

September 17, 2009

# Massachusetts Institute of Technology

Department of Physics - Classical Mechanics 8.09 - Fall 2009

## Homework set No. 2 (Due: September 24, 2009)

### Reading:

1. Goldstein, Chapter 1.2

### Problems:

1. (10 points) Calculate the horizontal deflection of a free falling object of mass  $m$  at a geographic latitude  $\phi$ . The object is initially at rest at height  $h$ .

Hint: Ignore any terms of order  $\omega^2$ . Choose a coordinate system that is fixed to the earth with the  $x'$ -axis pointing south, the  $y'$ -axis pointing east and the  $z'$ -axis pointing upwards.

- (a) (1 point) Show that the angular velocity of the earth  $\vec{\omega}$  can be written as:

$$\vec{\omega} = -\omega \cos \phi \vec{e}'_1 + \omega \sin \phi \vec{e}'_3$$

- (b) (4 points) Show that the equations of motion ignoring terms of order  $\omega^2$  are given as follows:

$$\ddot{x}' = 2\omega \dot{y}' \sin \phi$$

$$\ddot{y}' = -2\omega (\dot{x}' \sin \phi + \dot{z}' \cos \phi)$$

$$\ddot{z}' = -g + 2\omega \dot{y}' \cos \phi$$

- (c) (4 points) Show that the horizontal deflection for height  $h$  is given as follows:

$$y'(h) = \frac{\omega}{3} \sqrt{\frac{(2h)^3}{g}} \cos \phi$$

- (c) (1 point) Determine the horizontal deflection for Boston for  $h = 100$  m in comparison to the one at the equator.
2. (20 points) In 1851, Jean Bernard Léon Foucault (1819-1868) performed his famous pendulum experiment in the Panthéon in Paris as a proof of the earth's rotation. Consider the coordinate system  $(x', y', z')$  as introduced in problem 1. The pendulum string  $l$  is very long and the oscillations are very small, i.e. use the following approximations:  $x'/l \ll 1$ ,  $y'/l \ll 1$  and  $z'/l \ll 1$ . The point mass  $m$  moves therefore entirely in the  $(x', y')$  plane. Ignore the mass of the pendulum string. The tension in the string is denoted as  $T$ .
- (a) (6 points) Show that the equations of motion are given as follows:

$$m\ddot{x}' = -T(x'/l) + 2m\omega\dot{y}' \sin \phi$$

$$m\ddot{y}' = -T(y'/l) - 2m\omega(\dot{x}' \sin \phi + \dot{z}' \cos \phi)$$

$$m\ddot{z}' = T(l - z')/l - mg + 2m\omega\dot{y}' \cos \phi$$

- (b) (2 points) Show that the tension in the string can be written in the following form using the above approximations:

$$T = mg - 2m\omega\dot{y}' \cos \phi$$

- (c) (4 points) Show that the equations of motion in the  $(x', y')$  can be written in the following form using the above approximations, ignoring any non-linear terms:

$$\ddot{x}' = -g(x'/l) + 2\omega\dot{y}' \sin \phi$$

$$\ddot{y}' = -g(y'/l) - 2\omega\dot{x}' \sin \phi$$

- (d) (6 points) Solve the coupled differential equations and show that the result is given as follows for the following initial conditions at  $t = 0$ :  $x' = 0$ ,  $\dot{x}' = 0$ ,  $y' = A$  and  $\dot{y}' = 0$ .

$$x' = A \cos(\sqrt{g/l} t) \sin(\omega \sin \phi t)$$

$$y' = A \cos(\sqrt{g/l} t) \cos(\omega \sin \phi t)$$

- (e) (2 point) Interpret the result that you obtained in part d. Would you be able to verify the rotation of the earth everywhere on earth? Explain!
3. (10 points) Two masses  $m_1$  and  $m_2$  are connected by a massless rod of length  $l$ . This dumbbell is thrown from the coordinate origin inside the earth's gravitational field restricted to the earth surface.
- (a) (2 points) Determine the equation of motion for the center-of-mass (CM) point.

- (b) (2 points) Which trajectory does the CM point describe for an initial velocity of  $\vec{v}_0$  of the CM point?
- (c) (2 points) Separate the total angular momentum into the center-of-mass contribution  $L_0$  and the contribution around the the center-of-mass  $L_{CM}$ . Determine  $L_0$  employing your solution in part (b).
- (d) (2 points) Determine the equation of the relative motion around the center-of-mass point. What can you say about  $L_{CM}$ ?
- (e) (2 points) Show that the masses  $m_1$  and  $m_2$  move on circles around CM with constant angular velocity. What is the ratio of their radii?
4. (10 points) Consider the elastic collision of two hard spheres of masses  $m_1$  and  $m_2$  of identical radius  $r_A$  as shown below. The second sphere is at rest in the laboratory system O. Its center lies on the x-axis. Sphere 1 has an initial momentum of  $\vec{p}_1 = m_1\vec{e}_1$  with  $p_1 > 0$ . The trajectory of sphere 1 is shifted by a distance  $r_A$  from the x-axis.

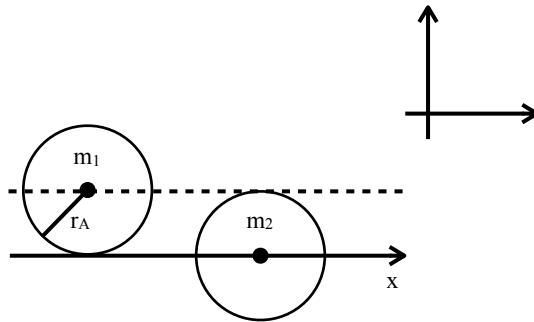


Figure 1: Elastic collision of two hard spheres of masses  $m_1$  and  $m_2$ .

- (a) (5 points) Determine the momenta of sphere 1 and 2 after the collision in the laboratory system.
- (b) (5 points) Determine the momenta of sphere 1 and 2 before and after the collision in the center-of-mass system.