November 11, 2011

Massachusetts Institute of Technology

Department of Physics - Classical Mechanics 8.09 - Fall 2011

Homework set No. 10 (Due: Monday, November 21, 2011)

Reading:

1. Goldstein, Chapter 3-6

Problems:

1. (10 points)

   (a) (5 points) Find the Lagrangian equations of motion for the coplanar double oscillator shown in Figure 1 in the vibration limit, assuming massless strings or connecting rods. From them find the normal frequencies of the system.

   (b) (5 points) Now consider a simple pendulum of mass m, again in the small-vibration limit. Suppose the string of length l is shortened very slowly by being pulled up through a frictionless hole in the support, so that the fractional change in l over one period is small. How does the amplitude of vibration of m vary with l?

2. (10 points) Four identical masses are connected by four identical springs and constrained to move on a frictionless circle of radius b as shown in Figure 2.

   (a) (5 points) How many normal-modes of small oscillations are there?

   (b) (5 points) What are the frequencies of small oscillations?

3. (10 points)

   (a) (5 points) Find the central force which results in the following orbit for a particle

   $$r = a(1 + \cos\theta)$$

   (b) (5 points) A particle of mass m is acted on by an attractive force whose potential is given by $U \propto r^{-4}$. Find the total cross section for capture for the particle coming from infinity with an initial velocity V_∞.

   Note: Parts (a) and (b) may refer to different forces.

4. (10 points)

   (a) (5 points) A particle of mass m moves under a conservative force with potential energy $V(x) = cx/(x^2 + a^2)$, where c and a are positive constants. Find the position of stable equilibrium and the period of small oscillations about it.

   (b) (5 points) If the particle starts from this point with velocity v, find the range of values of v for which it (1) oscillates, (2) escapes to $-\infty$, (3) escapes to $+\infty$.

5. (10 points) A thin square plate as shown in Figure 3 with side length a rotates at a constant angular frequency ω about an axis through the center tilted by an angle θ with respect to the normal to the plate.

   (a) (4 points) Find the principal moments of inertia.

   (b) (3 points) Find the angular momentum $\mathbf{J}$ in the laboratory system.

   (c) (3 points) Calculate the torque on the axis.

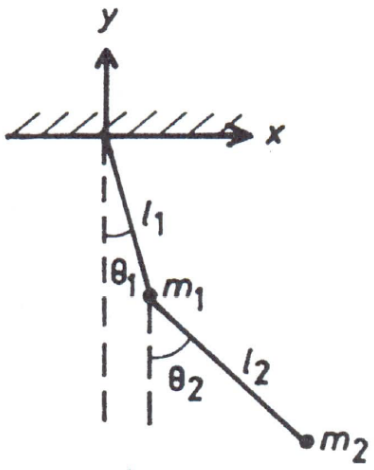

Figure 1: Double pendulum.

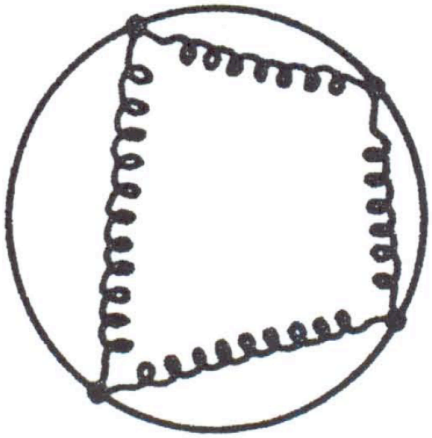

Figure 2: Four masses on a circle connected by springs.

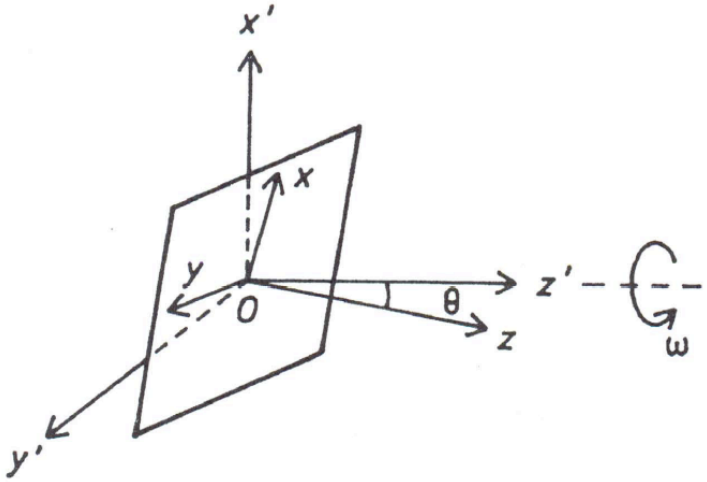

Figure 3: Rotating square plate.