

September 08, 2011

# Massachusetts Institute of Technology

Department of Physics - Theoretical Classical Mechanics 8.09 - Fall 2011

## Homework set No. 1 (Due: September 16, 2011)

### Reading:

1. Goldstein, Chapter 1.1

### Problems:

1. (10 points) Consider a point mass  $m$ , which has the following plane motion:

$$x_1 = a \sin \omega t, \quad x_2 = b \cos \omega t$$

- (a) (3 points) Determine the velocity of this point mass as a function of time.
- (b) (3 points) At which locations does the point mass  $m$  come to rest?
- (c) (3 points) Which force field gives rise to such a motion?
- (d) (1 point) Draw and discuss the plane motion of this point mass.

2. (10 points) A point mass experiences the following force:

$$\vec{F} = -mg\vec{n} - mb\vec{v}$$

with  $b > 0$  and  $g > 0$  and  $\vec{n}$  pointing into a fixed direction. The starting location  $\vec{r}_0$  and velocity  $\vec{v}_0$  are known.

- (a) (8 points) Determine  $\vec{v}(t)$ .

Hint: First solve the homogeneous differential equation  $\frac{d\vec{v}_h}{dt} + b\vec{v}_h = 0$ . Find then a solution for the inhomogeneous equation using  $v_s(t) = \vec{u}(t)e^{-bt}$ . The general solution is then given by:  $\vec{v}(t) = \vec{v}_s(t) + c\vec{v}_h(t)$ .

- (b) (2 points) Discuss the motion in terms of  $\vec{r}(t)$  and  $\vec{v}(t)$ , in particular for the case of  $t \rightarrow \infty$ .

3. (10 points) Calculate for the following force field the line integral:

$$\vec{F}(\vec{x}) = \begin{pmatrix} x_1x_2 \\ x_2x_3 \\ x_3x_1 \end{pmatrix}$$

(a) (2 points) Along a straight line from

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ to } \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

(b) (2 points) Along the following three straight segments:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ to } \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ to } \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \text{ to } \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

(c) (2 points) Is the above force field a conservative force field?

(d) (2 points) Find an alternative way to answer this question.

(e) (2 points) Calculate the line integral for the following force field

$$\vec{F}(\vec{x}) = \vec{\omega} \times \vec{x}$$

along the path of a circle with radius of 1 m which lies in a plane perpendicular to  $\vec{\omega}$ .

4. (10 points) Consider the following forces:

$$\vec{F}_1(\vec{r}) = f(\vec{r})\vec{r} \text{ and } \vec{F}_2(\vec{r}) = f(r)\vec{r}$$

(a) (2 points) What is the difference between the two forces?

(b) (4 points) Show that the force  $\vec{F}_2$  is conservative.

(c) (4 points) Determine the potential for  $\vec{F}_2$  with  $f(r) = -\alpha r^2$ . The potential should vanish at the coordinate origin.

5. (10 points) The position vector describing a helix in space is given as follows:

$$\vec{r}(t) = \begin{pmatrix} R \cos \omega t \\ R \sin \omega t \\ b \omega t \end{pmatrix}$$

(a) (2 points) Determine the tangent vector.

(b) (2 points) Determine the normal vector.

(c) (2 points) Determine the binormal vector.

(d) (2 points) Determine the derivatives of the above three vectors in  $s$  (path length).

(e) (2 points) Determine the curvature and torsion as a function of  $R$  and  $b$ .