

November 12, 2009

Massachusetts Institute of Technology

Department of Physics - Classical Mechanics 8.09 - Fall 2009

Homework set No. 10 (Due: Friday, November 20, 2009)

Reading:

1. Goldstein, Chapter 3-6

Problems:

1. (10 points)
 - (a) (5 points) Find the Lagrangian equations of motion for the coplanar double oscillator shown in Figure 1 in the vibration limit, assuming massless strings or connecting rods. From them find the normal frequencies of the system.
 - (b) (5 points) Now consider a simple pendulum of mass m , again in the small-vibration limit. Suppose the string of length l is shortened very slowly by being pulled up through a frictionless hole in the support, so that the fractional change in l over one period is small. How does the amplitude of vibration of m vary with l ?
2. (10 points) Four identical masses are connected by four identical springs and constrained to move on a frictionless circle of radius b as shown in Figure 2.
 - (a) (5 points) How many normal-modes of small oscillations are there?
 - (b) (5 points) What are the frequencies of small oscillations?
3. (10 points)
 - (a) (5 points) Find the central force which results in the following orbit for a particle

$$r = a(1 + \cos \theta)$$

- (b) (5 points) A particle of mass m is acted on by an attractive force whose potential is given by $U \propto r^{-4}$. Find the total cross section for capture for the particle coming from infinity with an initial velocity V_∞ .

Note: Parts (a) and (b) may refer to different forces.

4. (10 points)

(a) (5 points) A particle of mass m moves under a conservative force with potential energy $V(x) = cx/(x^2 + a^2)$, where c and a are positive constants. Find the position of stable equilibrium and the period of small oscillations about it.

(b) (5 points) If the particle starts from this point with velocity v , find the range of values of v for which it (1) oscillates, (2) escapes to $-\infty$, (3) escapes to $+\infty$.

5. (10 points) A thin square plate as shown in Figure 3 with side length a rotates at a constant angular frequency ω about an axis through the center tilted by an angle θ with respect to the normal to the plate.

(a) (4 points) Find the principal moments of inertia.

(b) (3 points) Find the angular momentum \mathbf{J} in the laboratory system.

(c) (3 points) Calculate the torque on the axis.

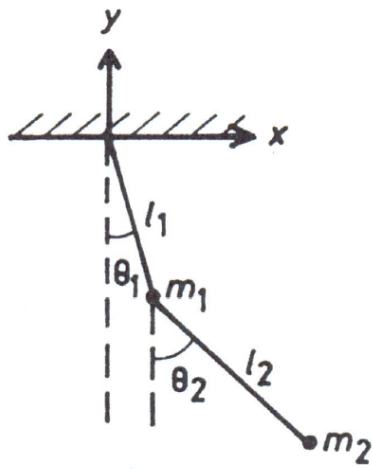


Figure 1: Double pendulum.

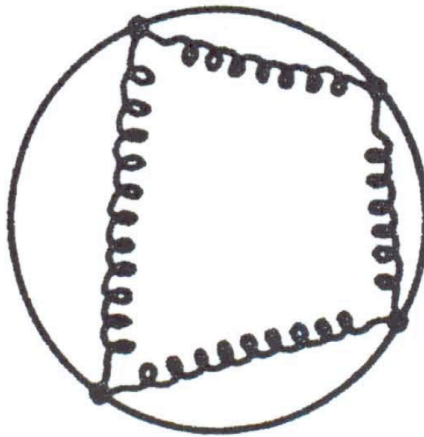


Figure 2: Four masses on a circle connected by springs.

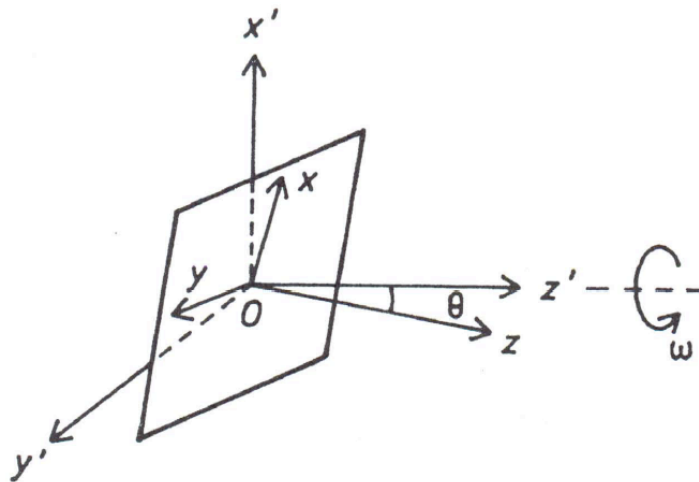


Figure 3: Rotating square plate.