

September 15, 2011

Massachusetts Institute of Technology

Department of Physics - Classical Mechanics 8.09 - Fall 2011

Homework set No. 2 (Due: September 23, 2011)

Reading:

1. Goldstein, Chapter 1.2

Problems:

1. (10 points) Calculate the horizontal deflection of a free falling object of mass m at a geographic latitude ϕ . The object is initially at rest at height h .

Hint: Ignore any terms of order ω^2 . Choose a coordinate system that is fixed to the earth with the x' -axis pointing south, the y' -axis pointing east and the z' -axis pointing upwards.

- (a) (1 point) Show that the angular velocity of the earth $\vec{\omega}$ can be written as:

$$\vec{\omega} = -\omega \cos \phi \vec{e}'_1 + \omega \sin \phi \vec{e}'_3$$

- (b) (4 points) Show that the equations of motion ignoring terms of order ω^2 are given as follows:

$$\ddot{x}' = 2\omega \dot{y}' \sin \phi$$

$$\ddot{y}' = -2\omega (\dot{x}' \sin \phi + \dot{z}' \cos \phi)$$

$$\ddot{z}' = -g + 2\omega \dot{y}' \cos \phi$$

- (c) (4 points) Show that the horizontal deflection for height h is given as follows:

$$y'(h) = \frac{\omega}{3} \sqrt{\frac{(2h)^3}{g}} \cos \phi$$

- (c) (1 point) Determine the horizontal deflection for Boston for $h = 100$ m in comparison to the one at the equator.
2. (20 points) In 1851, Jean Bernard Léon Foucault (1819-1868) performed his famous pendulum experiment in the Panthéon in Paris as a proof of the earth's rotation. Consider the coordinate system (x', y', z') as introduced in problem 1. The pendulum string l is very long and the oscillations are very small, i.e. use the following approximations: $x'/l \ll 1$, $y'/l \ll 1$ and $z'/l \ll 1$. The point mass m moves therefore entirely in the (x', y') plane. Ignore the mass of the pendulum string. The tension in the string is denoted as T .
- (a) (6 points) Show that the equations of motion are given as follows:

$$\begin{aligned} m\ddot{x}' &= -T(x'/l) + 2m\omega\dot{y}' \sin \phi \\ m\ddot{y}' &= -T(y'/l) - 2m\omega(\dot{x}' \sin \phi + \dot{z}' \cos \phi) \\ m\ddot{z}' &= T(l - z')/l - mg + 2m\omega\dot{y}' \cos \phi \end{aligned}$$

- (b) (2 points) Show that the tension in the string can be written in the following form using the above approximations:

$$T = mg - 2m\omega\dot{y}' \cos \phi$$

- (c) (4 points) Show that the equations of motion in the (x', y') can be written in the following form using the above approximations, ignoring any non-linear terms:

$$\begin{aligned} \ddot{x}' &= -g(x'/l) + 2\omega\dot{y}' \sin \phi \\ \ddot{y}' &= -g(y'/l) - 2\omega\dot{x}' \sin \phi \end{aligned}$$

- (d) (6 points) Solve the coupled differential equations and show that the result is given as follows for the following initial conditions at $t = 0$: $x' = 0$, $\dot{x}' = 0$, $y' = A$ and $\dot{y}' = 0$.

$$\begin{aligned} x' &= A \cos(\sqrt{g/l} t) \sin(\omega \sin \phi t) \\ y' &= A \cos(\sqrt{g/l} t) \cos(\omega \sin \phi t) \end{aligned}$$

Hint: Formulate the coupled differential equations using a description in terms of complex variables, i.e. $z = x + iy$.

- (e) (2 point) Interpret the result that you obtained in part d. Would you be able to verify the rotation of the earth everywhere on earth? Explain!

3. (10 points) Two masses m_1 and m_2 are connected by a massless rod of length l . This dumbbell is thrown from the coordinate origin inside the earth's gravitational field restricted to the earth surface.
- (2 points) Determine the equation of motion for the center-of-mass (CM) point.
 - (2 points) Which trajectory does the CM point describe for an initial velocity of \vec{v}_0 of the CM point?
 - (2 points) Separate the total angular momentum into the center-of-mass contribution L_0 and the contribution around the the center-of-mass L_{CM} . Determine L_0 employing your solution in part (b).
 - (2 points) Determine the equation of the relative motion around the center-of-mass point. What can you say about L_{CM} ?
 - (2 points) Show that the masses m_1 and m_2 move on circles around CM with constant angular velocity. What is the ratio of their radii?
4. (10 points) Consider the elastic collision of two hard spheres of masses m_1 and m_2 of identical radius r_A as shown below. The second sphere is at rest in the laboratory system O. Its center lies on the x-axis. Sphere 1 has an initial momentum of $\vec{p}_1 = m_1\vec{e}_1$ with $p_1 > 0$. The trajectory of sphere 1 is shifted by a distance r_A from the x-axis.

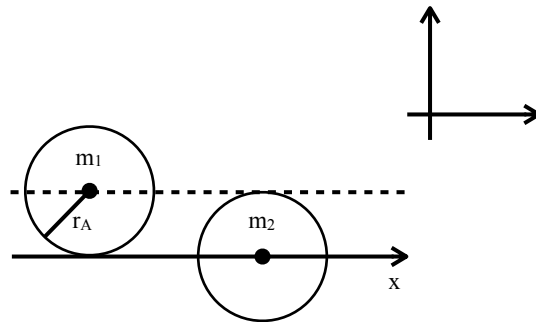


Figure 1: Elastic collision of two hard spheres of masses m_1 and m_2 .

- (5 points) Determine the momenta of sphere 1 and 2 after the collision in the laboratory system.
- (5 points) Determine the momenta of sphere 1 and 2 before and after the collision in the center-of-mass system.