November 17, 2011

Massachusetts Institute of Technology

Department of Physics - Classical Mechanics 8.09 - Fall 2011

Homework set No. 11 (Due: Tuesday, December 05, 2011)

Happy Thanksgiving

Reading:

1. Goldstein, Chapter 8

Problems:

1. (10 points)

   The internal energy $U = U(S, V)$ of a gas has the following total differential:

   $$dU = TdS - pdV,$$

   with temperature T, entropy S, pressure p and volume V. Formulate three other thermodynamic potentials using a Legendre transformation:

   (a) (2 points) Enthalpy: $H(S, p)$

   (b) (2 points) Helmholtz free energy: $F(T, V)$

   (c) (2 points) Gibbs free energy: $G(T, p)$

   (4 points) Derive the Lagrange equation 2^{nd}-type using a Legendre transformation starting with the canonical equations.

2. (10 points)

   (a) (4 points) Formulate the canonical equations of a point mass m inside a potential V in Cartesian, cylindrical and spherical coordinates.

   (b) (6 points)

   A wagon moves with constant velocity v_0 along the x-axis as shown in Figure 1. A point mass m is connected through a spring with spring constant k with the wagon. The point mass moves without friction on the wagon.

1) (3 points) Determine the Hamilton function in the laboratory system m and evaluate if the Hamilton function is a conserved quantity and equal to the total mechanical energy E.

2) (3 points) Use $x = x' + v_0 t$ to transform from the laboratory system to the body system of the point mass m. Determine again the Hamilton function in the body system and evaluate if the Hamilton function is a conserved quantity and equal to the total mechanical energy E.

3. (10 points)

   (a) (4 points) Formulate the canonical equations including friction.

   A hanger of mass m (Figure 2) for a car with a spring-damping system is moving along the x-direction with constant velocity v. It performs sinusoidal oscillations in y according to: $y_s = a\cos\left(\frac{2\pi}{l}x\right)$. The masses of both tires can be ignored.

   (b) (3 points) Formulate the equation of motion for the y-coordinate of the hanger using the canonical equations.

   (c) (3 points) How does the spring constant D and the friction coefficient c have to be dimensioned to ensure that the hanger is moving as quietly as possible?

4. (10 points) Proof the Jacobi identity:

$$[f,[g,h]] + [g,[h,f]] + [h,[f,g]] = 0$$

5. (10 points) Determine the following Poisson brackets and compare those to the well-known quantum mechanical commutator expressions:

   (a) (5 points) $[L_i, x_j]$, $[L_i, p_j]$, $[L_i, L_j]$ and $[\vec{L}^2, L_i]$ with $i, j = 1, 2, 3$

   (b) (5 points) $[L_i, \vec{r}\cdot\vec{p}]$ and $[p_i, r^n]$ with $i, j = 1, 2, 3$

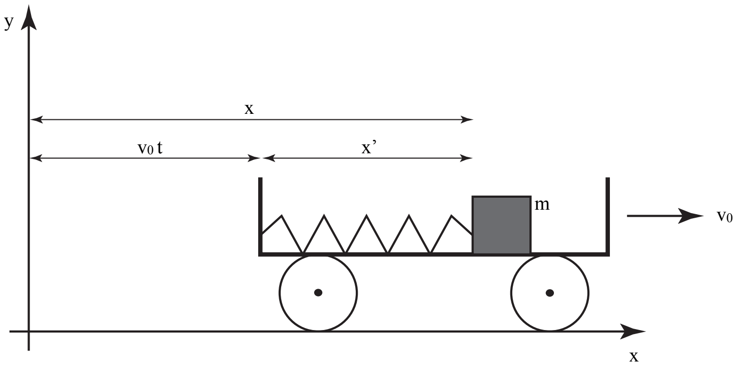

Figure 1: Mass m on a wagon connected by a spring with spring constant k.

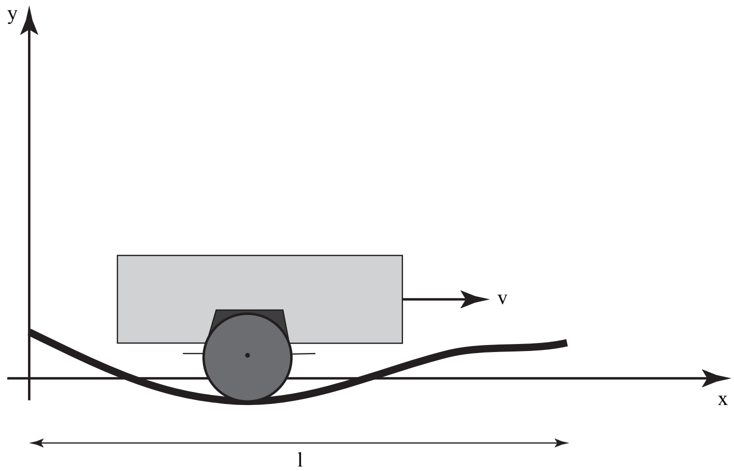

Figure 2: Hanger on a bumpy road.