

October 22, 2009

Massachusetts Institute of Technology

Department of Physics - Classical Mechanics 8.09 - Fall 2009

Homework set No. 7 (Due: October 29, 2009)

Reading:

1. Goldstein, Chapter 3

Problems:

1. (a) (4 points) Show that the motion in the gravitational potential $V(r) = -\alpha/r$ has an additional conserved quantity besides energy E and angular momentum \vec{L} , called *Lenz vector* $\vec{\Lambda}$:

$$\vec{\Lambda} = \frac{\vec{p} \times \vec{L}}{m\alpha} - \frac{\vec{r}}{r}$$

- (b) (3 points) Show that the Lenz vector $\vec{\Lambda}$ points in the direction of the perihelion.
- (c) (3 points) Determine the magnitude of $\vec{\Lambda}$ and show that its value is given by the eccentricity ϵ .
2. (a) (2 points) Under what conditions can an orbiting mass reach the center of a given field for $L \neq 0$:

$$V(r) = -\alpha r^{-n}$$

Hint: Think first of three conditions on combinations of n and α .

- (b) (4 points) Are the number of turns finite or infinite on the path to the center of the field?
- (c) (4 points) Show that the time required to reach the center of a given field is always finite and the velocity and angular velocity infinite.
3. (10 points) Circular orbits are possible in all attractive and rotationally symmetric central force fields. The stability of circular orbits is particularly important.
 - (a) (6 points) Formulate a condition for the stability of circular orbits in an arbitrary, rotationally symmetric central force field.

Hint: Start with the effective potential and think of a condition involving the effective potential.

(b) (2 points) For which numbers n does the potential $V(r) = -\alpha/r^n$ yield stable circular orbits?

(c) (2 points) For which parameter r_0 does the potential

$$V(r) = -\frac{\alpha}{r} e^{-r/r_0}$$

yield stable circular orbits.

4. (10 points) The Kepler problem of the type $V(r) = -\alpha/r$ yields to closed orbits and a fixed perihelion direction. Using the Kepler-type potential only is clearly an approximation. The Kepler potential differs from the actual gravitational potential according to the theory of general relativity, by a correction of the form $\delta V = \beta/r^2$. The addition of a small correction to the Kepler potential leads to a shift of the perihelion $\delta\varphi$.

(a) (6 points) Determine the perihelion shift in an approximative way by using an expansion for small values of δV .

Calculate the actual value for the perihelion shift for:

(b) (2 points) $\delta V = \beta/r^2$ and

(c) (2 points) $\delta V = \gamma/r^3$.

5. (10 points) Determine the differential cross-section for the scattering of a mass m on a central force field with $V(r) = \beta/r^2$ and $\beta > 0$.