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Massachusetts Institute of Technology
Physics Department

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Junior Physics Laboratory Experiment #2

Millikan Oil Drop Experiment

1 PURPOSE

The purpose of this experiment is to measure the charge of the electron.

2 PREPARATORY QUESTIONS

The Millikan apparatus in Junior Lab has two parallel plates separated by approximately 1 cm and a high-voltage power supply. Each time you start a new observation with zero field and a squirt from the atomizer you will see a myriad of droplets falling through the field of view. Your problem will be to pick a droplet that is of a size such that, if it carries a charge of a few electrons (e.g. 1–3e), you will be able to pull it upward with the available electric force. To judge which droplet to pick you must estimate the terminal fall velocity and holding voltage of a suitable droplet. The velocity of a droplet is determined from a measurement of the time it takes it to fall the distance between the levels corresponding to the two fiducial lines in the eyepiece reticle. That distance is approximately 0.3 cm.

Problem: Calculate the time of fall of a droplet which, if it carries a charge of 1e, can be held stationary with a voltage of 2000 volts. (According to the CRC tables, the viscosity of air at 18°C at a pressure of 1 atm is 182.7 micropoises.)

What is the ratio of the number of electrons to the number of protons in the droplet of problem 1?

Describe another completely independent method for determining the value of the electron charge.

Plot the expected value of ${e'}^{2/3}$ against $1/r'$ (quantities defined below).

N.B. You can obtain results of impressive accuracy in this experiment provided you take care in reducing random errors of measurement. The most important thing of all is to select appropriately sized droplets carrying very few elementary charges—1e to 3e or 4e. Make a preliminary analysis of the data for each droplet immediately after you obtain it so that you can perfect your judgement as to which droplets to select and what voltages to use. The timing measurements are like a video game in which practice makes perfect. N repetitions of any given measurement will reduce the random error of the mean in proportion to $N^{1/2}$. The most important source of systematic error is probably the voltage measurements. To estimate the error you should check the voltmeter against other similar ones and against the electrostatic voltmeter.

A video system can be used to display the moving droplets on a TV monitor. Some experimenters may find it easier to get accurate data watching the video display than by observing directly through the telescope.

3 INTRODUCTION

Greek philosophers debated whether matter is infinitely divisible or is, instead, composed of discrete atoms (in Greek atomos meaning "uncuttable") as taught by Democritus in the fifth century B.C. During the nineteenth century A.D. the atomic idea was the basis for various successful theories of chemistry and physics, most notably the theory of chemical combination (Dalton) and the kinetic theory of gases (Maxwell). Yet there remained competent scientists who doubted the reality of atoms as late as 1900. They followed Ernst Mach (1836–1916) in strict adherence to the philosophical principle of empiricism whereby physical theory must exclude any concept that is not directly observable. In fact, it was not until the present century that the atomicity of matter or of any of its attributes (e.g. charge) was directly observed. After all, one atom of lead weighs only 3.2×10^{-22} g; one electronic charge is $4.8 \cdot 10^{-10}$ statcoulombs; the quantum of angular momentum is $6.7 \cdot 10^{-27}$ cm^2 s^{-1}; and the energy of one quantum of red light is $3.2 \cdot 10^{-12}$ ergs. Whereas the units of these measures are physical quantities of palpable magnitude, readily measured with the instruments invented by nineteenth century physicists, the atomic quantities are such minute fractions of their units of measure as to have defied measurement by the best experimentalists of that century.

Then, in 1907, Robert Millikan's young collaborator, Harvey Fletcher, got the idea of watching the motion of a single, charged, microscopic oil droplet under the influence of gravity and a uniform electric field between two parallel metal plates. He produced microscopic droplets with an "atomizer", like a common nasal spray, illuminated the droplets from behind, observed individual droplets as unresolved pinpoints of diffracted light, and timed their motion between fiducial marks in the focal plane of a horizontal microscope as they switched the voltage between the plates on and off. Some droplets carried a few more electrons than protons, or vice versa, and could be suspended or drawn upward against the force of gravity by application of an electric field of the order of a thousand volts per cm. Droplets were tracked up and down many times, sometimes for hours, to reduce random errors in the measurements of the terminal velocities of the motions under the forces of gravity, viscous drag, and electricity in the presence of the fluctuations of Brownian motion.

The oil drop experiment proved the atomicity of electricity (i.e. that the droplets always carried integer multiples of a charge quantum, e), determined the value of e with an estimated error of ±0.2%, provided thereby the key to the accurate determination of the mass of the electron and Avagadro's number (from the previously measured values of e/m and $F = Ne$), and won Millikan the Nobel Prize. It turned out that his result was off by a systematic error of -0.6% due to an error in the viscosity of air which was corrected in 1930. The uncertainty of the currently accepted value is $\pm 0.0003\%$ (3 ppm).

For an interesting presentation of the physics and history of the discoveries of the electron and other fundamental particles, see Weinberg (1983).

4 THEORY OF THE EXPERIMENT

Consider a spherical oil droplet of radius r and density d falling at a terminal velocity u_d through air of density d_a and viscosity coefficient η. If r is large compared to the mean free path of the air molecules, then the gravity and buoyancy forces acting on the droplet are balanced by a drag force $6\pi\eta r u_d$ according to Stoke's law for the streamline motion of a sphere through a viscous medium. Actually, the experiment must employ oil droplets which are light enough to be suspended or drawn upward by the electric force exerted on just a few (1 to $\sim$10) electronic charges by a field of a thousand volts/cm. Such droplets have radii that are typically not very large compared to the mean free path of air molecules, which is $2.2 \cdot 10^{-6}$ cm at normal temperature and pressure according to the CRC Tables on page F–49. In this case the expression for the drag force is diminished by factor $(1 - a/r)$, where a is a constant of the order of the mean free path. Thus during free fall the equation representing the balance of forces is

$$\frac{4}{3}\pi r^3 \left(d - d_a\right) g = 6\pi\eta r u_d \left(1 - a/r\right) \tag{1}$$

where g is the acceleration of gravity. The term in brackets on the right hand side of equation (1) approaches unity as the radius becomes large compared to a. It turns out that the radii of the typical droplets used in this experiment are large enough so that $a/r \ll 1$. Thus, to high accuracy, we can replace r in the correction term by

$$r' = \left[\frac{9\eta u_d}{2\left(d - d_a\right) g}\right]^{1/2}. \tag{2}$$

Then,

$$r = r'(1 - a/r')^{1/2}. \tag{3}$$

If the droplet carries a charge ne and is moving upward with terminal velocity u_u under the influence of an electric field V/s between two parallel plates separated by the distance s and a potential difference V, the equation of motion is

$$\frac{4}{3}\pi r^3 (d - d_a) g - Vne/s + 6\pi\eta r u_u (1 - a/r') = 0. \tag{4}$$

Subtracting equation (1) from equation (4) and solving for ne, we obtain

$$ne = \frac{6\pi\eta r' s}{V}\left(u_d + u_u\right)\left(1 - a/r'\right)^{3/2}. \tag{5}$$

If we knew the value of a, the factor in Stoke's law that corrects for the effects of the granularity of air, then for each droplet we could find the value of ne and seek the greatest common divisor of the set of values, which would be the likely value of e. The problem is how to determine a. For this purpose we define the quantity

$$\begin{aligned} e' &= \frac{6\pi\eta r' s}{nV}\left(u_d + u_u\right) \\ &= \frac{6\pi\eta s}{nV}\cdot\left[\frac{9\eta u_d}{2\left(d-d_a\right)g}\right]^{1/2}\cdot\left(u_d+u_u\right). \end{aligned} \tag{6}$$

Substituting into equation (5) and rearranging we obtain

$$(e')^{2/3} = e^{2/3}(1 + a/r'). \tag{7}$$

Many measurements of ne' for drops of various radii will yield a collection of values; presumably, each is close to an integer multiple of the fundamental unit of charge. When the numbers of unit charges involved in each of the measurements has been figured out, then each measurement, divided by the proper integer number, yields a value of e'. A plot of $(e')^{2/3}$ against $1/r'$ should show data points clustered around a line with a slope equal to $ae^{2/3}$ and an intercept on the $1/r' = 0$ axis (corresponding to $r \rightarrow \infty$) equal to the best estimate of $e^{2/3}$ from which e can be determined.

5 EXPERIMENT

A schematic diagram of the apparatus is shown in Figure 1.

Adjust the eyepiece so the reticle is in sharp focus for your eye. Remove the cover of the Millikan apparatus, unscrew the high voltage connector from the top plate and lift the top plate out of the assembly by the handle of the shutter that covers the tiny hole in the center.

Measure the distance in the object plane of the telescope corresponding to the separation of the top and bottom lines of the reticle located in the eyepiece at the image plane of the telescope objective lens. For this you can use the special and expensive glass slide ruled with 10 lines/mm and mounted in a plastic stand. Place it on the bottom plate at the object plane, focus the telescope on it, and then count the number of grooves. Check the calibration by measuring the distance between the grooves with the traveling microscope in room 4-310.

Measure the diameters of the spacer balls with the micrometer provided.

Position the three balls in a triangular array on the bottom plate and reassemble the top plate. (Don't screw the high voltage terminal so tight as to lift the top plate off of the ball supports.) Level the plates by the bubble level.

Stick a fine wire through the hole, illuminate it with the collimated light, and focus the telescope on it by eye. Replace your eye by the video camera. Remove the wire and put the cover on.

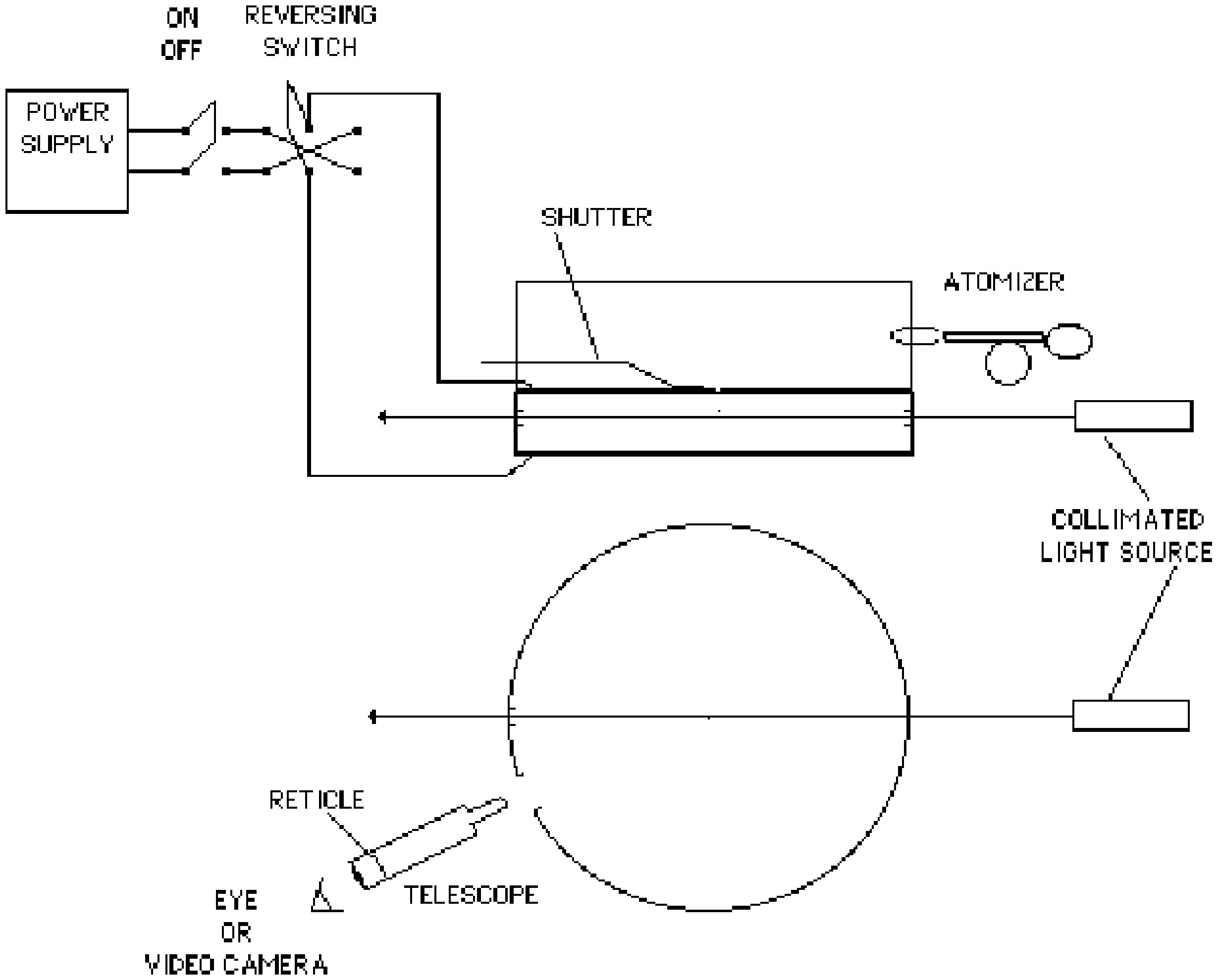

Figure 1: Schematic side and top view of the Millikan Oil Drop apparatus.

Spray droplets of oil from the atomizer through the port into the top chamber (take it easy—one small squirt is generally sufficient after you have primed the atomizer with a few squirts into a towel). Close the port. As the droplets drift down some pass through the tiny hole into the fiducial volume where they are illuminated by a collimated light as illustrated. When you see a droplet through the microscope as an unresolved point of diffracted light drifting slowly downward, attempt to arrest the droplet's fall by applying a voltage across the parallel plates. If the droplet moves downward more slowly under the influence of the electric force, then clearly the droplet is charged and an increase in voltage may arrest or reverse its motion. If the droplet accelerates downward under the action of electric force, the voltage should be reversed. If the electric field has no effect, then the droplet is not charged at all. Try another droplet. When you catch one that works, i.e. it drifts slowly downward (∼15 s fall time indicates the drop has about the optimum weight) with no voltage and can be pulled up by applying a voltage (∼1800 V indicates that there are only a few charge quanta on the drop) of one or the other sign, close the shutter and go to work.

Using as your race track the gap between two horizontal lines of the reticle in the focal plane of the telescope, measure the free-fall times with the voltage off and the rise times with the voltage on for one droplet. Repeat as many times as possible or as long as you have patience. When you think you have your first really good set of repeatable data for one droplet, stop and analyze it, and derive the value of the charge on the droplet. If everything seems reasonable and your value is close to a small (∼1–5) integer multiple of the known value of e, proceed to get data on more drops, working with each one as long as possible, and analyzing the data after each droplet is finished. Try to observe several with the shortest free-fall time (largest radius) you can measure accurately in order to have a good basis for extrapolating your values of e' to $r' \to \infty$.

Air currents can be a problem in this experiment. Take care that the chamber is well sealed, and reduce as much as possible any movement of air in the room.

If you are working alone you may find the following procedure more convenient:

- Pull the droplet above the top line by adjusting the voltage.
- Switch the voltage to zero and measure the time to fall from the top to the bottom line of the reticle. Keep your left hand on the timer controls and your right hand on the voltage controls
- After the droplet has passed the bottom line arrest the downward motion by switching on the voltage.
- Read and record the fall time, taking care that you will be able to identify the droplet after you have looked away for a moment to read and record the clock.
- Measure and record the voltage that renders the droplet exactly stationary ($u_u = 0$).
- Pull the droplet up and above the top line.

Repeat the sequence many times to reduce the random errors of the time and voltage measurements.

Tabulate your data in a format that will allow you to reduce it in an orderly fashion in adjacent columns. Be sure to record the barometric pressure and the temperature for each session.

You will find that it takes a considerable amount of practice to achieve high accuracy in this experiment. Both members of a team should perfect their skill at making all the various measurements. The more droplets you measure and the more data you accumulate on each droplet, the more accurate will be your final result.

6 ANALYSIS

The density of the oil was measured and found to be 1.07 ± 0.03 g/cm^3.

For each droplet find the value of r' and e' and the errors. Plot $(e')^{2/3}$ against $1/r'$. Determine the slope $(ae^{2/3})$, intercept $(e^{2/3})$ and errors by linear regression. The program `LINFIT` on the Junior Lab Macintosh in the folder labeled `Millikan` can be used for this purpose. From these results compute a and e and the errors. Take special care in understanding and evaluating the random and systematic errors.

REFERENCES

[1] P. R. Bevington & D. K. Robinson 1992, *Data Reduction and Error Analysis for the Physical Sciences*, 2nd Edition, McGraw Hill.

[2] R. A. Millikan 1911, "On the Elementary Electrical Charge and the Avagadro Constant." Phys. Rev. 32, 349.

[3] S. Weinberg 1983, "The Discovery of Subatomic Particles." Scientific American Books, New York.

SUGGESTED THEORETICAL TOPICS

1. Viscosity, and how it can be measured.
2. Stoke's law, and how you could verify it.
3. The equality of the magnitude of the charge of the electron and the charge of the proton, and how it can be checked.
4. Quarks have charges of $\pm(1/3)e$ and $\pm(2/3)e$. What is the explanation for the fact that such charges have never been observed?