A Uniform Sphere of Charge

PROBLEM 1: The Electric Field, Potential, and Energy of a Uniform Sphere of Charge

(a) Using Gauss’s Law, calculate the electric field \( \vec{E} \) outside and total charge \( Q \) for a uniform sphere of charge with radius \( R \). The charge enclosed is \( \rho \) everywhere.

(b) Derive an expression for the capacitance of this capacitor in terms of the quantities given. What is the capacitance per unit length?

(c) Let the gap between the outer shell and the inner cylinder be small compared to the radii.

(d) When the gap is small compared to the radii, the sphere is like the area of the equivalent parallel plate capacitor. The energy stored in the capacitor by integrating the energy density \( \epsilon_0 E^2 \) over the volume where \( E \) is nonzero and compared with the result you get using the capacitor's energy density, \( \epsilon_0 \kappa V^2 \), must have the form

\[
\frac{\epsilon_0}{2} \int |\vec{E}|^2 dV = \frac{\epsilon_0}{2} \int |\vec{E}|^2 dV.
\]

(e) must have the form

\[
\frac{\epsilon_0}{2} \int |\vec{E}|^2 dV = \frac{\epsilon_0}{2} \int |\vec{E}|^2 dV.
\]

(f) Since both the inner and outer cylinders have nontrivial electric fields, and total potential difference across the gap is \( \Delta V \), we imagine a parallel plate capacitor (see Griffiths Eq. (2.54) on p. 106).

Problem 2: The Electric Field, Potential, and Energy of a Cylindrical Capacitor

(a) Compute the energy stored in the capacitor by integrating its energy density over the volume where \( E \) is nonzero and compared with the result you get using the capacitor’s energy density, \( \epsilon_0 \kappa V^2 \), must have the form

\[
\frac{\epsilon_0}{2} \int |\vec{E}|^2 dV = \frac{\epsilon_0}{2} \int |\vec{E}|^2 dV.
\]

(b) When the gap is small compared to the radii, the sphere is like the area of the equivalent parallel plate capacitor. The charge enclosed is \( \rho \) everywhere.

(c) Show that in this case that your answer for part (c) reduces to that for a parallel plate capacitor (see Griffiths Eq. (2.54) on p. 106).
\[
\frac{\partial}{\partial \rho} = (\rho H) \frac{\partial}{\partial \rho} \frac{\partial}{\partial \rho} = \rho \frac{\partial}{\partial \rho} \frac{\partial}{\partial \rho} = \rho \frac{\partial}{\partial \rho} \int \frac{\partial}{\partial \rho} \frac{\partial}{\partial \rho} = \text{constant}
\]

Integrating uniformly:

\[
\text{all surface elements are at the same distance from the center point, so the}
\]

\[
\text{potential at the center is straightforward,}
\]

\[
(3.1)
\]

\[
\text{whereas for a charge distribution we have}
\]

\[
\frac{\partial}{\partial \rho} \frac{\partial}{\partial \rho} = \Lambda
\]

\[
(3.2)
\]

\[
\text{Problem 3: Potential of a Hemispherical Bowl (10 points)}
\]

\[
\text{For a Gaussian sphere of radius } r, \text{ the charge enclosed is } Q, \text{ so the}
\]

\[
\text{electric potential is calculated per uniform.}
\]

\[
(3.3)
\]

\[
\text{Using the expression above,}
\]

\[
\int \frac{\partial}{\partial \rho} \frac{\partial}{\partial \rho} = \Lambda
\]

\[
(3.4)
\]
the potential on the next page. Thus we have

\[
\frac{V^0_{\mathcal{D}}}{\mathcal{D}} = \mathcal{D}
\]

If you have trouble following the calculation in the middle of this equation, see

\[
\frac{\partial V^0_{\mathcal{D}}}{\partial \mathcal{D}} = \frac{p \cdot \partial V^0_{\mathcal{D}}}{\partial \mathcal{D}} = \frac{p \cdot \partial V^0_{\mathcal{D}}}{\partial \mathcal{D}} + \frac{p \cdot \partial V^0_{\mathcal{D}}}{\partial \mathcal{D}}
\]

The change in the capacitance \(C\) is the change in the charge \(Q\). The work that we did equal the change in the capacitance, so that the change in the capacitance is equal to the change in the charge:

\[
\frac{\partial V^0_{\mathcal{D}}}{\partial \mathcal{D}} + \frac{p \cdot \partial V^0_{\mathcal{D}}}{\partial \mathcal{D}} = \mathcal{D}
\]

The charge on the plates are fixed. In this case, the capacitance is split

\[
\frac{\partial V^0_{\mathcal{D}}}{\partial \mathcal{D}} = \mathcal{D}
\]

Thus, we obtain:

\[
\frac{\partial V^0_{\mathcal{D}}}{\partial \mathcal{D}} = \frac{p \cdot \partial V^0_{\mathcal{D}}}{\partial \mathcal{D}} = \mathcal{D}
\]

Therefore, the force on the plates of the capacitor is

\[
\frac{\partial V^0_{\mathcal{D}}}{\partial \mathcal{D}} = \mathcal{D}
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Thus, the potential on the plates of the capacitor is

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Therefore, the force on the plates of the capacitor is

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Thus, the potential on the plates of the capacitor is

\[
\frac{\partial V^0_{\mathcal{D}}}{\mathcal{D}} = \mathcal{D}
\]
They are both independent of \(x\); the requested relation between \(x\) is 1)

\[
\frac{\mathbf{V}}{I} = (x) a(x) \iff |ad| = \frac{\mathbf{V}}{I}
\]

The above equation with \(I\) is the requested relation between \(x\) and \(a\).

\[
\frac{\mathbf{V}}{I} = (x) A \mathbf{a} = (x) \frac{e\mathbf{v} A}{2}
\]

The Taylor series for \(1 + \frac{x}{a}\) is given by

\[
\left(\frac{p}{p^2 - 1}\right) \frac{p}{I} \approx \left(\frac{q + 1}{p}\right) \frac{p}{I} = \frac{p}{I}
\]

Therefore

\[
\sum \frac{e - e x + x - 1}{I} = \frac{x + 1}{I}
\]
The problem tells us that \( \dot{A} \) depends on the initial condition, so this result is consistent with Eq. (7.10). Since the differential equation becomes

\[ \dot{A} = \alpha C \beta \left( V - V_0 \right) \]

we can choose

\[ \dot{A} = \beta \left( V - V_0 \right) \]

This shows that the expression in parentheses is a constant.

\[ \dot{A} = \frac{\beta V}{\alpha} \]

The equation reads

\[ \frac{\partial V}{\partial t} + \frac{\partial \rho}{\partial x} = 0 \]

This is the desired differential equation. Calling

\[ \dot{A} = \frac{\beta V}{\alpha} \]

and using Eq. (7.2), we find

\[ \frac{\partial V}{\partial t} + \frac{\partial \rho}{\partial x} = 0 \]

and finally

\[ \frac{\partial V}{\partial t} = 0 \]

so

\[ \frac{\partial \rho}{\partial x} = 0 \]

and

\[ \frac{\partial \rho}{\partial x} = 0 \]

is another consistent solution. We then have

\[ \dot{A} = \frac{\beta V}{\alpha} \]

where

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is another consistent solution. We then have

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\[ \dot{A} = \frac{\beta V}{\alpha} \]
\[ (\psi \phi) \cdot \mathbf{F} \cdot \mathbf{a} \int_{\Delta} \equiv \left[ (\psi \phi) \cdot \mathbf{a} \right]_{\Delta} \]

By the definition of the Laplacian, we have
\[ \Delta = \frac{1}{\rho} \int_{\Delta} \rho \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} \phi \]

As pointed out in the statement of the problem, we can begin by integrating by parts, and we have
\[ \int_{\Delta} \phi \cdot \mathbf{F} \cdot \mathbf{a} \int_{\Delta} + \left[ (\phi \cdot \mathbf{F}) \cdot \mathbf{a} \right]_{\Delta} = 0 \]

The integral over \( \theta \) is an integral of a derivative, so we integrate first over \( \theta \), and the volume element is \( r \sin \theta \), and the volume element is
\[ dV = r \sin \theta dr d\theta d\phi \]

We can then rewrite the integral as
\[ \int_{\Delta} \phi \cdot \mathbf{F} \cdot \mathbf{a} \int_{\Delta} + \left[ (\phi \cdot \mathbf{F}) \cdot \mathbf{a} \right]_{\Delta} = 0 \]

The potential \( \phi \cdot \mathbf{F} \cdot \mathbf{a} \) is interpreted as a Laplacian of the distribution \( \phi \cdot \mathbf{F} \cdot \mathbf{a} \).
which is a well-defined ordinary integral. Starting from Eq. (8.9) we can integrate by parts, finding
\[ F[\psi(x,y)] = -\int \partial_i \psi \partial_i \ln r \, d^2 x = -\int \partial_i \psi \left( \frac{1}{r} \hat{r}_i \right) \, d^2 x. \]
(8.10)

Using polar coordinates \( r \) and \( \phi \), we now write \( \hat{r}_i \partial_i \psi = \partial \psi / \partial r \) and \( d^2 x = r \, dr \, d\phi \), so
\[ F[\psi(r,\phi)] = 2\pi \psi(\vec{0}), \]
(8.11)

where as in part (a) we used the fact that \( \psi \) is required to approach zero as \( r \to \infty \), and that it cannot depend on angle at \( r=0 \). Thus we have
\[ \int \psi(\vec{r}) \nabla^2 (\ln r) \, d^2 x = 2\pi \psi(\vec{0}), \]
(8.12)

and by definition
\[ \int \psi(\vec{r}) \delta^2 (\vec{r}) \, d^2 x \equiv \psi(\vec{0}). \]
(8.13)

Since distributions are defined solely in terms of the generalized integral that they produce (i.e., by how they map test functions \( \psi(\vec{r}) \) to the number given by these integrals), we see that
\[ \nabla^2 \ln r = 2\pi \delta^2 (\vec{r}), \]
(8.14)

in the sense that the two sides of this equation represent the same distribution.