DETACH THESE SHEETS & USE AT YOUR CONVENIENCE

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\varepsilon_o}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B}=\mu_o\mathbf{J}+\mu_o\varepsilon_o\frac{\partial\mathbf{E}}{\partial t}$$

$$\nabla\cdot\mathbf{B}=0$$

$$\phi(\mathbf{r},t)=\frac{1}{\left[1-\hat{\mathbf{n}}(t'_{ret})\cdot\boldsymbol{\beta}(t'_{ret})\right]}\frac{q}{4\pi\varepsilon_o}\frac{1}{\left|\mathbf{r}-\mathbf{X}(t'_{ret})\right|}$$

$$\mathbf{A}(\mathbf{r},t)=\frac{\mathbf{u}(t'_{ret})}{c^2}\phi(\mathbf{r},t)$$

$$t'_{ret}=t-\left|\mathbf{r}-\mathbf{X}(t'_{ret})\right|/c$$

$$\mathbf{B}=\nabla\times\mathbf{A}$$

$$\mathbf{E}=-\nabla\phi-\frac{\partial\mathbf{A}}{\partial t}$$

$$\mathbf{E}(\mathbf{r},t)=\left[\frac{q}{4\pi\varepsilon_o}\frac{\hat{\mathbf{n}}-\boldsymbol{\beta}}{\gamma_u^2(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^3R^2}\right]_{ret}+\left[\frac{q}{4\pi\varepsilon_o}\frac{1}{c}\frac{\hat{\mathbf{n}}\ \mathbf{x}\left\{(\hat{\mathbf{n}}-\boldsymbol{\beta})\ \mathbf{x}\ \dot{\boldsymbol{\beta}}\right\}}{(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^3R}\right]_{ret}$$

$$\mathbf{B}(\mathbf{r},t)=\frac{1}{c}\left[\hat{\mathbf{n}}\ \mathbf{x}\ \mathbf{E}\right]_{ret}$$

$$\frac{dW_{rad}}{d\Omega\,dt'}=\frac{q^2}{(4\pi)^2c\varepsilon_o}\frac{\left|\hat{\mathbf{n}}\times\left[(\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]\right|^2}{(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^5}$$

$$\frac{dW_{rad}}{dt'}=\frac{1}{4\pi\varepsilon_o}\frac{2q^2}{3c}\gamma^6\left[\left|\dot{\boldsymbol{\beta}}\right|^2-\left|\boldsymbol{\beta}\times\dot{\boldsymbol{\beta}}\right|^2\right]$$

$$\begin{pmatrix}\bar{x}^0\\ \bar{x}^1\\ \bar{x}^2\\ \bar{x}^3\end{pmatrix}=\begin{pmatrix}\gamma & -\gamma\beta & 0 & 0\\ -\gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x^0\\ x^1\\ x^2\\ x^3\end{pmatrix}$$

$$\begin{pmatrix}x^0\\ x^1\\ x^2\\ x^3\end{pmatrix}=\begin{pmatrix}\gamma & \gamma\beta & 0 & 0\\ \gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\bar{x}^0\\ \bar{x}^1\\ \bar{x}^2\\ \bar{x}^3\end{pmatrix}$$

$$J^\mu=(c\rho,J_x,J_y,J_z)$$

$$A^\mu=(\frac{\phi}{c},A_x,A_y,A_z)$$

$$\bar{E}_x=E_x$$

$$\bar{E}_y=\gamma\left(E_y-\mathrm{v}\,B_z\right)$$

$$\bar{E}_z=\gamma\left(E_z+\mathrm{v}B_y\right)$$

$$\bar{B}_x=B_x$$

$$\bar{B}_y=\gamma\left(B_y+\frac{\mathrm{v}}{c^2}E_z\right)$$

$$\bar{B}_z=\gamma(B_z-\frac{\mathrm{v}}{c^2}E_y)$$

Contra-variant 4 vector transforms as

$$\begin{pmatrix}\bar{S}^0\\ \bar{S}^1\\ \bar{S}^2\\ \bar{S}^3\end{pmatrix}=\begin{pmatrix}\gamma & -\gamma\beta & 0 & 0\\ -\gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}S^0\\ S^1\\ S^2\\ S^3\end{pmatrix}$$

Co-variant 4 vector transforms as

$$\begin{pmatrix}\bar{S}_0\\ \bar{S}_1\\ \bar{S}_2\\ \bar{S}_3\end{pmatrix}=\begin{pmatrix}\gamma & \gamma\beta & 0 & 0\\ \gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}S_0\\ S_1\\ S_2\\ S_3\end{pmatrix}$$

$$\begin{pmatrix}S^0\\ S^1\\ S^2\\ S^3\end{pmatrix}=\begin{pmatrix}-S_0\\ S_1\\ S_2\\ S_3\end{pmatrix}$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = (\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$$

$$\partial^\mu = \frac{\partial}{\partial x_\mu} = (-\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}$$

$$G^{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z/c & E_y/c \\ -B_y & E_z/c & 0 & -E_x/c \\ -B_z & -E_y/c & E_x/c & 0 \end{pmatrix}$$

$$\partial_\mu F^{\mu\nu} = -\mu_o J^\nu \qquad \partial_\mu G^{\mu\nu} = 0$$

$$X^\mu(t) = \begin{pmatrix} c\,t \\ \mathbf{X}(t) \end{pmatrix}$$

$$\mathbf{u}(t) = \frac{d\,\mathbf{X}(t)}{dt}$$

$$d\tau = d\,t\sqrt{\left(1 - \frac{\mathbf{u}^2(t)}{c^2}\right)}$$

$$\eta^\mu = \frac{d}{d\tau} X^\mu(\tau) = \begin{pmatrix} \gamma_u c \\ \gamma_u \mathbf{u}(t) \end{pmatrix}$$

$$m\frac{d}{d\tau}\eta^\mu = qF^{\mu\sigma}\eta_\sigma$$

$$m\frac{d}{d\tau}\begin{pmatrix} \gamma_u c \\ \gamma_u \mathbf{u}(t) \end{pmatrix} = m\gamma_u \frac{d}{dt}\begin{pmatrix} \gamma_u c \\ \gamma_u \mathbf{u}(t) \end{pmatrix}$$

$$= q\begin{pmatrix} \gamma_u \dfrac{\mathbf{E}\cdot\mathbf{u}}{c} \\ \gamma_u\left(\mathbf{E} + \mathbf{u}\,\mathbf{x}\,\mathbf{B}\right) \end{pmatrix}$$

$$Energy\ of\ \ particle = m\gamma_u c^2 = mc^2 / \sqrt{1 - \frac{u^2}{c^2}}$$

$$\mathbf{F}_{\text{radiation reaction}} = \frac{1}{4\pi\varepsilon_o}\frac{2}{3}\frac{q^2}{c^3}\frac{d^2\mathbf{V}}{dt^2}$$

Electrostatics:

$$\mathbf{E} = -\nabla\phi \qquad \phi(\mathbf{r}) = -\int_{\mathbf{r}_o}^{\mathbf{r}} \mathbf{E}(\mathbf{r}')\cdot d\,\mathbf{l}'$$

$$\phi(\mathbf{r}) = \int_{all\,space} \frac{\rho(\mathbf{r}')\,d^3x'}{4\pi\varepsilon_o\left|\mathbf{r} - \mathbf{r}'\right|}$$

$$W_N = \sum_{i=1}^{N}\sum_{j<i}\frac{q_i q_j}{4\pi\varepsilon_o\left|\mathbf{r}_i - \mathbf{r}_j\right|}$$

$$W = \int \frac{1}{2}\rho(\mathbf{r})\phi(\mathbf{r})\,d^3x = \int\frac{1}{2}\varepsilon_o E^2(\mathbf{r})d^3x$$

$$\nabla^2\phi(\mathbf{r}) = -\rho(\mathbf{r})/\varepsilon_o$$

$$\int_{\substack{\text{closed}\\\text{surface}}} \mathbf{E}\cdot\hat{\mathbf{n}}\,da = \frac{q_{\text{enclosed}}}{\varepsilon_o}$$

In general in spherical coordinates:

$$\phi(r,\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left[A_{lm}r^l + B_{lm}r^{-l-1}\right]Y_{lm}(\theta,\phi)$$

$$Y_{lm}(\theta,\phi) = \sqrt{\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{+im\phi}$$

For ϕ symmetry in spherical coordinates:

$$\phi(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

$$\int_{-1}^{1} P_l(x)P_m(x)dx = \frac{2}{2l+1}\delta_{lm}$$

Boundary conditions on E:

$$E_{2n} - E_{1n} = \hat{\mathbf{n}}\cdot\left(\mathbf{E}_2 - \mathbf{E}_1\right) = \sigma/\varepsilon_o$$

$$\mathbf{E}_{2t} = \mathbf{E}_{1t}$$

Problem 1: Relativistic particle motion

(a) A spaceship with speed $V < c$ moves directly toward an observer in the laboratory frame, firing laser blasts with a time separation $\Delta t'$ at the observer, where $\Delta t'$ is the time separation of the blasts as measured by our infinite grid of observers at rest in the laboratory frame. What is the time separation Δt between these laser blasts when they arrive at the observer, in terms of *V*, *c* and $\Delta t'$? Justify your answer.

(b) The four acceleration vector is defined by $\Xi^\mu = \frac{d}{d\tau}\eta^\mu$ and is given in terms of ordinary acceleration and velocity by $\Xi^\mu = \gamma_u{}^2 \begin{pmatrix} \gamma_u{}^2 \left(\frac{\mathbf{u}\cdot\mathbf{a}}{c}\right) \\ \mathbf{a}(t) + \gamma_u{}^2 \mathbf{u}\left(\frac{\mathbf{u}\cdot\mathbf{a}}{c^2}\right) \end{pmatrix}$. From this expression for Ξ^μ, how would you compute the acceleration of the particle in its instantaneous rest frame given quantities measured in the lab frame? This is not a question about Lorentz transformations, it is a question about Lorentz invariants. You do not have to carry out the computation, just indicate how you would do it.

Problem 2: Dynamics of Magnetic Monopoles

Let the mass of a magnetic monopole be m_m, its 4 vector position be $X^\mu(t) = \begin{pmatrix} ct \\ \mathbf{X}(t) \end{pmatrix}$, its ordinary velocity be $\mathbf{u}(t) = \frac{d\,\mathbf{X}(t)}{dt}$, and its 4 vector velocity be $\eta^\mu = \frac{d}{d\tau}X^\mu(\tau)$. We have for an electric monopole that $m\frac{d}{d\tau}\eta^\mu = qF^{\mu\sigma}\eta_\sigma$.

(a) What manifestly covariant equation do you think would be appropriate for a magnetic monopole, assuming that the non-relativistic form is $m_m\mathbf{a} = q_m\mathbf{B}$.

(b) Write your manifestly covariant equation from (a) in ordinary coordinate form, that is in terms of the ordinary position and velocity of the monopole, with any time derivatives in terms of coordinate time *t*. Give both the time and space components of your 4 vector equation in terms of these quantities. Your final equations should be relativistically correct, even though written in in terms of the ordinary position and velocity of the monopole and differentials with respect to coordinate time *t*.

Problem 3: Electrostatics

Suppose we have a sphere or radius R which has a surface charge density on its surface that is equal to $\sigma_o \cos\theta$, where θ is the spherical polar angle θ. We are going to guess that the electric field due to this surface charge density is of the form

$$\mathbf{E} = \begin{cases} \dfrac{2p\cos\theta}{4\pi\varepsilon_o r^3}\hat{\mathbf{r}} \\ +\dfrac{p\sin\theta}{4\pi\varepsilon_o r^3}\hat{\boldsymbol{\theta}} \quad r > R \\ \quad -E_o\hat{\mathbf{z}} \quad r < R \end{cases} \tag{0.1}$$

Using the boundary condition on **E**, find the value of E_o and p in terms of R and σ_o and fundamental constants. You must justify your answer to get credit, not simply state the results.