

Massachusetts Institute of Technology

Department of Physics - Classical Mechanics 8.09 - Fall 2009

Homework set No. 6 (Due: October 22, 2009)

Reading:

1. Goldstein, Chapter 2

Problems:

1. (10 points) Consider a closed N-particle system with the following Lagrange function:

$$L = \sum_{i=1}^N \frac{m_i}{2} \dot{\vec{r}}_i^2 - \sum_{i < j} V_{ij}(|\vec{r}_i - \vec{r}_j|)$$

Show that the invariance of the Lagrange function under

- (a) (2 points) time translations lead to energy conservation,
 - (b) (2 points) rotations lead to angular momentum conservation,
 - (c) (2 points) translations lead to linear momentum conservations and
 - (d) (4 points) Galilei transformations - relative motion with constant velocity - lead to a conserved quantity, which is characteristic for the center-of-mass theorem.
2. (10 points) Consider the Atwood machine in Figure 1. Determine the tension in the string wrapped around the disk with mass m_2 as function of g and the masses m_i ($i = 1, \dots, 4$).
Hint: First think about which of the following coordinates x_1, \dots, x_4 should be considered as independent. Ignore the moment of inertia of both disks with masses m_1 and m_2 .
 3. (10 points) Consider the motion of a pearl of mass m on a helix with fixed radius R as shown in Figure 2 with $z = a\varphi$. Determine the constraint forces as function of m , g , R and a . One constraint force has also an explicit dependence on t . Which one?
 4. (10 points) Consider the motion of a pearl of mass m on a helix with varying radius $r = bz$ and $\varphi = az$ as shown in Figure 3.
 - (a) (5 points) Integrate the equation of motion and determine the constraint torque Z_φ as a function of a , b , m and z .
 - (b) (5 points) Show that the particle energy is constant and that the angular momentum increases with increasing z .

5. (10 points) Solve two basic problems involving calculus of variations:

(a) (5 points) The Brachistochrone problem is shown in Figure 4. A point mass m , which is initially at rest, starts at point $P_1(x_1, 0)$ and moves without friction to point $P_2(x_2, y_2)$. For which curve does the point mass m move from P_1 to P_2 at the shortest amount of time?

(b) (5 points) A soap film is set up between two rings as shown in Figure 5 and forms a minimal surface. Determine a condition for this surface, i.e. what functional form do you get between both rings?

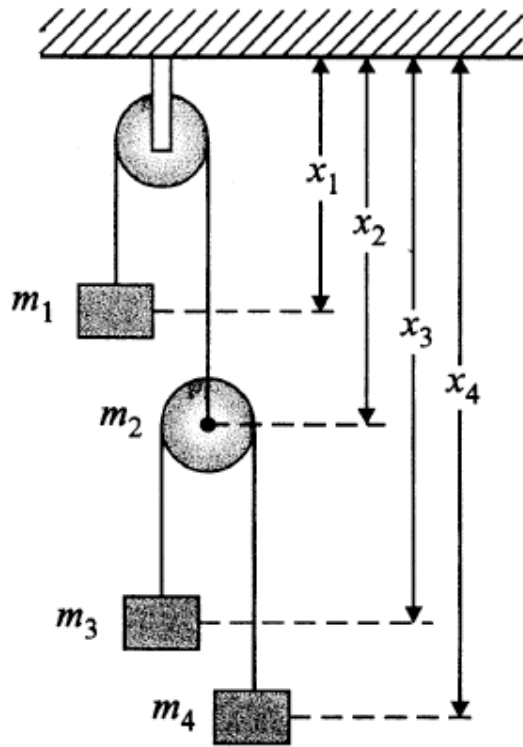


Figure 1: Atwood machine.

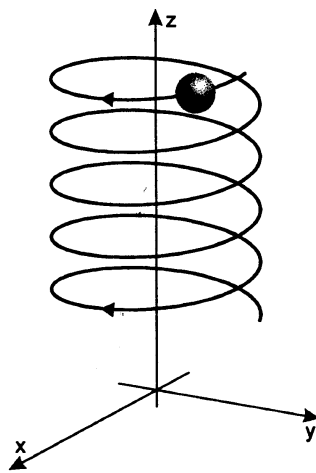


Figure 2: Pearl of mass m on a helix with fixed radius R .

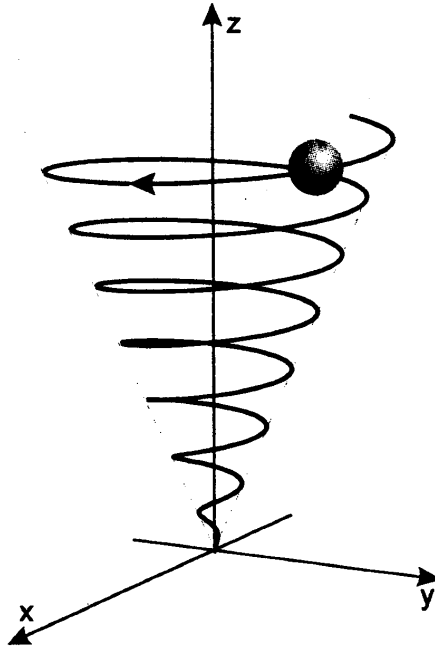


Figure 3: Pearl of mass m on a helix with varying radius r .

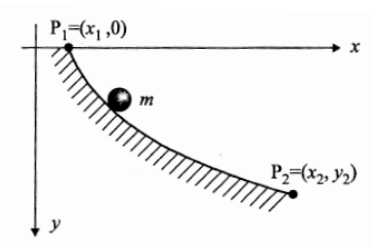


Figure 4: Brachistochrone problem.

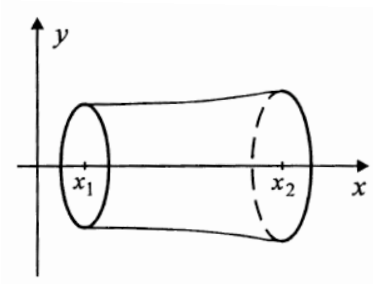


Figure 5: Minimal surface of soap film between two rings.