

**Massachusetts Institute of Technology
Physics Department**

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Junior Physics Laboratory: Experiment #8

THE FARADAY EFFECT

PURPOSE

The purpose of this experiment is to observe the effect of a magnetic field on the transmission of linearly polarized light through a dispersive medium, to measure the Verdet constant of dense flint glass at several wavelengths, and to test the validity of the classical theory of magnetic circular birefringence, known as the Faraday Effect.

PREPARATORY QUESTIONS

1. Describe the following phenomena:
 - linear birefringence
 - optical activity
 - magnetic circular birefringence
2. Derive Becquerel's expression for the Verdet constant.
3. Define the angle of minimum deviation of a prism and derive the relation between it and the index of refraction.
4. The material used in this experiment to display the Faraday effect is an expensive piece of heavy flint glass in the form of a long prism. What property of this material makes it specially useful for demonstrating the Faraday effect?
5. Plot the expected relation between the rotation angle and the wavelength for the Faraday effect in a substance with an optical resonance at 2000 \AA .

QUANTITIES TO BE MEASURED OR CALCULATED IN THIS EXPERIMENT.

1. The angle and sense (clockwise or counterclockwise relative to the direction of propagation and the magnetic field) by which the plane of polarization of linearly polarized light is rotated in traversing a measured thickness of flint glass under the influence of a magnetic field, for several wavelengths and several field strengths.
2. The Verdet constant $V = \psi / BD$ for each of the several wavelengths, from the data obtained in 1.

3. The index of refraction of flint glass for each of the five bright lines of the emission spectrum of mercury vapor.
4. The constants of the formula which expresses the index of refraction as a function of wavelength, from the data obtained in 3.
5. The quantity $dn/d\lambda$ at each of the several wavelengths for which the Verdet constant has been determined.
6. The effective ratio of charge to mass of the particles with which light interacts in the flint glass for several wavelengths.

INTRODUCTION

This experiment is concerned with the measurement and interpretation of the Faraday effect - magnetically induced circular birefringence. The experiment reveals a fundamental connection between optics and electromagnetism, and leads to an evaluation of the effective ratio of charge to mass of the particles in flint glass with which light most strongly interacts, i.e., electrons.

Any transparent substance whose molecules are not mirror symmetric and which is not a racemic mixture of its enantiomorphous stereoisomers is 'optically active' to some degree, i.e. it exhibits the phenomenon of circular birefringence whereby the plane of polarization of a beam of linearly polarized light passing through the substance is rotated by an amount proportional to the distance traveled. Dextrose (otherwise known as *d*-glucose, one of the 16 stereoisomers of pentahydroxyaldehyde) and *d*-tartaric acid (used in soft drinks) are nutritious examples of optically active substances. Pure or in solution, they both rotate the plane of polarization counterclockwise relative to the direction of propagation; their mirror images do the opposite and, incidentally, are not nutritious. Some crystals, e.g. natural quartz, exhibit both optical activity and birefringence.

In 1845 Faraday (1846) discovered that a magnetic field in the direction of propagation of a light beam in a transparent medium produces the effects of circular birefringence. Thus he established for the first time a direct connection between optics and electromagnetism - a connection he had long suspected to exist and for which he had searched for many years. However, a half century passed before an explanation of the Faraday effect was formulated in terms of Maxwell's electromagnetic theory of light (developed in the 1860's) and the concept of the atomicity of charge which had evolved during the 19th century under the influence of electrochemistry and been validated by Thomson's discovery of the electron in 1897.

CLASSICAL THEORY OF THE FARADAY EFFECT

Representation of Linearly Polarized Light as a Superposition of Circularly Polarized Components

The electric vector \mathbf{E} of a plane wave of linearly polarized light of frequency ω traveling in the +X direction with its polarization in the Y direction can be represented by the expression

$$\mathbf{E}(x,t) = \mathbf{E}_r + \mathbf{E}_l, \quad (1a)$$

where

$$\mathbf{E}_r(x,t) = \{0, (A/2)\cos[\omega(t - nx/c)], (A/2)\sin[\omega(t - nx/c)]\} \quad (1b)$$

and

$$\mathbf{E}_l(x,t) = \{0, (A/2)\cos[\omega(t - nx/c)], -(A/2)\sin[\omega(t - nx/c)]\} \quad (1c)$$

represent the electric vectors in plane waves of right and left circularly polarized light, respectively. (Note that the sum of the z components is zero.) The amplitude of \mathbf{E} is A , and the velocity of propagation is c/n where n is the index of refraction. Thus a beam of linearly polarized light can be considered as a linear superposition of right and left circularly polarized components.

Under certain circumstances the velocities of right and left circularly polarized light may be different, as in *d*-glucose or in a substance in which there is a magnetic field. In such a circumstance one can calculate separately the propagation of the two components and then recombine them to obtain the electric vector of the resultant linearly polarized wave. The net effect, as we now show, is a rotation of the plane of polarization.

To indicate a difference in velocities for right and left circularly polarized waves we replace the index n in equations (1b) and (1c) by n_r and n_l , respectively. As illustrated in Figure 1, the electric vector on the y-z plane at $x=D$ is then

$$\mathbf{E}(D,t) = \mathbf{E}_r(D,t) + \mathbf{E}_l(D,t). \quad (2a)$$

Using the identities

$$\cos(\alpha) + \cos(\beta) = 2 \cos(\alpha/2 + \beta/2) \cos(\alpha/2 - \beta/2),$$

and

$$\sin(\alpha) - \sin(\beta) = 2 \cos(\alpha/2 + \beta/2) \sin(\alpha/2 - \beta/2),$$

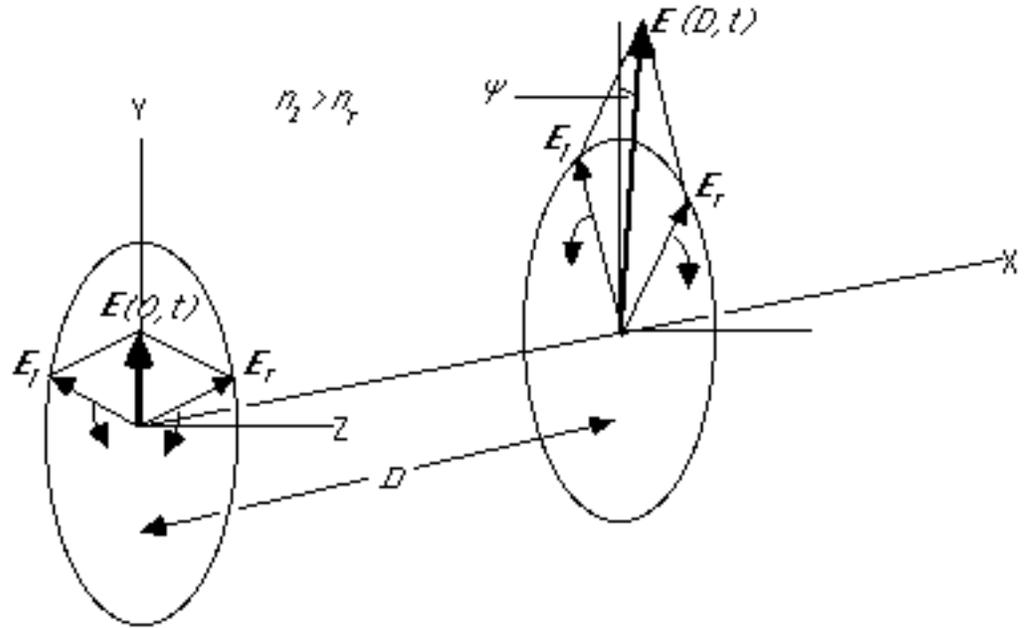


Figure 1. Addition of right and left circularly polarized components on two planes defined by $x=0$ and $x=D$, respectively. Because $n_l > n_r$, E_l is more retarded in phase at $x=D$ relative to $x=0$ than E_r , with the result that their sum at $x=D$ is rotated clockwise by the angle $\psi = \pi(n_l - n_r)D/\lambda$.

we find for the electric field at $x = D$ the expression

$$\begin{aligned} E(D, t) = \{ & 0, A \cos[\omega(t - (n_r + n_l)D/2c)] \cos[\omega(n_l - n_r)D/2c], \\ & A \cos[\omega(t - (n_r + n_l)D/2c)] \sin[\omega(n_l - n_r)D/2c] \}. \end{aligned} \quad (2b)$$

This represents an oscillating electric field of amplitude A inclined with respect to the y -axis by an angle

$$\psi = \tan^{-1} \left(\frac{E_z}{E_y} \right) = \frac{\omega (n_l - n_r) D}{2c} = \frac{\pi (n_l - n_r) D}{\lambda}, \quad (3)$$

where λ is the wavelength of the light in vacuum. Thus circular birefringence can be explained as an effect of a difference in the propagation velocities of right and left circularly polarized light. The difference is generally very small, amounting to only a few parts per million in optically active materials as well as in the Faraday effect at moderate magnetic field strengths. However, in a

typical setup $D/\lambda \gg 1$ so that even minute differences between n_l and n_r yield values of ψ which are large enough to be measured accurately.

MAGNETIC CIRCULAR BIREFRINGENCE – BECQUEREL'S THEORY OF THE FARADAY EFFECT

Experiments established early on that the Faraday rotation angle ψ is proportional to the product of D and the magnetic field B . Thus

$$\psi = VDB, \quad (4)$$

where V is a proportionality factor called the Verdet constant. The question remained as to how one might calculate the value of V from the fundamental physics of light, magnetism, and matter.

Up to this point in our development we have only used the two ideas; 1) light is a linearly superposable transverse wave with velocity c/n , and 2) $n_r - n_l$. The first step in understanding the Faraday effect in terms of electromagnetic theory and the atomic structure of matter was taken by H. Becquerel, in 1897. His theory was based on the concept of the newly discovered electron, but it did not include the idea of quantized energy states introduced by Planck in 1900, or the concept of light quanta, developed by Einstein in 1905. Even though the Becquerel theory does not make use of quantum principles, it does provide a simple conceptual framework for a preliminary understanding of the phenomenon, and yields quantitative predictions for the Verdet constant that are remarkable close to the measured values.

The next substantial steps beyond this theory were not taken until the early 1930's, but then only in gases (Serber, 1932; Van Vleck, 1932). A large amount of theoretical and experimental work has been done on the Faraday effect in solids since 1960, because of its application to semiconductors and the construction of microwave switching devices. Recent theoretical treatments are very complex, as a perusal of reviews (e.g. Mavroides 1972) will show.

Incidentally, the theory of the Faraday effect in ionized gas, where the active agents are free electrons, is not so complicated. The effect is important in radio astronomy because it provides a measure of the average value of the product $B_r n_e$ along the line of sight to a source of linearly polarized radio emission, where B_r is the component of the magnetic field along the line of sight and n_e is the free electron density (typically $B_r \sim 10^{-6}$ gauss, $n_e \sim 0.1 \text{ cm}^{-3}$).

Following Becquerel and subsequent presentations of his theory (see e.g. Rossi 1957), we assume that a transparent material contains particles of mass m and charge q embedded in a continuum of opposite charge and restrained by elastic force to vibrate about fixed sites. We consider the situation in which there is a steady magnetic field \mathbf{B} of magnitude B in the +X direction and a plane right circularly polarized electromagnetic wave which, at a given point,

produces a rapidly varying electric field \mathbf{E} of constant magnitude E rotating clockwise with respect to \mathbf{B} in a plane perpendicular to \mathbf{B} . In the steady state the charged particles move in a circle of radius r governed by the equation (cgs units)

$$-m\omega^2 r = -kr + Eq + Bq\omega r/c, \quad (5)$$

where k is the "spring constant" of the restraining force. For left circularly polarized light the sign of the magnetic term is reversed. Equation (5) is readily solved to obtain for r the formula

$$r = (Eq/m)/(\omega_0^2 - \omega^2 - Bq\omega/mc), \quad (6)$$

where

$$\omega_0 = (k/m)^{1/2}.$$

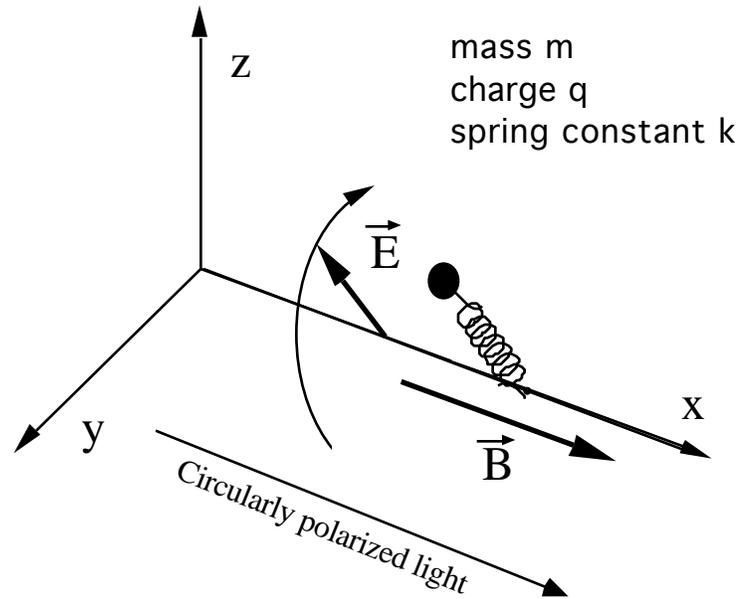


Figure 2. The particle, with mass m and charge q , is fixed to the end of a spring, with spring constant k , attached rigidly to the x axis. The directions of light propagation and the \mathbf{B} field are parallel to the x axis; the \mathbf{E} field is perpendicular to the x axis and rotates in the yz plane.

Displacement of a charge q from its oppositely charged equilibrium site by the distance r creates an electric dipole of magnitude qr . If there are N such dipoles per unit volume, then the polarization \mathbf{P} has the magnitude Nqr , and the permittivity is $\epsilon = (1 + 4\pi\mathbf{P}/\mathbf{E})$. Therefore, in a dielectric where the permeability $\mu=1$ and the propagation velocity $v=c/\epsilon^{1/2}$ and in the presence of a magnetic field, we obtain for the index of refraction for right circularly polarized light the expression

$$n_r(\omega) = c/v = [1 + (4\pi Nq^2/m)/(\omega_0^2 - \omega^2 - B\omega q/mc)]^{1/2} \quad (7a)$$

and for left circularly polarized light

$$n_l(\omega) = c/v = [1 + (4\pi Nq^2/m)/(\omega_0^2 - \omega^2 + B\omega q/mc)]^{1/2} \quad (7b)$$

Equations (7) show that the indices "blow up" as the frequency of the light approaches a value such that $\omega_0^2 - \omega^2 \pm B\omega q/mc = 0$. (One can avoid this unphysical consequence of our simple assumptions by adding to the equation of motion a damping term representing a drag force proportional to the velocity of the charged particle. Such a term can give a classical account of the phenomenon of absorption. Both the indices of refraction and the absorption coefficient have maxima near ω_0 . Fortunately, omission of a damping term has very little effect on the accuracy of the theory at frequencies far from ω_0 , as in the present case.) The sign of the effect of the magnetic field on the indices of refraction for circularly polarized light depends on the sign of the product $B\omega q$. In particular, for right circular polarization with \mathbf{B} in the direction of propagation ($B_x > 0$) and negatively charged particles, $B\omega q < 0$, which implies a decrease in n_r and a corresponding increase in n_l . Therefore, under these assumptions $(n_l - n_r) > 0$ and, according to equation (3), $\psi > 0$.

The change of n_r caused by turning on a magnetic field B is equal to the change of n_r when the frequency of the light is changed from ω to $\omega + \Delta\omega_r$, where $\Delta\omega_r$ is defined by the quadratic equation

$$\omega^2 + B\omega q/mc = (\omega + \Delta\omega_r)^2 .$$

The solution of this equation is

$$\Delta\omega_r = Bq/2mc. \quad (8a)$$

Similarly,

$$\omega^2 - B\omega q/mc = (\omega + \Delta\omega_l)^2$$

and

$$\Delta\omega_l = -Bq/2mc. \quad (8b)$$

These $\Delta\omega$'s are very small compared to ω . Thus the difference in the indices of refraction can be written with high accuracy as

$$n_l - n_r = (dn/d\omega)(\Delta\omega_l - \Delta\omega_r) = (dn/d\lambda)(\lambda^2/2\pi c)(Bq/mc), \quad (9)$$

where $\lambda=2\pi c/\omega$ is the vacuum wavelength of the light. Combining equations (3) and (9) we obtain for the angle of Faraday rotation

$$\psi = \frac{dn}{d\lambda} \frac{\lambda}{2c^2} \frac{q}{m} DB. \quad (10)$$

Thus the Verdet constant is related to the constants of the interacting particles by the formula

$$V = \frac{dn}{d\lambda} \frac{\lambda}{2c^2} \frac{q}{m}. \quad (11)$$

Equation (11) is the formula derived by H. Becquerel. It was characterized by Van Vleck (1932) as "rather too simple". Indeed, the quantum treatment of the problem in even the simplest of materials is complicated, and the results depend critically on the nature of the medium, i.e. whether it is dielectric, semiconducting, diamagnetic, paramagnetic, ferromagnetic, etc. Of course this means that measurements of the Faraday effect can be of value in the study of the electronic structure of matter provided an adequate theoretical framework for its interpretation is available.

We will bury the complications by defining a constant C such that equation (11) is written as

$$V = - \frac{dn}{d\lambda} \frac{\lambda}{2c^2} C \frac{e}{m_e}, \quad (12)$$

where $-e$ and m_e are the charge and mass of the free electron. It is reassuring that for visible light in ordinary molecular hydrogen gas, H_2 , the value of C is 0.99. For other substances and other spectral bands the values of C can be substantially different from unity and, in some cases, even negative. In the case of the interstellar medium where the effect is due to the presence of free electrons one has $\omega_0=0$ and $C=1$.

To get a rough idea of what to expect in this experiment we estimate the angle of rotation under the following set of assumptions:

- a) Suppose $n=2.0$ at $\lambda = 5000 \text{ \AA}$, and the peak absorption is at $\lambda_0=2000\text{\AA}$.
- b) $B=2000$ gauss and $D=10$ cm.
- c) $C=1.0$.

For convenience we rewrite equation (7) to express the dependence of the index of refraction on wavelength ($B=0$) and obtain the formula

$$n = \left\{ 1 + \frac{K}{1 - (\lambda_0/\lambda)^2} \right\}^{1/2} \quad (13)$$

where $\lambda_0 = 2\pi c/\omega_0$ and $K = 4\pi Nq^2/m\omega_0^2$. Assumption a) implies $K = 2.52$. Differentiating equation (13) we find $\lambda(dn/d\lambda) = -0.286$. Substituting this in equation (10) with $q = -4.8 \times 10^{-19} \text{esu}$ and $m = 9.1 \times 10^{-31} \text{kg}$, we find $\psi = +1.67$ radians or $+96^\circ$.

EXPERIMENT

A sample of high-dispersion flint glass is placed between the pole pieces of an electromagnet, as shown in Figure 3. A beam of approximately monochromatic light, linearly polarized by passage through an adjustable Polaroid filter, is directed through a hole in one pole piece, through the glass in the direction of the field, through a hole in the other pole piece, through a motor-driven rotating second Polaroid filter, and finally to a photomultiplier, PM1. The intensity registered by PM1 varies as $\cos^2\theta = 1/2 + 1/2\sin(\omega t + \delta)$, where θ is the angle between the plane of polarization of the emergent beam and the transmission axis of the rotating Polaroid, ω is the angular velocity of the rotating Polaroid, and δ is the phase of the intensity signal relative to a periodic fiducial pulse from PM2 generated by light from a trigger lamp that passes through a slot in the mounting wheel of the rotating

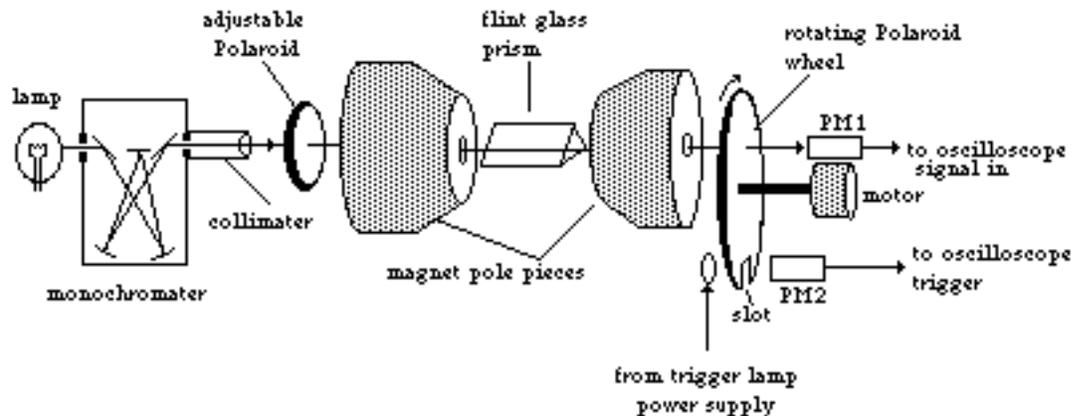


Figure 3. Schematic diagram of the apparatus for measuring Faraday rotation.

Polaroid. When the strength of the field is changed, the phase angle δ changes by an amount δ equal to the change in the Faraday rotation angle ψ . The change in δ is determined by a measurement of the angle through which the adjustable Polaroid must be counter-rotated to compensate for the Faraday effect.

A. MEASUREMENT OF THE VERDET CONSTANT.

Calibration of the magnet.

1. Remove the pin which holds up the flint glass sample located between the pole pieces of the magnet, and carefully slide the sample to its lower position out of the gap.

2. Turn on the cooling water for the magnet. (Note the pressure-controlled switch that prevents the magnet current from being turned on if the water is not on).

3. Turn on the DC power to the magnet. **In general, one should never interrupt a magnet current suddenly by opening a wall switch or flipping a reversing switch.** Such a rapid change in circuit resistance may produce a very large negative di/dt and, according to Faraday's law of induction, a corresponding high voltage (Ldi/dt) that may cause an arc in which the energy stored in the magnetic field is suddenly dissipated as heat in the switch or your hand. Incidentally, turning the magnet current on suddenly is not dangerous. Why?

4. Run the current up to +6 amperes and down to zero. Flip the reversing switch. Run the current up to -6 amperes and down to zero. Repeat the cycle a few times ending at zero after a negative run up, thereby putting the magnet at an easily reproducible place on its hysteresis plot (B vs I). (A current "crowbar" on the supply cuts out a little above 6 amperes. To restore operation, reduce the current control and switch the supply off and on.)

5. With the gaussmeter probe centered in the gap between the pole pieces, measure the field for several increasing values of the current up to +6 amperes. Then measure the field for several decreasing values of the current. Plot the data as you tabulate it to see how bad the effects of hysteresis are. To get back where you started, run the current to -6 amperes and back to zero.

6. With the current set at +6 amperes measure the field at several measured positions between the pole pieces from one side of the gap to the other along the axis of the pole pieces. Repeat with the current at an intermediate value. Again, plot your data as you tabulate it. (Note that in your determination of the Verdet constant you will need to know the average value of B along the optical path through the flint glass sample.)

MEASUREMENT OF THE FARADAY ROTATION.

The intensity of the beam that emerges from the rotating Polaroid filter is measured by the signal photomultiplier, PM1. The oscilloscope sweep is triggered once per revolution of the rotating filter by the signal generated when the slit in the filter wheel goes past the trigger photomultiplier, PM2, and admits a flash of light from the trigger lamp. Thus the start of each oscilloscope sweep is synchronized with the angular position of the rotating filter. The observed waveform represents the intensity of the beam transmitted by the filter as a function of its angular

position. When the magnetic field is turned on the resulting horizontal displacement on the oscilloscope of the sinusoidal signal is proportional to the angle through which the plane of polarization has been rotated by the Faraday effect in the flint glass sample.

1. Explore the operation of the monochromator. Calibrate the wavelength dial by observing the dial settings which permit the various lines of the mercury spectrum to pass through. To do this, remove the monochromator from its support and place it on a wooden block on the bench so that light from the mercury discharge tube enters the input slit. Place your eye so that you can see light emerging from the output slit as you (gently) turn the crank that controls the transmitted wavelength. When the green line (5460\AA) comes through, note the dial reading. Also record the dial readings for the purple line (4358\AA) and the yellow doublet (5769.6\AA , 5789.7\AA).

2. Turn on the incandescent source light and set the voltage to 7.0 v. Check that the fan is operating to keep the lamp from melting. Replace the monochromator on the wooden support, and position it with respect to the white light lamp so that a monochromatic beam emerges from the exit slit, passes through the output lens, and falls on a white card. Dial through the colors. Adjust the output lens to form a well collimated beam. Position the whole light and monochromator assembly so that the output beam goes through the holes in the magnet pole pieces and into the detector.

3. Position the flint glass prism between the pole pieces so the light coming through the holes in the pole pieces passes through the length of the prism. Take care not to touch the prism surfaces, because it is very difficult to clean. The prism is 3.125" long plus or minus .005". The base of the prism is a triangle with the angles 90° , 50° and 40° .

4. Connect the output from the phase reference "trigger" photomultiplier to the TIME BASE B trigger input. Connect the output from the signal photomultiplier to the channel 2 input of the scope dual-beam preamplifier.

5. Open the monochromator slit to 2.0 (at the red and blue ends of the visible spectrum you will need all the light you can get: the spread in wavelength won't cost much in accuracy) and set the wavelength dial for 5700\AA .

6. Check that the phase reference light (trigger light) turned on when the magnet power supply was turned on.

7. Turn the variac controlling the motor that drives the rotating Polaroid to 80 volts.

8. Turn on the high voltage for the photomultipliers and set the voltage at NEGATIVE 1100.

9. The following oscilloscope set up is recommended:

CHANNEL 1

(press 1)

source: phase-detecting PM1 yielding sinusoid

coupling: AC

Volts/div 200mv
 CHANNEL 2
 (press 2)
 source: trigger-PM2
 coupling: DC
 Volts/div 2.V
 Time/div: 5.00ms
 Trigger Source: Channel 2
 (press *source*)
 Trigger Mode: Normal
 (press *mode*)

10. You should now see a sinusoidal wave form that represents the intensity of light reaching the phase-detecting PM1 as a function of the angle between the transmission axes of the two Polaroid filters. If all is well you can work with the room lights on. If stray room light affects your measurement, track down the light leaks and cover them so you can avoid working in the dark.

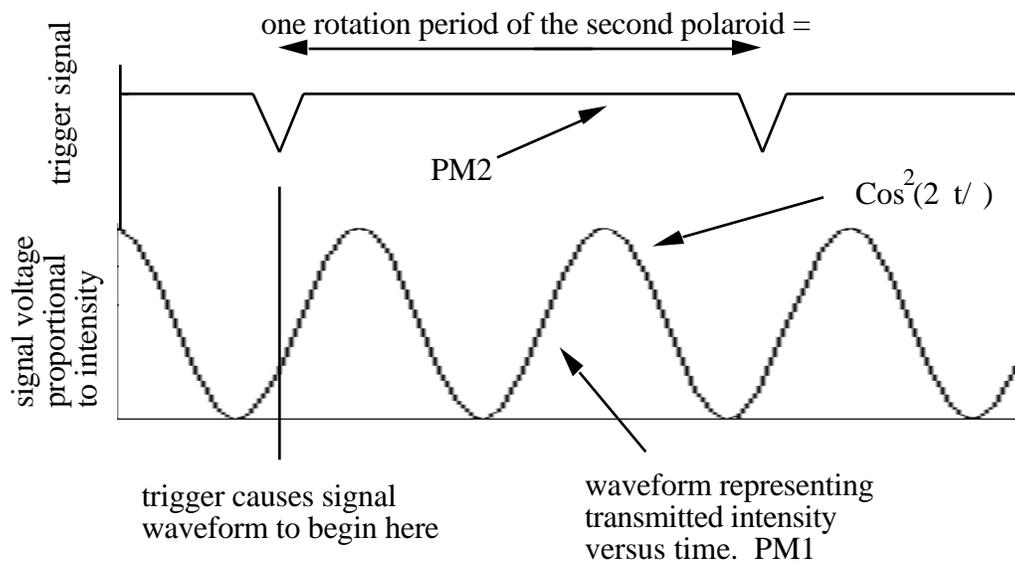


Figure 4. Sinusoidal waveform generated by detection of polarized light transmitted through rotating Polaroid.

11. Turn on the magnet current and observe how the phase of the waveform relative to the sweep trigger signal changes as you a) rotate the first polarizer, b) vary the magnet current, or c) vary the wavelength. **The Faraday rotation angle is the angle through which the**

first polarizer must be counter-rotated to restore the phase to its zero-field value.

12. Measure the Faraday rotation at a fixed wavelength for several values of the current. (In each run reduce the current to the minimum, flip the reversing switch, a few times, and then increase the current to successively larger values so that your calibration curve will be valid.) Is a linear relation verified?

13. At a fixed high value of the current measure the value and sense (right-handed or left-handed) of the Faraday rotation for several wavelengths spaced over the complete range of wavelengths for which the transmitted intensity is large enough to permit a reasonably accurate measurement of the phase. Adjust the input preamplifier gain as required to optimize your measurements. Determine the direction of the magnetic field! (You need to know this to determine the sign of the charges responsible for the interaction between the light and the glass.)

14. Repeat your measurements several times to establish an empirical basis for estimating the random errors of measurement.

15. Compute, tabulate and plot the values of the Verdet constant *versus* the wavelength.

B. DETERMINATION OF THE DISPERSION AS A FUNCTION OF WAVELENGTH.

The purpose of the following measurements is to determine the dispersion, $dn/d\lambda$, of the flint glass prism at several wavelengths. One approach might be to measure the ratio

$$n' = [n(\lambda_1) - n(\lambda_2)] / [\lambda_1 - \lambda_2] \quad (14)$$

for various pairs of close values of λ . This would be poor experimental strategy - the fractional errors in the measurements of n would necessarily be large because $n' \ll n$.

A better strategy is to determine the values of the two constants K and λ_0 in equation (13) by fitting the formula to measurements of n *versus* λ . Rearranging equation (13) we obtain

$$[1/(n^2 - 1)] = a + b (1/\lambda)^2, \quad (15)$$

where

$$a = 1/K$$

and

$$b = -\lambda_0^2/K.$$

Thus the classical theory of the index of refraction predicts that a plot of $1/(n^2 - 1)$ versus $(1/\lambda)^2$ should be fitted well by a straight line with an intercept and slope from which one can

determine a and b and, in turn, K and λ_o . Given these quantities, one can then derive a value of $dn/d\lambda$ for any desired wavelength by analytical differentiation of equation (13).

The index of refraction of a substance can be determined from a measurement of the angle of minimum deviation of a light ray passing through a triangular prism of the substance. The condition of minimum deviation, illustrated in Figure 5, occurs when the ray inside the prism is perpendicular to the bisector of the apex angle α . We call γ the angle of incidence on the entrant face of the prism which, in this symmetric situation, equals the angle of refraction from the emergent face. The angle of deviation is θ . From geometry we have the relations

$$\beta = \alpha/2, \quad (16a)$$

and

$$\theta = 2(\gamma - \beta). \quad (16b)$$

Thus

$$\gamma = 1/2(\theta + \alpha). \quad (17)$$

Assuming that the index of refraction of air is not significantly different from unity, we have according to Snell's law the relation

$$\sin \gamma = n \sin \beta. \quad (18)$$

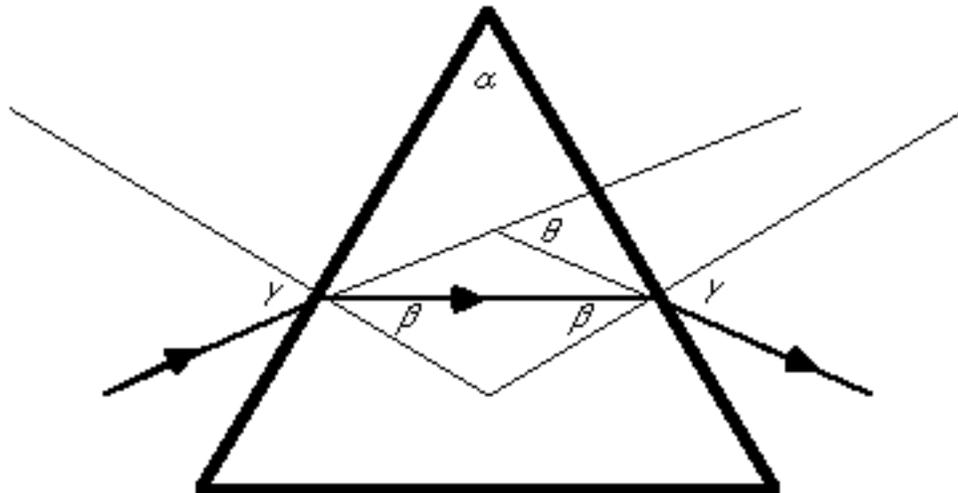


Figure 5. Light ray passing through a prism at minimum deviation.

Combining the last two equations we obtain the expression

$$n = \frac{\sin\left(\frac{\theta + \alpha}{2}\right)}{\sin\frac{\alpha}{2}} \quad (19)$$

The angle of minimum deviation and the apex angle of the flint glass prism are measured with a prism spectroscope having a collimator, telescope, and prism table mounted with vernier scales in such a way that the relative orientations can be measured to accuracies of ± 1 arc minute. Figure 6 illustrates the optical arrangement. (If you can't figure out how to read the vernier scales, ask your instructor for help.)

1. Adjust the eyepiece of the telescope so that the cross hair is in sharp focus. Then focus the telescope on a distant object so that the images of the object and the cross hair are simultaneously sharp. This adjustment insures that parallel light entering the telescope will be focussed to a point in the plane of the cross hair. To accomplish this you can set the spectrometer on a stool in the hall outside the darkroom

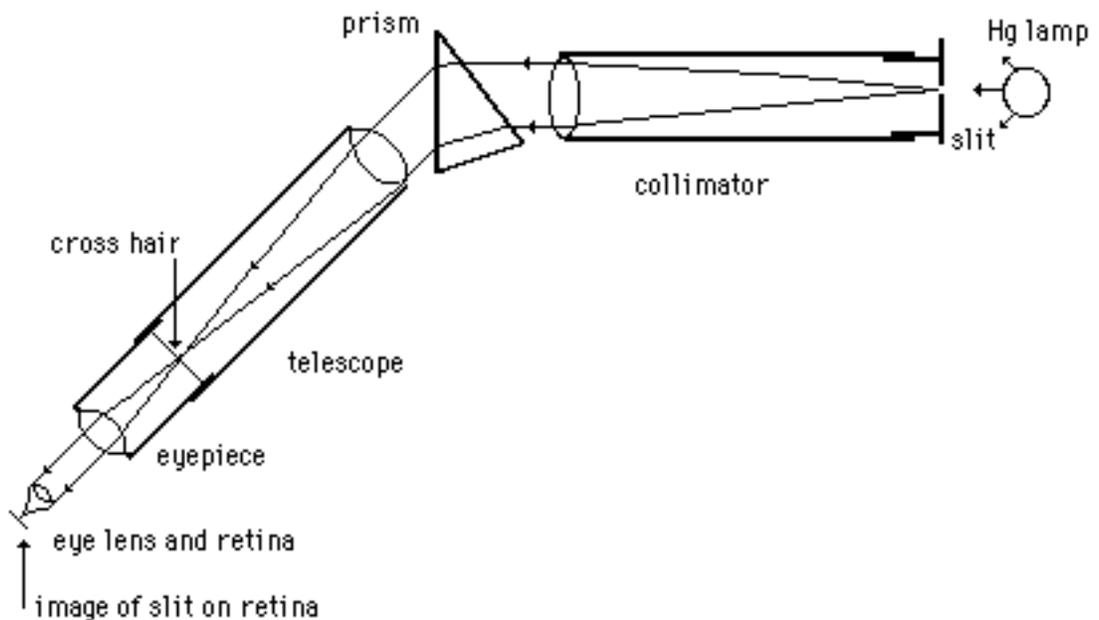


Figure 6. Schematic diagram of the spectrometer for measuring minimum angle of deviation.

and, with no prism in the way, observe something at the end of the hall through the telescope. Then, with the telescope looking into the collimator, adjust the position of the tube carrying the slit until the slit is in sharp focus. (This renders the rays emanating from a point at the slit parallel as

they enter the prism.) Line up the slit and cross hair by rotating their holders without losing the focus condition. Note that if the beam entering the prism is not parallel, then no adjustment of the telescope will produce sharply focused monochromatic images of the slit, i.e. spectral "lines".

2. Make the slit narrow and align the telescope and collimator so that the cross hair and slit images coincide. Record the position of the telescope using the vernier scale to achieve a precision of ± 1 arc minute. Put the prism in place, and swing the telescope around until you see the lines of the mercury spectrum. Without moving the telescope rotate the prism table and notice how the lines move. Find the angular position of the prism where the green line has its minimum deviation, i.e. where the green line moves in the same direction regardless of which way the prism is turned. This is the position where the green light passing through the prism suffers the minimum angle of deviation. With the prism at this precise position move the telescope to place the cross hair on the green line. Record the telescope position. Measure the minimum deviation of at least 4 lines widely separated in wavelength, e.g. the two lines of the yellow doublet, the famous green line, and the prominent blue and purple lines. You must set up the minimum deviation condition for each line in turn by rotating the prism holder in the central table until you see the lines move to and then away from their position of minimum deviation. Repeat the measurements several times with different relative positions of the various rotatable elements to reduce the random error and to obtain an estimate of its magnitude.

3. Determine the apex angle of the prism from measurements of the orientations of the prism table at which images of the slits reflected off of the prism faces can be seen through the telescope when it and the collimator are fixed in position. This is best done with the telescope positioned about 45° from the collimator and the apex of the prism close to the center of the prism table. Lock the telescope in place, and rotate only the prism table to bring the reflections of the slit in the prism faces into view.

(Hint: typical values of n for the flint glass are ~ 2.00).

Given a set of measured values of $(1/\lambda)^2$ and $1/(n^2 - 1)$, determine values and error estimates of the parameters a and b in equation (15) by fitting a straight line to a plot, or better by a linear regression analysis. From these quantities compute values and errors of K and λ_0 which can be substituted into your expression for the derivative of equation (13).

DETERMINATION OF Q/M AND C

From each measured value of the Verdet constant and the corresponding value of (dn/dl) calculated from the derivative of equation (13) compute a value for q/m and the "fudge" factor $C=(q/m)/(e/m_e)$. Determine the sign of the interacting charge from your observation of the relation between the direction of Faraday rotation and the direction of the magnetic field. Evaluate the

experimental errors, comment on the degree to which the Becquerel theory conforms to the observations. Most importantly, to what degree are your values for the "effective" (q/m) independent of wavelength, as predicted by the Becquerel formula?

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SUGGESTED THEORETICAL TOPICS

1. The Becquerel theory of the Faraday effect.
2. The minimum angle of deviation.
3. Circular birefringence in molecules.
4. Electromagnetic theory of refraction.