Transmission and Attenuation of Electromagnetic Pulses

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(Dated: August 22, 2023)

The purpose of this experiment is to acquaint you with some of the principles involved in the manipulation of electrical pulses. You will study reflections and measure termination resistances and signal propagation velocities for different cables.

PREPARATORY PROBLEMS

1. Derive an expression for the characteristic impedance of a coaxial cable with a central wire of radius a, a cylindrical sheath of radius b, and with the space between the inner and outer conductors filled by a non-conducting medium with a dielectric constant κ. See for example References [1–3].

2. A certain transmission line attenuates pulses at a rate of 2.5% per meter. Derive an exact formula for the pulse amplitude as a function of distance along the cable. Make a plot of the amplitude of a pulse as a function of position along the transmission line from 0 to 200 meters. (The formula is the solution of an elementary differential equation.)

3. Draw the predicted shape and amplitude of an ideal rectangular pulse of amplitude -1 volt and duration 10 ns *after* it has traversed a coaxial cable 10 m long and returned following reflection from a shorted end.

I. INTRODUCTION

Many experiments involve the production and measurement of electrical pulses. Depending on the length of the signals Δt, the approaches are very different:

$\Delta t \geq 1$ ms: do whatever you want with your wires. If the signals are small, you may want to use shielded cables.

1 ms $\geq \Delta t \geq 0.1$ μs: typical for computers. Use ribbon cable, "twisted pair," or cables without termination.

100 ns $\geq \Delta t \geq 0.1$ ns: region of interest for "fast" signals to be studied here. Cables must be terminated or else undesirable reflections will occur. In Junior Lab, these types of signals are often present in photomultipliers, but subsequent amplifiers slow the signals into the middle category.

Given a signal source outputting a series of pulses, one may wish to measure the pulses' rate, distribution of amplitudes, occurrence times relative to other pulses, *etc.* Such measurements are done with oscilloscopes, multichannel analyzers, amplifiers, discriminators, coincidence circuits, *etc.*[1], which may or may not be working properly. It is essential, therefore, to gain facility in the use of test equipment such as pulse generators and oscilloscopes so that the performance of a pulse-measuring apparatus can be checked, point by point. Electrical pulses are piped around a laboratory via transmission lines of one sort or another, with consequent delays, attenuations, and reflections. It is important to understand these effects and to know how to measure them. This experiment is a study of pulses in transmission lines. You will learn that you must terminate all transmission lines properly so as not to be deceived by false signals.

II. EQUIPMENT

In Junior Lab, you will most often be connecting equipment using coaxial cables constructed according to a standard set of parameter specifications known as RG-58. For this experiment, you will also use RG-59 cable, which is similar in construction to RG-58, but has a larger outer diameter and therefor a different characteristic impedance. Coaxial cables can be connected to equipment and to each other via a variety of connector types. The most common connector for laboratory work is a type of "bayonet" connector known as BNC, which connects with a 90° push-twist motion. Connectors may be "male" or "female", named according to the disposition of the inner conductor. Usually, the male connector is on the cable, and the female connector is on the chassis of the equipment being connected. A wide variety of cable adapters are available, with differing numbers of male and female terminals to suit different tasks. For example, two cables could be connected by a female-to-female BNC connector. Three cables can be connected by an all-female "T connector". A more useful type of T connector — with two female and one male terminals — can be used to connect two cables to a single output or input terminal on a piece of equipment. There are

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[1] Oscilloscopes display the signals $V(t)$ *vs.* t as described in the Appendix. Multichannel analyzers (MCAs) sort signal events of different amplitudes into bins: that is, they make histograms. Amplifiers enlarge the signal but may also alter its shape. Charge sensitive pre-amplifiers produce an output pulse proportional to the total electric charge in the input pulse. Discriminators emit a logical (square $V(t)$) signal if the input exceeds a certain threshold. Coincidence units produce a logical pulse if two (or more) inputs overlap in a certain window of time.

also adapters which allow transitioning between different kinds of cable, such as BNC-to-"banana plug" for connecting the the two conductors of a coaxial cable to two individual wires.

III. EXPERIMENTS

III.1. Reflection of electrical pulses from discontinuities in a transmission line

Connect the output of a pulse generator to the input of a digital oscilloscope by means of a T connector and attach a long RG-58 cable to the third side of the T, as shown in Figure 1. Various terminating resistors will be connected to the far end of the long cable.

Use the pulse generator to produce pulses of amplitude about 5 V and the shortest possible duration (about 4–30 ns), creating as close as possible to a voltage "spike" of infinitesimal duration. Be careful to set the repetition rate slow enough that you are not confused by overlapping pulses. Set the oscilloscope controls so that you can observe the pulse produced by the pulse generator. Set the oscilloscope to whatever settings help you best visualize the signal of interest. For example, try setting the vertical scale at 2 V/division, the sweep speed (commonly called the time base) at 50 ns/division, and the trigger at internal, normal, and positive slope. Be sure to record the observed waveforms, including the settings used to view them, in your lab notebook.

Observe the primary pulse from the pulser and describe the pulse reflected from the end of the cable when the end is 1) open, 2) shorted, and 3) terminated with a variable resistor with values in the range from about 10–200 Ω. You will typically see one reflection[2]. Use the variable resistor to determine the characteristic impedance of RG-58 cable, and then repeat the above steps on RG-59 cable. Take enough statistically independent measurements to find a mean and a variance of each characteristic impedance. When measuring the resistance of the variable resistor with an ohmmeter, the resistor must be disconnected from the cable, or else you will be measuring the combined effective resistance of the entire circuit.

Connect another piece of cable to the end of the first cable by clip leads[3] (*i.e.* a very bad connector) and observe the effects of the discontinuity in the transmission line. **Remember to graph the observed waveforms in your notebook and label both time and amplitude axes!**
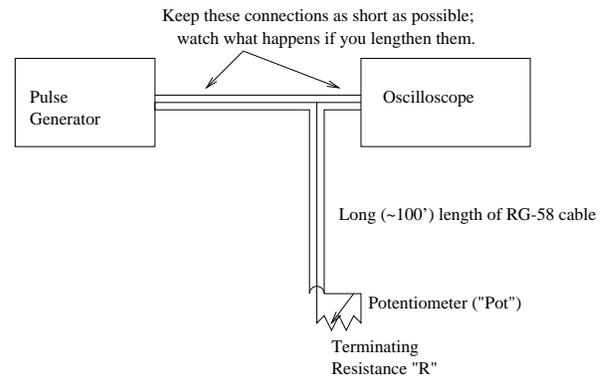

FIG. 1. Experimental setup to test the effect of various terminating resistances on the transmission and reflection of electrical pulses in coaxial cable.

III.2. Speed and attenuation of pulses in transmission lines.

Determine the velocity of pulses in RG-58 and RG-59 cable by measuring the difference in the arrival time of the direct and reflected pulses at the oscilloscope. Record sufficient data and other information to permit an accurate assessment of the random and systematic errors in your determinations. Note that each pulse measured by a sweep of the oscilloscope is, in principle, a new independent measurement. Compare the velocity in the cable to the velocity of light in vacuum, and explain the cause of the difference. Add at least three cables of known length to make a plot of time versus length.

Measure the attenuation of RG-58 and RG-59 cable by comparing the amplitude of the pulse reflected from the open end of the cable with and without an additional length joined by a BNC connector[4]. Note that this strategy isolates the effect of the delay in the cable from possible complicating effects of the discontinuities in the circuit at the connections to the oscilloscope and pulser.

III.3. Propagation of CW in a transmission line

Utilizing a function generator[5], explore the phenomena of a sinusoidal continuous wave (CW) propagating in a transmission line with various terminations and frequencies. Use two channels on the oscilloscope to measure the

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[2] Why not $n + 1$? Hint: Pulse generators have an internal resistance, too (50 Ω).

[3] Clip leads are also known as "alligator clips".

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[4] Also observe how the shape of the pulse changes.

[5] A function generator is a voltage source which can produce periodic voltage signals of various kinds, such as sinusoids, square waves, triangle waves, sawtooth waves, and others. Since a pulse generator outputs a periodic voltage signal (a short pulse followed by a time delay of constant voltage), it could be thought of as just a special case of function generator, but in practice pulse generators are built to higher tolerances for producing sharp voltages steps than function generators of comparable cost.

wave amplitudes V_{input} and V_{output}. Be sure the output is terminated. Why is this necessary?

Produce a graph of attenuation versus frequency for RG-58 and RG-59 cable. Obtain attenuation values for frequencies ranging from 1 Hz to 10 MHz. Think about how this frequency dependence for CW signals explains the change in shape of pulsed signals observed in the previous section.

The function generator should default to a 1 kHz sine wave with an amplitude of 100 mV when you turn it on. Make sure you are using the OUTPUT terminal and not the SYNC terminal. To adjust the frequency, first press the Freq button. The current frequency will then be displayed. The flashing digit can be adjusted with the knob in the upper right hand corner. You can also enter the desired frequency by pressing Enter Number, entering the numeric value of the frequency and then pressing the the button for the appropriate units. For instance, you can hit the ∧ button for MHz.

A common means of expressing attenuation is in dB/100 ft where $dB = 20 \log_{10} (V_{out}/V_{in})$ when the cable termination is matched to its characteristic impedance. (*E.g.* -3 dB corresponds to a voltage ratio of 0.707.)

IV. ANALYSIS

1. Determine the characteristic impedance Z for RG-58 and for RG-59 coaxial cables. Compare your values versus published values. Assess the random and systematic errors.

2. Determine V_{prop} for RG-58 and RG-59 coaxial cables. Compare your values versus published values. Assess the random and systematic errors.

3. Determine the attenuation in dB/m for RG-58 and RG-59 coaxial cables. Compare your coefficients versus published values. Assess the random and systematic errors.

Tables of standard values for the properties of RG-58 and RG-59 cable are readily available from any number of references. The websites of manufacturers are an especially good place to check for equipment specifications.

V. POSSIBLE TOPICS FOR ORAL EXAM

1. The step function voltage on an open cable.

2. The partial differential equations for the voltage between the inner and outer elements of a coaxial transmission line carrying a signal.

3. The fraction of the energy of a pulse reflected at the junction of two coaxial cables with different radii of the inner and outer conductors.

4. Estimate the attenuation at low frequencies (Ohm's law). Why does the attenuation grow rapidly at very high frequencies?

VI. STATISTICAL EXERCISE

1. With the setup described in Section III.1, obtain 25 independent (explain how you ensure their independence) measurements of the characteristic impedance for RG-58 and RG-59. Plot the two distributions and calculate their mean and variance.

2. Measure and tabulate the time delay of signals in RG-58 in at least 4 different cable lengths. Again, make sufficient measurements to obtain an estimate of the (random) error for the next step.

3. Plot time delay Δt *vs.* cable length ℓ for RG-58. Prove a linear relationship by fitting your data to the model

$$\Delta t = m\ell + t_0 \tag{1}$$

using linear curve fitting techniques and examining the resulting χ^2 value. Quote your best-fit values and uncertainties for the slope m and offset t_0 parameters.

4. Determine the value and uncertainty of the signal velocity in each cable by propagating the uncertainty from your values for the slope m.

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[1] A. French, *Vibrations and Waves* (Norton, 1971)
[2] G. Bekefi and A. Barrett, *Electromagnetic Vibrations, Waves and Radiation* (MIT Press, 1977)
[3] W. Leo, *Techniques of Nuclear and Particle Experiments* (Springer, 1992)

Appendix A: Visualization of Electrical Signals by Oscilloscopes

A schematic of an analog oscilloscope is shown in Figure 2. A hot wire in the cathode ray tube emits electrons, which are accelerated by an annular anode by about 20 kV. Two pairs of crossed capacitors then deflect the electron beam (AA' horizontally, BB' vertically). The fluorescent screen emits green light when hit by the electron beam. By applying a linearly rising voltage to AA',

a horizon line is produced on the screen (aa'). The (amplified or attenuated) signal applied to BB' produces a vertical deflection bb' for the time duration of $\Delta\tau$. This process is started by the trigger, which is initiated by a signal becoming larger than the threshold (adjusted by a knob control), which can be positive or negative.
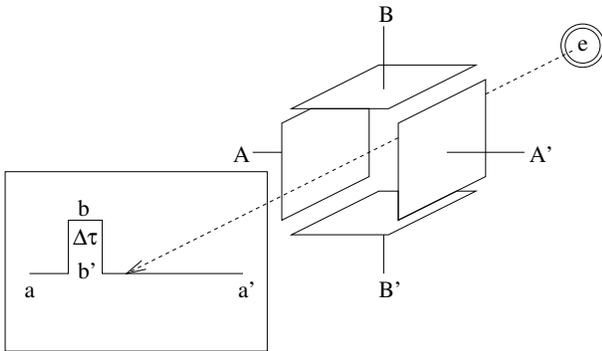

FIG. 2. Schematic of how an analog oscilloscope works.

Modern digital oscilloscopes utilize a similar principle, but operate differently. They sample the incoming voltage periodically (after receiving a trigger event). Then, a computer is used to display the resulting data as a function of time at various user-selectable voltage and time-scales. You may be using such a modern oscilloscope in this experiment. More information about the digital oscilloscopes used in Junior Lab is available in the online e-library.

Appendix B: Equipment List

Manufacturer	Description	URL
HP-Agilent	100 MHz Scope	agilent.com
BNC	8010 Pulse Generator	berkeleynucleonics.com
Various	100' RG/58U Cable	
Various	100' RG/59U Cable	