits only on the side of the apparatus where observer I lives. It is made of matter and absorbs $\overline{K}^0$'s. Remember, $\overline{K}^0$'s interact strongly with matter while K^0's just pass through. What do observers I and II see now? After filtration the $K\overline{K}$ wavefunction is

$$|K\overline{K}\rangle \quad = \quad |K^0(\vec{p})\ \overline{K}^0(-\vec{p})\rangle \tag{21}$$

[25] Such apparati actually exist. ϕ's can be made by electron positron annihilation — $e^-e^+ \to \phi$. There are several colliders where electrons and positrons of equal energies collide to produce hadrons at rest in the laboratory. A facility primarily dedicated to ϕ physics, known as DAPHNE is located in Italy.

[26] Note that "seeing a K_S" means seeing 2 pions, while "seeing a K_L" means seeing 3 pions.

Each of the two observers still has a 50/50 chance to observe either K_S or K_L, because both K^0 and $\overline{K}^0$ are 50/50 linear superpositions of K_S and K_L. However, now the two observations are *entirely uncorrelated*: if I observes a K_L, II still has a 50/50 chance to observe a K_S or K_L. The classical spring-loaded K_S/K_L machine we described to fit the original experimental results cannot handle this situation.

In fact there are several more chapters to this thread of discussion. It turns out that it *is* possible to construct a classical model that fits *all* the measurements we have just described. In it, each produced kaon carries labels that tell it not only whether it is a K_L or K_S, but also whether (if asked) it will behave like a K^0 or a $\overline{K}^0$. Although cumbersome, such a classical system can mimic quantum mechanics. Classical physics finally and irrevocably fails when three different quantum observables are considered. There are three independent observables in a two-state systems, corresponding to the Pauli matrices σ_1, σ_2, and σ_3. If we choose the K_L/K_S basis as the one where σ_3 is diagonal, and associate the $K^0/\overline{K}^0$ basis with σ_1, then in principal there is an additional observable associated with the σ_2 direction. A sequence of measurements of the *three* observables yields predictions that cannot be mimicked by any classical model. This was proved by the great Irish theoretical physicist John Bell in the 1960's, and the result, known as Bell's Theorem, has been verified experimentally. The whole apparatus of quantum measurement theory: states, superposition, eigenvalues, and so forth, does seem to be the way Nature works rather than a relic of our ignorance of some underlying classical dynamics.