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Junior Physics Laboratory Experiment #002

Blackbody Radiation

The Stefan-Boltzman Law

PURPOSE This experiment is an exercise in electrical measurements, data analysis, error evaluation, and assessment of physical significance in the context of a measurement of the dependence on temperature of the power radiated by incandescent tungsten. The results are to be compared with the classical law governing the radiated power of an ideal black body.

1 PREPARATORY PROBLEMS

1. What is a 'black body', and what is the Stefan-Boltzmann law?

2. The resistivity of tungsten at 300 K is 5.65 $\mu\Omega\ cm$. Compute the resistance of a tungsten wire 0.001 cm in diameter and 1 cm long.

3. Suppose a voltmeter with an internal resistance of 10^6 ohms registers 1 mV when it is connected across a 1-Ω precision resistor in series with a pilot light to measure the current through the filament, as illustrated in Figure 1. What is the exact current in the filament?

4. Suppose the error in a measured value of P is 2%, and the error in the corresponding measured value of T is 1%. What is the error in the calculated quantity PT^{-4}?

5. In this experiment you will measure V_2 as a function of V_1. For each pair of voltage values you will compute a value of T according to Equation (2). On the basis of the Stefan-Boltzmann law, predict the shape of a plot of the quantity $\frac{V_1 V_2}{T^4}$ against V_1.

2 INTRODUCTION

All bodies emit and absorb radiant energy. A "black body", by definition, absorbs all radiation that falls on it. The best practical approximation to a black body is a small hole in a cavity with its interior surface covered with the blackest material available. If the entire cavity is maintained at a uniform temperature, then by the second law of thermodynamics, the radiation escaping from the hole must have almost exactly the properties of ideal black body radiation at that temperature. The failure of 19th century physics to explain the complete spectral distribution of black body radiation was the key problem whose solution by Planck in 1900 initiated the development of quantum mechanics. Substantial progress had already been made, however, in understanding the properties of black body radiation. For example, in 1879 Stefan observed that the power radiated per unit area by an ideal black body is proportional to the fourth power of the absolute temperature according to the relation

$$P = \sigma T^4 \tag{1}$$

where P is the emitted radiant power in $ergs\ cm^{-2}s^{-1}$, T is the absolute temperature in Kelvin, and σ is a constant. Five years later Boltzmann derived this relation, the Stefan-Boltzmann law, from Maxwell's equations of the electromagnetic field and the first and second laws of thermodynamics. His "classical" derivation could not predict the value of σ in terms of the more fundamental constants k (Boltzmann's Constant), c (The velocity of light), and h (Planck's Constant). Moreover, the spectrum of the radiated power, (i.e., the power emitted with frequency between ν and $\nu + \Delta\nu$), defied classical explanation. Planck found the mathematical formula that describes the spectrum and discovered that the formula could be derived on the assumption that the exchange of energy between matter and the electromagnetic field is quantized in units of $h\nu$, where h was a new physical constant. By integrating Planck's formula one can find the formula that expresses the Stefan constant σ in terms of fundamental constants. The formula yields $\sigma = 5.67x10^{-5} erg\ cm^{-2}s^{-1}K^{-4}$ In the present experiment electrical measurements with simple equipment will be used to determine the quantity $\frac{P}{T^4}$ for an incandescent tungsten filament from just above room temperature to over 3000 K, and thereby to test how well tungsten mimics the behavior of an ideal black body. The data analysis will provide an exercise in error propagation.

3 EXPERIMENT

3.1 Equipment

The circuit for the measurements is shown in Figure 1. The radiating element is the tungsten filament of a #47 6-volt pilot-light bulb. #47 bulbs, in contrast to household light bulbs, contain no filling gas, i.e. they are evacuated. Thus, except for a small amount of heat conducted away by the filament supports, the electrical energy supplied to the bulb is carried away by electromagnetic radiation.

The resistivity ρ of tungsten at various temperatures is tabulated in the Handbook of Chemistry and Physics where one finds it increases by a factor of ~ 20 from room temperature to 3000 K. The resistance R of a filament of cross sectional area A and length L is directly proportional to the resistivity, i.e., $R = \frac{\rho L}{A}$. The resistance of the filament can therefore be used as a measure of its temperature provided the ratio $\frac{L}{A}$ dosn't change significantly with temperature. A polynomial fit to the tabulated data, convenient in the later data analysis, is:

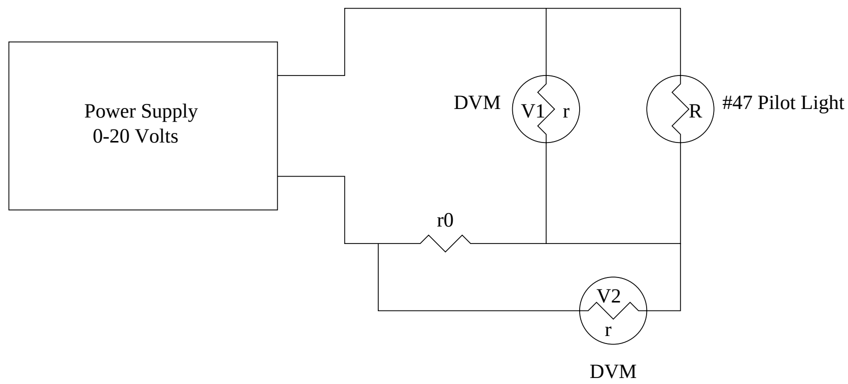

Figure 1: Circuit for measurement of the radiated power versus temperature in tungsten. The internal resistances of the DVMs is represented by r, the precision shunt resistance by r_0 (=1.00 Ω), and the filament resistance by R. Note that not all of the current in r_0 is in R.

$$T = a_0 + a_1\rho^{'} + a_2\rho^{'2} + a_3\rho^{'3} \tag{2}$$

where T is the temperature in K, $\frac{\rho}{\rho_{300}}$ is the ratio of the resistivity at temperature T to its value at 300 K, and the coefficients are: $a_0 = 103.898$, $a_1 = 214.93$, $a_2 = -2.9944$, $a_3 = 0.04328$.

The deviation of the tabulated values from the formula values is 5% at 300 K, no more than 1% from 400 to 700 K, and generally about 0.1 to 0.2% from 700 to 3655 K. If $\frac{L}{A}$ is constant, then $\frac{R}{R_{300}} = \frac{\rho}{\rho_{300}}$. The other equipment consists of a variable voltage, regulated power supply, and two digital voltmeters (DVMs), one to measure the voltage across the bulb, the other to measure the current through it. The current ranges of the small DVMs when used directly as ammeters extend only to 200 mA. Since a greater range is required, we make use of a scheme common in measuring currents, namely, a current shunt. In our case a precision 1%, 1-Ω resistor is connected in series with the bulb, and the voltage across it is measured with a DVM.

3.2 PROCEDURE

Connect the apparatus as shown in Figure 1, and explore its operation. At about 3 times the rated voltage of the pilot lamp (6 v) the filament will evaporate rapidly and quickly fail. You will see the deposited tungsten darken the bulb's interior. Clearly, at the higher voltages you will have to move quickly to get useful measurements before the physical characteristics of the filament change significantly. Begin your measurements of V_1 versus V_2 at very low voltages so as to get a room temperature reference value of the resistance. (Better yet, use the DVM as an ohmmeter to get a value of the resistance, $\frac{V}{I}$, at the highest sensitivity of the meter, i.e. at the lowest value of I that will activate the current-measuring circuit inside the DVM.) Take a series of measurements at, say, roughly 1 volt increments, up to about twice the rated voltage to get the hang of working quickly. Compute and plot the temperature versus the voltage from Figure 1 as you go along to see how well your points space out in temperature. Repeat each of your measurements several times to obtain a direct estimate of the random errors. When you are well organized you can do a series of measurements, perhaps a total of six series consisting of three series on each of two different bulbs. This way you will be able to make good estimates of the random and systematic errors.

4 ANALYSIS

The experiment will work well with 10 or 20 points on the V_1, V_2 curve. A key idea is that the resistivity, $\rho = \frac{AR}{L}$, of the tungsten is directly proportional to the resistance, R, of the filament, i.e., $\rho = \rho_{300}\frac{R}{R_{300}}$, provided that the ratio $\frac{A}{L}$ is constant (which it is to a reasonably good approximation). Try to explain the deviations of your data from your predicted plot of V_2 against V_1. Identify and assess the sources of error in your measurements. Distinguish between random and systematic errors. Check the DVMs for systematic errors against the precision voltage source which is available in the laboratory from any of the Technical Staff. Work out an error for $\sigma = \frac{P}{T^4}$ for each of your different voltages by propagating the errors of the measured quantities. If you find the value is approximately constant over a substantial portion of the temperature range, work out a global average for this range with an error estimate. Estimate the dimensions (length and cross section area) of the filament by cracking a bulb and examining the spiral filament under a microscope with some other object of known size in the field of view, e.g. a wire of known diameter. You can determine the length of the filament by stretching the spiral. Then, given your measured values of $\frac{P}{T^4}$ and the accepted value of Stefan's constant, you can estimate the emissivity of incandescent tungsten, i.e. the fraction which the emitted power is of the ideal black body value.

REFERENCES

P. R. Bevington and D. K. Robinson 1992, Data Reduction and Error Analysis for the Physical Sciences, 2nd Edition, McGraw Hill.

S. Gasiorowicz 1974, Quantum Physics, Wiley.

A. Melissinos, A. 1966, Experiments in Modern Physics, Academic Press.

Handbook of Physics and Chemistry, 65th Edition, CRC Press, p. E-381.

SUGGESTED THEORETICAL TOPICS FOR PRESENTATION AT THE ORAL REVIEW

1. Derivation of Stefan's constant σ in $P = \sigma T^4$ from the Planck formula for the spectrum of blackbody radiation.

2. Derivation of the T^4 dependence of the power emitted by a black body from the classical laws of thermodynamics.