

September 29, 2011

# Massachusetts Institute of Technology

Department of Physics - Classical Mechanics 8.09 - Fall 2011

## Homework set No. 4 (Due: October 07, 2011)

### Reading:

1. Goldstein, Chapter 2

### Problems:

1. (10 points) Double pendulum (Figure 1) / Spring pendulum (Figure 2):
  - (a) (4 points) Determine the equation of motion using the  $2^{nd}$  Lagrange equation for the double pendulum (Figure 1).
  - (b) (4 points) Solve the equation of motions for the double pendulum for the case of  $m_1 = m_2 = m$ ,  $l_1 = l_2 = l$  using the small angle approximation.
  - (c) (2 points) Determine the equation of motion using the  $2^{nd}$  Lagrange equation for the spring pendulum (Figure 2). The equilibrium length with an attached mass  $m$  refers to  $l_0$ .
2. (10 points) A small pearl of mass  $m$  slides without friction on a parabolic ( $z = ar^2$ ) rotating wire (Figure 3) which is rotating with an angular velocity  $\omega$  around the  $z$ -axis.
  - (a) (5 points) Determine the equation of motion using the  $2^{nd}$  Lagrange equation.
  - (b) (5 points) Solve the equation of motion.
3. (10 points) A rope of length  $l$  is thrown in the air vertically as shown in Figure 4. The rope is fully flexible such that the kink can move along the rope.
  - (a) (4 points) Use the distances  $h_1$  and  $h_2$  as generalized coordinates and determine the equations of motion.
  - (b) (3 points) Substitute  $x = h_1 - h_2$  and determine the equation of motion of  $x$ , i.e. for the movement of the kink location. What can you say about  $\dot{x}$  for  $x \rightarrow l$ ? Explain!
  - (c) (3 points) Calculate the tension in the rope  $Z = \rho s(\ddot{h}_2 + g)$  for an arbitrary location P. The density per unit length refers to  $\rho$ . Explain the catastrophic effect a flipping rope can move!

4. (10 points) Derive the following relations:

(a) (3 points)

$$\frac{\partial \vec{r}}{\partial q_i} = \frac{\partial \dot{\vec{r}}}{\partial \dot{q}_i}$$

(b) (3 points)

$$\frac{\partial \dot{\vec{r}}}{\partial q_i} = \frac{d}{dt} \frac{\partial \vec{r}}{\partial q_i}$$

(c) (4 points) Show that the following potential  $V = q(\Phi - \vec{v} \cdot \vec{A})$  yields the electromagnetic force  $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$  with  $\vec{E} = -\vec{\nabla}\Phi - \frac{\partial \vec{A}}{\partial t}$  and  $\vec{B} = \vec{\nabla} \times \vec{A}$ .

5. (10 points) Consider the following problems involving friction:

(a) (4 points) Determine the equation of motion of a particle of mass  $m$  that is sliding down an inclined plane with angle  $\alpha$  including the impact of sliding friction.

(b) (3 points) Determine the dissipation function for a spherical pendulum of length  $l$  with an attached mass of mass  $m$  in a viscous medium assuming that the pendulum moves overall with small velocity.

(b) (3 points) Determine the dissipation function of a dumbbell with two equal masses of mass  $m$  and length  $2l$  which moves with small velocity freely in a viscous medium.

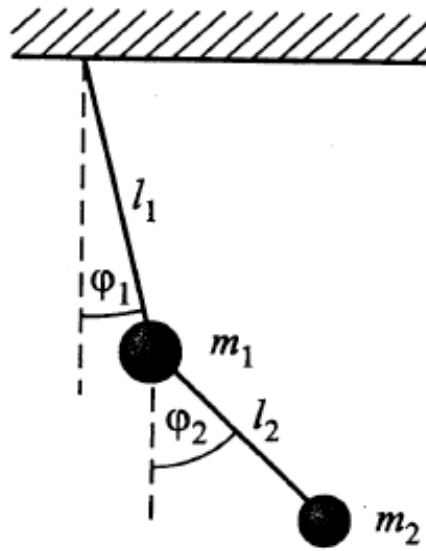


Figure 1: Double pendulum.

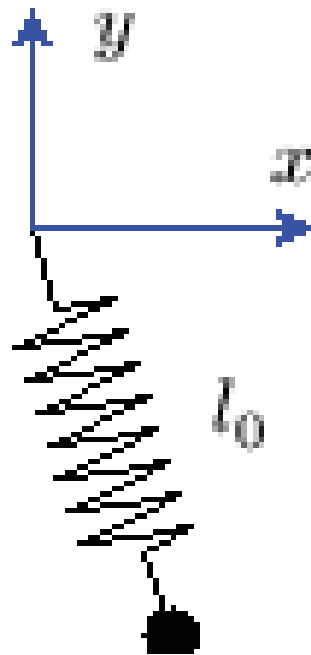


Figure 2: Spring pendulum.

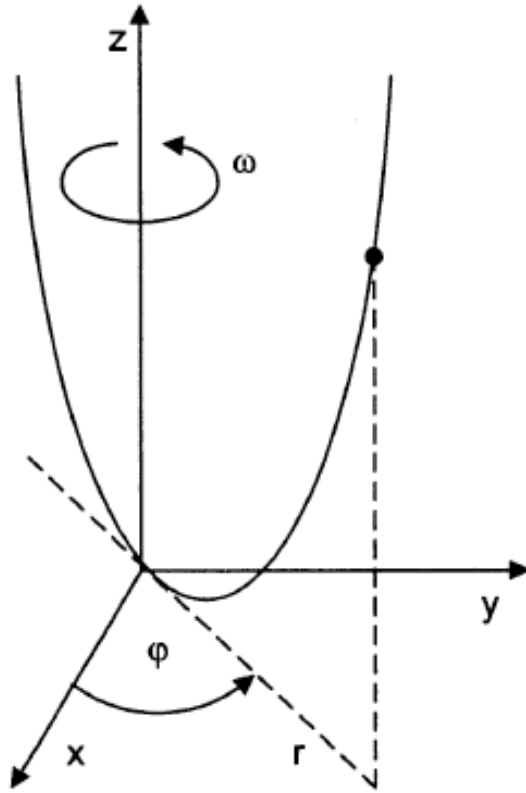


Figure 3: Pearl on a parabolic rotating wire.

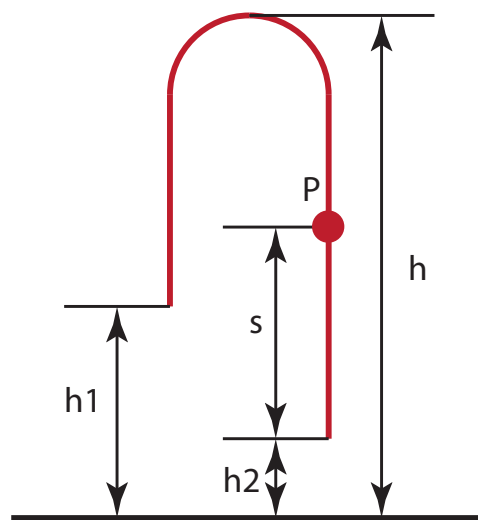


Figure 4: Thrown rope in air. The kink is located at height  $h$  with respect to ground.