Rutherford Scattering

MIT Department of Physics
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This is an experiment which studies scattering alpha particles on atomic nuclei. You will shoot alpha particles, emitted by ^{241}Am, at thin metal foils and measure the scattering cross section of the target atoms as a function of the scattering angle, the alpha particle energy, and the nuclear charge. You will then measure the intensity of alpha particles scattered by thin metal foils as a function of the scattering angle for several elements of very different atomic number.

PREPARATORY QUESTIONS

Please visit the Rutherford Scattering chapter on the 8.13x website at mitx.mit.edu to review the background material for this experiment. Answer all questions found in the chapter. Work out the solutions in your laboratory notebook; submit your answers on the web site.

SUGGESTED PROGRESS CHECK FOR END OF 2nd SESSION

Plot the rate of alpha particle observations for $10°$ and $30°$ *vs.* $1/\sin^4(\theta/2)$. Comment on the agreement.

I. INTRODUCTION

Little was known about the structure of atoms when Geiger and Marsden began their experiments scattering alpha particles on thin metal foils in 1909 at the Cavendish Laboratory. A decade earlier at the Cavendish, J. J. Thomson had discovered the electron and determined the ratio of its charge to its mass by measuring the deflections of electron beams (cathode rays) by electric and magnetic fields. In 1909, Millikan measured the charge of the electron in the oil drop experiment. Thus by 1909 both the charge and mass of the electron were known with considerable accuracy. Furthermore, Thomson's interpretation of X-ray scattering from carbon and other light elements had established that the number of electrons per atom of a given element was equal not to its atomic weight, but to its atomic number, determined by its position in the periodic table. Since the mass of an electron is much less than the mass of the lightest atom, hydrogen, it was clear that most of the mass in any atom is associated with the positive charge. The central problem was to figure out how the positive and negative parts of an atom are held together, in such a way as to produce optical emission spectra with the regularities expressed by the Balmer formula (discovered by Johann Balmer in 1885) for hydrogen and the combination rules and series limits for the complex spectra of multielectron atoms.

Thinking within the limitations of Newtonian mechanics and Maxwell's electromagnetic theory, Thomson imagined the atom as a sphere of positive charge within which the electrons occupy certain positions of equilibrium, like raisins in a pudding. Set in motion, the electrons should vibrate harmonically, radiating electromagnetic energy with characteristic sharp frequencies that would be in the optical range if the radii of the atomic spheres were of the order of 10^{-8} cm. However, the "raisin pudding" model yielded no explanation of the numerical regularities of optical spectra, *e.g.* the Balmer formula for the hydrogen spectrum and the Ritz combination principle [3] for spectra in general.

At this point, Ernest Rutherford got the idea that the structure of atoms could be probed by observing the scattering of alpha particles. Alpha particles, as Rutherford himself had recently demonstrated, are the positively charged emanations of radioactive substances. They are also bare helium nuclei. According to the raisin pudding model, an alpha particle traversing a thin gold film should experience many small angle deflections as it passes close to or through the positive spheres of the gold atoms. Rutherford showed [4] that the fraction of particles scattered in this way through an angle θ or greater should decrease exponentially according to the equation

$$F_\theta \approx \exp(-\theta^2/\theta_m^2), \qquad (1)$$

where θ_m is the mean multiple scattering angle. For a typical foil of gold leaf, $\theta_m \approx 1°$. Thus at $\theta = 30°$, one finds F_θ on the order of $\exp(-30)$ or 10^{-13}.

Rutherford's formula turned out to be correct for very small angles of scattering. Evidently there was substantial truth in the idea of multiple scattering. But in experiments initiated at Rutherford's direction, Geiger and Marsden (1909) found that 1 in 8000 alpha particles passing through a thin film of platinum was scattered through more than $90°$! It was as though bullets fired at a bale of cotton could occasionally ricochet backward. Such an observation might lead one to suspect rocks in the cotton.

At this point Rutherford (1911) advanced the hypothesis that the positive charge and most of the mass of an atom is concentrated in a "nucleus" with dimensions of the order of 10^{-12} cm (10,000 times smaller than the atom as a whole) with the electrons in some sort of configuration around it. Applying the principles of classical mechanics, he calculated the trajectories of alpha particles passing near such nuclei, and derived an expression for the differential scattering cross section which accounted accurately for the scattering data, thereby vali-
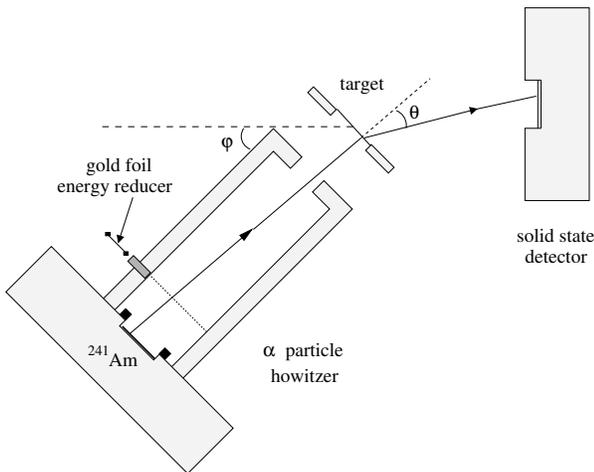
FIG. 1. Schematic of the howitzer, target, and detector in the vacuum chamber.

dating the hypothesis of the nuclear atom.

The Rutherford scattering differential cross section *per target atom* for any target atom is

$$\frac{d\sigma}{d\Omega} = \left(\frac{ZZ'e^2}{4E} \right)^2 \frac{1}{\sin^4(\theta/2)} \qquad (2)$$

where θ is the scattering angle, Ze is the charge of the target nuclei, $Z'e$ is the charge of the alpha particles and E is their kinetic energy. (For a derivation, see Reference [5].) Further excruciatingly tedious experiments by Geiger and Marsden confirmed the validity of the formula within the statistical errors of their measurements. Geiger hadn't invented the Geiger counter yet, and electronic detection methods were still 20 years in the future. They used a low power microscope to observe and count by eye the scintillations produced by the alpha particles when they impinged on a screen lightly coated with zinc sulfide dust.

References [5, 6] present the Rutherford theory and discuss the interpretation of data from a scattering experiment that is quite similar to that in the Junior Lab, with the exception of the specific detector and circuit arrangement. We will confine our discussion to the features of the experimental setup and procedures that are peculiar to our setup.

II.   APPARATUS

Figure 1 is a schematic drawing of the apparatus in the vacuum chamber. The source, ^{241}Am, emits alpha particles of various discrete energies, the most frequent of which are 5.486 MeV (86%), 5.443 MeV (12.7%), and 5.391 MeV (1.4%). All these decays lead to excited states of ^{237}Np. The half-life of ^{241}Am is 458 years. The source, deposited on a thin metal disk with the highest activity

of ^{241}Am per unit area commercially available (about 1.5 millicuries per square inch) and sealed with an evaporated gold coating 1.5 microns thick, is covered by a metal washer with a 0.64 cm diameter hole and enclosed in a "howitzer" with a 0.64 cm diameter aperture in its snout. Due to the gold coating, the alpha particles' initial energies are reduced to a value closer to 4.8 MeV. Under near-vacuum, a collimated beam of alpha particles emerges from the snout (the range of 5.5 MeV alpha particles in air at atmospheric pressure is only abuot 4 cm).

If the setup is under vacuum, then decide whether you want to take the time to break and then reestablish the vacuum, or whether the sample currently mounted is adequate for your purposes. If you do wish to break vacuum, then turn the bias voltage on the detector slowly down to zero, close the vacuum valve between the pump and the chamber, and slowly open the vacuum release valve, leaving the pump running. Place the black hood on a table where it can serve as a cushion for the plastic cover. Lift the cover off the steel cylinder and place it flat on the hood, being careful not to damage the underside of the plastic cover. Find out how you can adjust the relative positions of the howitzer and detector with respect to the target, and how you can turn, raise and lower the target with the control rod that protrudes from the bottom of the chamber. Note how you can simultaneously rotate the howitzer and the target about a vertical axis through the target with the lever attached to the outer cylinder under the chamber. This feature enables you to maintain a fixed relative orientation of incident beam and target while you vary the scattering angles of the detected alpha particles.

Two warnings: **Do not remove the black cover while the detector is still on.** The lights in the lab are far stronger than what the detector is designed to absorb. Exposure to the background light in the lab may damage the detector. **Do not release the vacuum too quickly.** A rapid change in pressure can cause the delicate foil targets to rupture.

III.   MEASUREMENTS

III.1.   Calibrate the Measurement Chain

Move the howitzer support arm so that the howitzer points directly at the detector through the empty target hole. Pump the system down with the bias voltage off. Using an oscilloscope, you can watch the pulses from the unbiased detector appear and gradually grow in amplitude as the amount of air between the source and detector diminishes. The pressure should reach 200 microns in 10 to 15 minutes. If it doesn't, you probably have leak a around the O-ring seal under the plastic cover or around the gasket under the stainless steel cylinder. If necessary, a good way to apply the necessary pressure to initially seal the O-ring and gasket is by weighting the plastic cover with 6 – 8 lead bricks available next to the exper-

iment. Also, there is vacuum grease available, but don't overuse it! A pea-sized amount is more than enough for the surface of the O-ring, and should last for years.

Cover the vacuum chamber with a thick black cloth to eliminate room lights and protect the sensitive detector. Connect a voltmeter to the small aluminum switch box marked "68 volt limiter" that is situated between the bias voltage supply and the preamp for the detector. When the pressure is below 200 microns, you may switch on the bias voltage. The voltmeter should read around 45 volts. Adjust the amplifier gain so that the alpha-particle (bipolar) pulses have an amplitude of about +7 volts. The bipolar amplifier output is better for handling high count rate detection such as the case with no foil.

Observe the pulse size distribution with the MCA and readjust the gains so that the peak of the alpha-particle pulse-size distribution lies at a convenient position within the full range of the multi-channel analyzer. You can assume that the amplitude of a detector pulse is proportional to the energy lost by the particle in the silicon detector. Thus a linear plot of the channel number versus energy, scaled to match the energy of the alpha-particles coming straight from the source, should be an accurate plot for the interpretation of the pulse sizes you will be measuring.

III.2.   Adjust the Position of the Target

Accurate vertical positioning of the target holder is essential to assure that the particle beam passes cleanly through one or another of the holes in the aluminum sheet that supports the foils, without touching the edge of the hole. Either you or your partner can manipulate the target holder from under the table while the other judges its position.

To aid you, there are 3 separate machined aluminum cylinders which can be placed on the vertical positioning shaft. These cylinders have been measured and machined to precisely place the desired target at the same height as the howitzer.

With the target holder in the open hole position, measure the counting rate as a function of the howitzer position angle from about -10° to about +10° relative to the nominal center position. **Plot the data as you proceed on a log scale. Log graph paper is available if desired.** Determine the exact pointer readings of the center position and at the positions of zero counting rate intercepts on either side of the center to determine the beam center and its total width.

III.3.   Measure the Effects of Target Foils on the Pulse-Height Spectrum of Alpha-Particles

Using the preset accumulation time feature of the MCA accumulate a size spectrum of the pulses produced by unscattered alpha particles with the howitzer position angle set at the center position. To characterize the width of the distribution, place the start and stop cursors on either edge of the distribution at the positions where the counting rate is half the maximum value, and note the channel numbers. Measure the channel number of the peak counting rate. Compare the results with no foil, the two gold foils, and the titanium foil, and figure out the most probable energy lost by the alpha particles in traversing each of the films. Describe and explain the changes in the shape of the size spectrum when a target foil is in the beam.

Determine the thicknesses of the target by reference to the range-energy data available on the web at [7]. Be sure to use the "projected range" data and not the CSDA range data. To interpolate this table plot the tabulated range versus energy. Call c_1 and c_2 the median channels of the pulse before and after passage through a target. You can assume with some confidence that the median channel number is proportional to the particle energy. From your range energy plot, read the range of the incident alpha particles E_0 and the range of particles with energy $(c_2/c_1) E_0$. The difference in range is the thickness of the target in $mg \cdot cm^{-2}$. Does your data fit better using $E_0 = 5.48$ MeV or using a reduced value accounting for the 1.5 μm gold coating on the source?

III.4.   Plan and Execute Your Scattering Measurements

Your data from this experiment should consist of *at least* the results of the following operations:

1. Determine the essential characteristics of the experimental setup.

   (a) Measure the counting rate as a function of howitzer position angle with the open hole.

   (b) Measure the loss in energy of the alpha particles in traversing each of the three targets and determine the thicknesses of the targets from the data.

2. Measure the angular dependence of the scattering cross section of gold. Measure the counting rate with a gold target for the full energy alpha particle beam as a function of howitzer position angle out to the largest angles that counting statistics and time limitations allow.

3. Determine the total cross section σ. Measure the intensity of the beam that emerges from the howitzer. Note that the howitzer is designed so that the entire emergent beam irradiates the target foils; *i.e.*, the beam is sufficiently narrow to pass through the foils without intercepting the target holder.

4. Measure the Z-dependence of the Rutherford cross-section. Compare your results from the gold targets

to counting rates measured using the provided targets containing other elements.

At large scattering angles the counting rates are very low, so the accuracy of your data will be severely limited by the (Poisson) statistical accuracy you can achieve. Your best strategy will be to make a complete set of measurements with short integrating times ($10 - 30$ minutes) in each configuration you plan to use, and to carry out a preliminary analysis of the resulting data during the first session. Then, in the light of what you have learned about the counting rates and the problems of analysis, you can make a run plan allotting enough time to each configuration to obtain enough counts to insure good statistical accuracy. Obviously, you cannot afford to measure the counting rates at $2°$ intervals of the howitzer angle. You may decide, for example, to settle for measurements of good statistical accuracy of the angular dependence of the counting rate with one target only at howitzer angles of, say, $20°$, $30°$, $40°$ and $60°$, and measurements of the Z and E dependence at only one howitzer angle, say $20°$.

You will probably want to attain at least $2 - 20\%$ ($> 4 - 400$ counts) statistical accuracy in each of your measured rates. If you occupy the first lab sessions getting acquainted with the experiment, then you will have a total of about 9 hours in the next three sessions to get your definitive scattering data plus a possible overnight run at a very large scattering angle to observe the amazing phenomenon of atomic bullets ricocheting nearly straight back. Clearly you cannot afford to creep along the curve of rate versus position angle at one-degree intervals. Rather, you must take large steps in position angle to define the general shape of the curve, and then fill in to refine your data as time permits.

III.5. Challenge Experiment: Cross-Section Energy Dependence

To measure the energy dependence of the cross section you can reduce the incident energy by inserting into the slot in the howitzer the titanium foil in the holder provided. You should make your measurement of the energy dependence of the differential cross section at a position angle large enough (say, $20°$) to reduce the complications caused by multiple small-angle Rutherford scattering. Your analysis will be simpler if you maintain the same geometrical relation between the howitzer and the scattering foil. This is feasible for measurements at position angles from $0°$ to $\approx 60°$.

IV. ANALYSIS AND INTERPRETATION

Some additional references that you may find useful include [7–10].

1. Using a log plot, plot the gold-foil counting rates against $\sin^{-4}(\theta/2)$, where θ is the howitzer position angle, and compare the result with the theoretical expectation based on the Rutherford formula. Compare also with the results given in [5]. Be sure to also complete a full error analysis.

To make an accurate comparison between the data plot and the predictions of the Rutherford theory, you should take into account the spread in the angular response of the apparatus. The inescapable fact of any scattering measurement is that events with a range of scattering angles contribute to the counting rate at any given position angle of the detector. Ideally one would like that range to be very small so that a plot of counting rate against position angle would be, in effect, a plot of counting rate against scattering angle. But then the counting rate would be impractically small. Thus, in designing a scattering experiment, one must strike a compromise between angular resolution and counting rate. To achieve acceptable counting rates in the present experiment, it was necessary to design it with a broad angular acceptance.

Consequently, your plot of counting rate against position angle should be compared to a convolution of the Rutherford cross section with the angular response function of the apparatus. This may be approached in one of several ways of differing sophistication. To see how this can be done, call $g(\theta, \phi)$ the angular response function such that $g(\theta, \phi)d\theta$ is the probability that a particle scattered at an angle between θ and $\theta + d\theta$ will be detected when the howitzer is at position angle ϕ. Then the expected counting rate at ϕ is

$$C(\phi) = C_0 \int_0^\pi g(\phi, \theta) \sin^{-4}(\theta/2) d\theta, \qquad (3)$$

where C_0 is a constant that includes the solid angle subtended by the detector at the point of scattering. For a crude approximation one might represent g by a triangular function defined by

$$g(\phi, \theta) = \begin{cases} (1 - \frac{|\theta - \phi|}{\theta_0}), & |\theta - \phi| < \theta_0 \\ 0, & |\theta - \phi| > \theta_0 \end{cases}, \qquad (4)$$

where θ_0 is the half-width of the base of a triangular function. The value of θ_0 can be estimated by analysis of Figure 2, which is a scale drawing of the howitzer, target holder and detector. One might set $2\theta_0$ equal to the difference between the extreme angles of scattering that detected particles can undergo when $\phi = 0$. The convolution should yield a curve of counting rate versus position angle that conforms more closely to the data than the function $\sin^{-4}(\phi/2)$ of the Rutherford cross section.
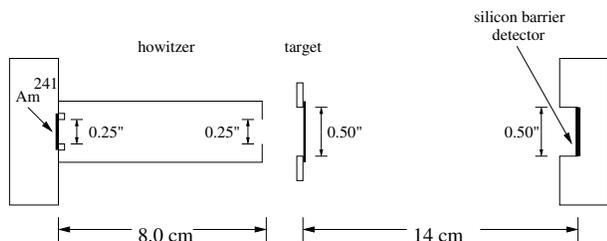
FIG. 2. Dimensioned drawing of the alpha-particle howitzer, target foil, and detector geometry. Dimensions which are easily verified by the user should be done so.

A second approximation may be made simply by using the beam profile obtained in Section III.2.

A little thought will convince you that it is a formidable job of geometrical analysis to construct an exact analytical expression for g. A more practical approach is to write a "Monte Carlo" program that simulates the experiment by following individual particles through the system, choosing positions and directions of emission and scattering with random numbers according to appropriate probability distributions, among which is the Rutherford scattering probability distribution to be tested. If you are an experienced programmer you may want to try this approach, which is also not a trivial job. You can probably get advice on how to proceed from one of your instructors.

2. Determine the differential scattering cross section per gold atom at an angle (say, $30°$) large enough to reduce substantially the problems of interpretation caused by multiple scattering.

3. There is a substantial problem with Rutherford's distribution that should by now be very apparent: equations of the form $1/\sin^4(\theta)$ approach ∞ in the limit $\theta = 0$. However, you will not observe an infinite number of particles emitted from the target when it is at the center position. Try to come up with an explanation to reconcile the Rutherford model with what you observe. A few things to think about: multiple scattering; the nature of scattering cross sections; quantum mechanics.

POSSIBLE THEORETICAL TOPICS

- The Rutherford scattering cross section.
- Energy loss of charged particles in matter.
- Multiple Coulomb scattering.
- Silicon barrier detector.

[1] A. Melissinos, "Experiments in modern physics: Solid-state detectors," (Academic Press, 2003) Chap. 8.5, pp. 344–354, 2nd ed.
[2] G. F. Knoll, "Radiation detection and measurement: Semiconductor diode detectors," (John Wiley and Sons, 2000) Chap. 11, pp. 353–404, 3rd ed., covers solid state detectors at a higher level of detail than Melissinos.
[3] W. Ritz, Astrophysical Journal **28**, 237 (1908).
[4] E. Rutherford, Philisophical Magazine **21**, 669 (1911).
[5] A. Melissinos, "Experiments in modern physics: Rutherford scattering," (Academic Press, 1966) Chap. 6, pp. 226–252.
[6] A. Melissinos, "Experiments in modern physics: Interaction of charged particles with matter," (Academic Press, 1966) Chap. 5, pp. 152–165.
[7] J. C. M.J. Berger and M. Zucker, *ESTAR, PSTAR, and ASTAR: Computer Programs for Calculating Stopping-Power and Range Tables for Electrons, Protons, and Helium Ions*, Tech. Rep. (NIST, Gaithersburg, MD, 1999-2002) http://physics.nist.gov/PhysRefData/Star/Text.
[8] P. Bevington and D. Robinson, *Data Reduction and Error Analysis for the Physical Sciences*, 3rd ed. (McGraw-Hil, 1992).
[9] S. Gasiorowicz, *Quantum Physics*, 2nd ed. (Wiley, 1974) p. 417.
[10] E. Serge, "Nuclei and particles," (Benjamin, 1977) Chap. 2.

Appendix A: Equipment List

Manufacturer	Description	URL
Canberra	PIPS α-particle detector	canberra.com
Canberra 2006	Charged Particle preamplifier	canberra.com
Canberra	Amplifier	canberra.com
Ortec	Multi-Channel Analyzer	ortec-online.com