November 03, 2011

Massachusetts Institute of Technology

Department of Physics - Classical Mechanics 8.09 - Fall 2011

Homework set No. 9 (Due: November 14, 2011)

Reading:

1. Goldstein, Chapter 4, 5 and 6

Problems:

1. Figure 1 shows a simple-minded realization of a camshaft with point masses m and $2m$ fixed on massless rods, all in one plane. It rotates with constant angular velocity ω around the axis OO' through the long shaft, held by frictionless bearings at O and O'. A body system centered at point C is oriented such that the z axis points along the axis from O to O'. The x-axis points upwards.

   (a) (3 points) Determine the moment of inertia tensor I_{ij} for the camshaft configuration shown in Figure 1 using the body system (x, y, z).

   (b) (2 points) Determine the angular momentum $\vec{L}$ in the body system for a rotation around the z-axis.

   (c) (2 points) Determine the respective torque $\vec{\tau}$ with respect to the midpoint of the long shaft exerted by the bearings.

   (d) (3 points) Determine the principal moments and axes of the inertia tensor that you found in part a. Provide a clear presentation on how these new axes are oriented with respect to the axes of the body system given above.

2. A heavy symmetrical top ($I_1 = I_2$) (Figure 2) with one point fixed is precessing at a steady angular velocity Ω about the vertical axis z_I. The top mass m and its center of gravity is at a distance h from the fixed point. Use the coordinate systems indicated in Figure 2. Assume that precession proceeds without any nutation, i.e. constant nutation angle.

   (a) (2 points) Determine the components of the torque in terms of Euler angles.

   (b) (2 points) Formulate the angular velocity in terms of Euler angles.

   (c) (6 points) Derive a minimum condition on ω'.

3. A thin rectangular plate, of mass M and sides a by $2a$, rotates with constant angular velocity ω about an axle through the two diagonal corners as shown in Figure 3. The axle is supported

at the corners of the plate by bearings which can exert forces only on the axle. Ignore gravitational and frictional forces.

(a) (3 points) Determine the moment of inertia tensor I_{ij} using the body system (x, y, z) as shown in Figure 3.

(b) (3 points) Determine the torque in the body system.

(c) (2 points) Determine the forces on the bearings in the body system.

(d) (2 points) Determine the torque in the laboratory system (x_I, y_I, z_I).

4. (10 points) Three particles of equal mass m move without friction in one dimension as shown in Figure 4. Two of these particles are connected to the third by a massless spring of spring constant k. Determine the normal modes and corresponding frequencies. Sketch the underlying motion for each of these three modes.

5. (10 points) Three equal point masses m move on a circle with radius b under the forces derivable from a potential of the form

$$V(\alpha, \beta, \gamma) = V_0 \left(e^{-\alpha} + e^{-\beta} + e^{-\gamma}\right)$$

where α, β and γ are their angular separations in radians as shown in Figure 5. The equilibrium position of this system is defined by $\alpha = \beta = \gamma = \frac{2\pi}{3}$. Find the normal mode frequencies using an approximation of linear oscillations.

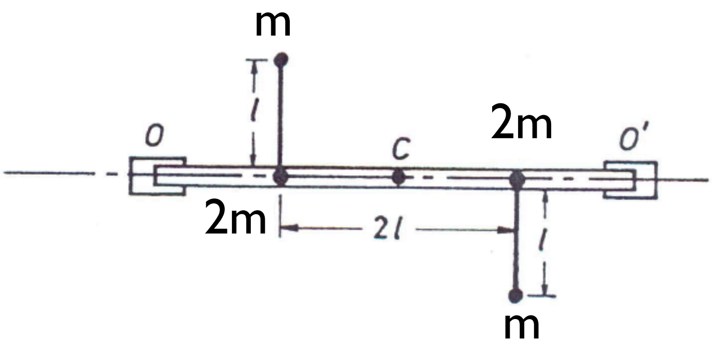

Figure 1: Camshaft configuration.

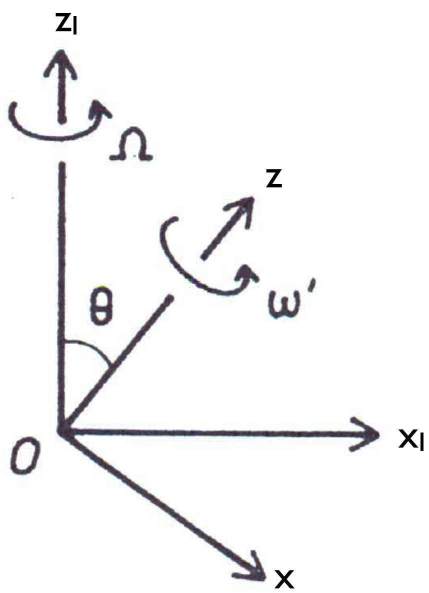

Figure 2: Heavy symmetrical top.

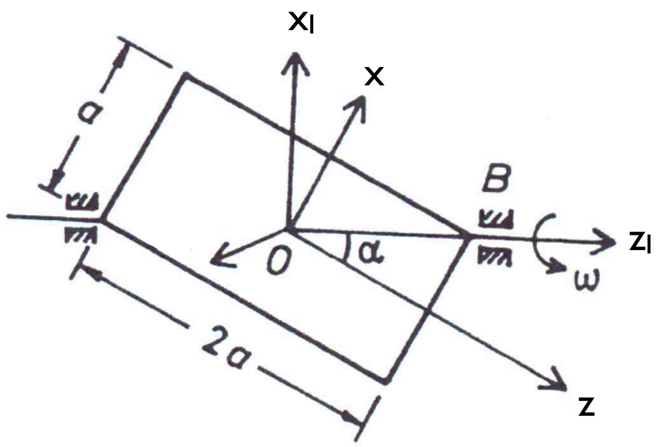

Figure 3: Rotating rectangular plate.

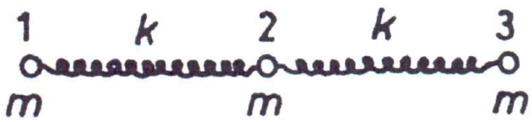

Figure 4: Three masses connected by a spring of spring constant k.

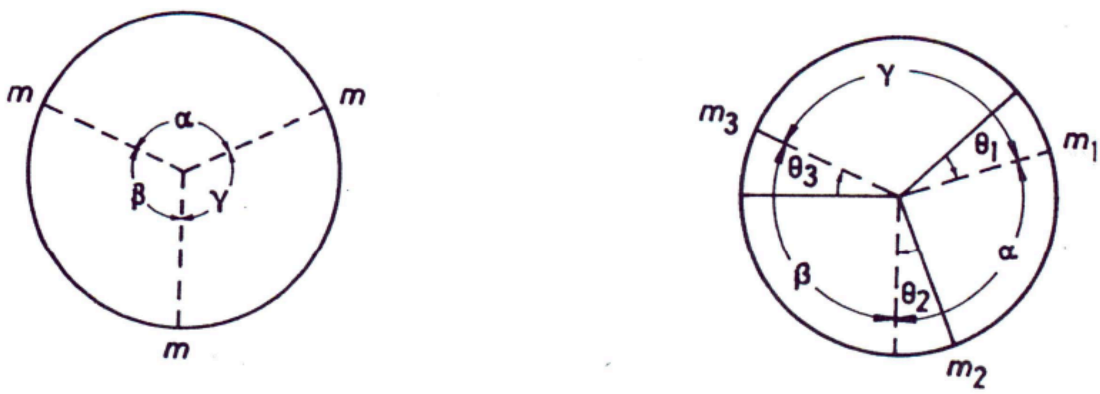

Figure 5: Three point masses constrained to move on a circle of radius b.