Massachusetts Institute of Technology
Physics Department

8.13/8.14 2000/2001

Junior Physics Laboratory Experiment #18

The Stern-Gerlach Experiment

Quantization of Angular Momentum

Purpose The purpose of this experiment is to perform a version of one of the most important experiments in the development of quantum physics, and to derive from the results information about the quantum properties of angular momentum, the magnetic moment of potassium atoms, and the Maxwell-Boltzmann distribution.

1 PREPARATORY QUESTIONS

1. Sketch the expected plot of beam intensity versus lateral deflection in a Stern-Gerlach experiment with a beam of atoms in a state with $j = 1$. The same for the case of $j = \frac{3}{2}$.
2. Make an energy level chart of the magnetic substates of the electronic ground state of the potassium atom in a weak magnetic field and in a strong magnetic field. Take into account the effects of the electronic and nuclear magnetic moments. In the light of this chart predict how many intensity peaks you will see in this experiment and explain your prediction.
3. What would be the result of passing one of the deflected beams from this experiment through a second inhomogeneous field that is identical to the first except for being rotated 90° around the axis?
4. How is the intensity of atoms with velocity between v and dv in the beam related to the density of atoms with velocity between v and dv in the oven?
5. Derive equation (8) from the preceding results.
6. Derive equation (21) from (19) and (20) in Appendix I.

WHAT YOU WILL MEASURE

1. The angular momentum quantum number of the ground state electronic configuration of potassium atoms.
2. The magnetic moment of the potassium atom.
3. The temperature inside the oven from which the atomic beam emerges into vacuum.

2 INTRODUCTION

The following sketch of the history of the Stern-Gerlach experiment is based on the much more complete account by B. Friedrich & D. Herschbach in *Daedalus*, **127**/1, 165 (1998).

The discovery of the Zeeman effect (1896) and its theoretical interpretation demonstrated that atoms have magnetic dipole moments. However, no constraint was placed on the orientation of the moments by the "classical" explanation of the *normal* Zeeman effect, in which the spectral lines of some elements in a magnetic field are split into three components. Bohr's theory (1913) of the hydrogen atom assumed circular orbits and required the quantization of angular momentum and, by implication, quantization of the associated magnetic moment. Sommerfeld (1916) generalized the Bohr theory to allow elliptical orbits described by three quantum numbers: n, k, and m. The number $n = 1, 2, 3...$, called the principal quantum number, corresponded to the quantum number of of the Bohr theory. The number $k = 1, 2, 3..n$ defined the shape of the orbit which was circular for $k = n$. The number $m = -k, -k+1, ..., k-1, +k, m \neq 0$, determined the projection of the vector angular momentum on any prescribed axis, a consequence of the theory that was called space quantization. Sommerfeld showed that his theory could account for the fine structure of the hydrogen atom (now expained in terms of spin-orbit coupling) when relativistic effects on the motion in the elliptical orbits were considered. The Sommerfeld theory also provided an alternative explanation of the normal Zeeman effect. Nevertheless, the question remained as to whether space quantization really occurs, e. g., whether the projections of the angular momentum and its associated magnetic moment on an axis defined by the direction of an imposed magnetic field are quantized.

Otto Stern proposed (1921) a defintive experiment to decide the issue. It would consist of passing a beam of neutral silver atoms through an inhomogeneous magnetic field and observing how the beam was deflected by the force exerted by the field on the magnetic dipole moments of the atoms. The detector would be a glass plate on which the silver atoms in the deflected beam would be deposited. Since the silver atom has one valence electron, it was assumed that $k = n = 1$ and $m = \pm 1$ in the ground state. If the magnetic moments were randomly oriented, then the distribution of deflections would decrease monotonically on either side of zero deflection, reflecting a random distribution of the dipole orientations. If space quantization was a reality, then the beam should be split into two distinct beams corresponding to the parallel and anti-parallel alignments of the magnetic moments with respect to the direction of the inhomogeneous magnetic field. Stern was clumsy with his hands and never touched the apparatus of his experiments. He enlisted Walther Gerlach, a skilled experimentalist, to collaborate in the experiment.

Stern predicted that the effect would be be just barely observable. They had difficulty in raising support in the midst of the post war financial turmoil in Germany. The apparatus, which required extremely precise alignment and a high vacuum, kept breaking down. Finally, after a year of struggle, they obtained an exposure of sufficient length to give promise of an observable silver deposit. At first, when they examined the glass plate they saw nothing. Then, gradually, the deposit became visible, showing a beam separation of 0.2 millimeters!

Apparently, Stern could only afford cheap cigars with a high sulfur content. As he breathed on the glass plate, sulfur fumes converted the invisible silver deposit into visible black silver sufide, and the splitting of the beam was discovered.

The new quantum mechanics of Heisenberg, Shrödinger, and Dirac (1926-1928) showed that the orbital angular momentum of the silver atom in the ground state is actually zero. Its magnetic moment is associated with the intrinsic spin angular momentum of the single valence electron the projection of which has values of $\pm\frac{\hbar}{2}$, consistent with the fact that the silver beam is split in two. If Stern had chosen an atom with $L = 1, S = 0$, then the beam would have split into three, and the gap between the m=+1 and m=-1 beams would have been filled in, and no split would have been visible! Vol. II, chapters 34 and 35, and Vol. III, chapters 5 and 6 of the Feynman Lectures gives a lucid explanation of the quantum theory of the Stern-Gerlach experiment. Platt (1992) has given a complete analysis of the experiment using modern quantum mechanical techniques. Here we present an outline of the essential ideas.

2.1 THEORY OF ATOMIC BEAM EXPERIMENTS

Within the framework of classical mechanics one can show that an electron in a circular orbit has an angular momentum $\vec{L} = m\omega r^2$ and an associated magnetic moment $\mu = -\frac{e}{2m_e}\vec{L}$, where m and e are, respectively, the mass and charge of the electron, and r and ω are the radius and angular velocity of the orbital motion. In a magnetic field $\vec{B}$ the atom will be acted on by a torque $\mu \times \vec{B}$ which causes $\vec{L}$ to precess about the direction of $\vec{B}$ with some fixed value of the projection $\mu_z = |\mu| cos\theta$ of its magnetic moment along the direction of the field. The atom will also have a potential energy $-\mu \cdot \vec{B}$, and if the field is inhomogeneous such that at a certain point it is in the z direction and varies strongly with z, then the atom will be acted on by a force $F_z = -\nabla_z(-\mu \cdot \vec{B}) = \mu_z \frac{\partial B_z}{\partial z}$ which may have any of a continuous set of values from $-|\mu|\frac{\partial B_z}{\partial z}$ to $+|\mu|\frac{\partial B_z}{\partial z}$ One would then expect a monoenergetic beam of atoms, initially randomly oriented and passing through an inhomogeneous magnetic field, to be deflected in the $+z$ and $-z$ directions with a distribution of deflection angles that has a maximum value at zero deflection and decreases monotonically in either direction. This is not what is observed. Instead, an atomic beam, passing through such a field, is generally split into several distinct beams, implying that the sideways force deflecting the beam is restricted to certain discrete values.

According to quantum mechanics, an atom can exist in a steady state (i.e. an eigenstate of the Hamiltonian) with a definite value of the square of the magnitude of its total angular momentum, $\vec{F} \cdot \vec{F}$ and a definite component F_z of its angular momentum in any particular direction such as that of the z axis. Moreover, these quantities can have only the discrete values specified by the equations

$$\vec{F} \cdot \vec{F} = f(f+1)\hbar^2 \tag{1}$$

and

$$F_z = m_f \hbar \tag{2}$$

where f, the angular momentum quantum number, is an integer or half integer, m_f , the magnetic quantum number, can have only the values $-f, -(f-1), ..., +(f-1), f$,and $\hbar = \frac{h}{2\pi}$. The magnetic moment associated with the angular momentum is:

$\mu = -g_f \frac{e}{2m_e c}\vec{F}$

where g , called the g-factor, is a quantity of the order of unity and characteristic of the atomic state. The projection of μ on the z axis can have only one or another of a discrete set of values $\mu_z = g_f m_f \mu_B$ where $\mu_B = \frac{e\hbar}{2m_e c}(= 0.92731x10^{-20}$ erg/gauss) is the Bohr magneton. In the presence of an inhomogeneous magnetic field in the z direction the atoms will be acted on by a force which can have only one or another of a discrete set of values $m_f g_f \mu_B \frac{\partial B_z}{\partial z}$. When a monoenergetic beam of such atoms, distributed at random among states with $2f+1$ possible values of m_f, passes through an inhomogeneous magnetic field, it is split into $2f+1$ beams which are deflected into $\pm z$ directions with deflection angles corresponding to the various possible discrete values of the force. Thus, if a beam of atoms of some particular species were observed to be split into, say, 4 beams in a Stern-Gerlach experiment, then one could conclude that the angular momentum quantum number associated with the magnetic moment responsible for the deflection is $\frac{4-1}{2} = \frac{3}{2}$.

Turning now to the present experiment in which a beam of potassium atoms passes through an inhomogeneous field, we note first that the total angular momentum is the sum of the spin and orbital momenta of the electrons and nucleons. The electronic ground state of potassium is designated as $^2S_{1/2}$, which means that the total orbital angular momentum of the electrons, L, is equal to 0 (i.e. the atom is in an S-state), the fine-structure multiplicity of higher states (i.e. those with non-zero orbital angular momentum) due to spin-orbit interactions is 2 (one unpaired electron with spin 1/2), and the total angular momentum $\vec{J} = \vec{L} + \vec{S} = \frac{\hbar}{2}$ .The magnetic moment associated with the spin of the electron is $-\frac{g_s \mu_B \vec{S}}{\hbar}$ where $\vec{S}$ is the spin angular momentum, and $g_s = 2.002319304$ is the gyromagnetic ratio of the electron. The nuclear angular momentum (total spin and orbital momenta of the nucleons lumped into what is called nuclear "spin") of ^{39}K (the most abundant isotope of potassium) is $\vec{I} = \frac{3}{2}\hbar$, and the nuclear magnetic moment is $\frac{g_n \mu_B \vec{I}}{\hbar}$, where g_n is much smaller than the $\frac{m_e}{m_p} \approx \frac{1}{1836}$. In field free space the interaction between the magnetic moments associated with the total electronic angular momentum $\vec{J} = \vec{L} + \vec{S}$ and nuclear angular momentum $\vec{I}$ causes them to precess with a frequency of the order of 100 MHz around their sum $\vec{F} = \vec{J} + \vec{I}$ which is the total angular momentum of the atom. According to the rules for combining angular momenta the quantum number of the sum, is $f = i \pm j = 1$ or 2. With each combination there is associated a magnetic moment whose value can be calculated by matrix mechanics or, more simply, by the "vector" model, as explained in Melissinos and other texts.

Potassium atoms emerging into a field-free region from an oven at a temperature of $\approx 200°$ will be

1. almost exclusively in the ground electronic state,

2. nearly equally distributed among the two hyperfine states with $f = 1$ and $f = 2$,

3. very nearly equally distributed among the degenerate "magnetic" substates of each of the hyperfine states, i.e. the states with the same f but different m_f , where the latter is the quantum number of the component of total angular momentum in the direction of the field.

A beam of such atoms passing through a weak inhomogeneous field would be split into as many beams as there are magnetic substates with different components of magnetic moment in the direction of the field. By "weak field" we mean an external field in which the torques exerted by the field on the magnetic moments associated with either $\vec{I}$ and $\vec{J}$ are small compared to the torque on each that results from the mutual interaction of their magnetic moments. In a strong inhomogeneous field the $\vec{I}$ and $\vec{J}$ are "decoupled", and both precess independently about the external field direction. The magnetic moment of the potassium atom is then dominated by the magnetic moment associated with the decoupled $\vec{J} = \vec{S}$ whose projection on the direction of the external field can have only the values $\mu_z = \pm\frac{g_s\mu_B}{2}$. The beam is thus split into two groups of components, with each group having a "hyperfine" splitting, due to the nuclear spin, which can only be resolved by very refined atomic beam techniques.

2.1.1 DEFLECTIONS OF ATOMS IN A BEAM BY A NON-UNIFORM MAGNETIC FIELD

Figure 1 shows the apparatus used in this experiment. The central axis of the beam is taken as the y axis, and the $\vec{B}$ direction as the z axis. The detector is a hot, straight platinum wire extending a short distance in the $\pm x$ direction about $x = 0, z = 0$. The beam, defined by a pair of parallel slits, also extends a few mm in the $\pm x$ direction.

To calculate the deflection of an atom of mass M and velocity V we assume that the deflecting force is constant in the region between the pole pieces traversed by the beam, and zero elsewhere. Calling z the deflection of an atom at the $x - z$ plane of the detector due to the force exerted during its passage between the pole pieces, we have as the solution for Newton's Second Law with constant acceleration the expression

$$z = \frac{1}{2}V_z t_1 + V_z t_2 \tag{3}$$

where $V_z = \frac{F_z}{M}t_1, t_1 = \frac{d_1}{V_y}$, and $t_2 = \frac{d_2}{V_y}$. The distances d_1 and d_2 are, respectively, the length of the path between the magnets and the distance from the edge of the magnet to the hot wire detector, as shown on Figure 1. It turns out that the deflection angle of the beam is so small that we can approximate V_y by $|\vec{V}| \equiv V$ to great accuracy and can neglect various small corrections caused by the deflection. The expression for the deflection can then be written

$$z = F_z\Big[\frac{d_1(d_2 + \frac{d_1}{2})}{MV^2}\Big] \tag{4}$$

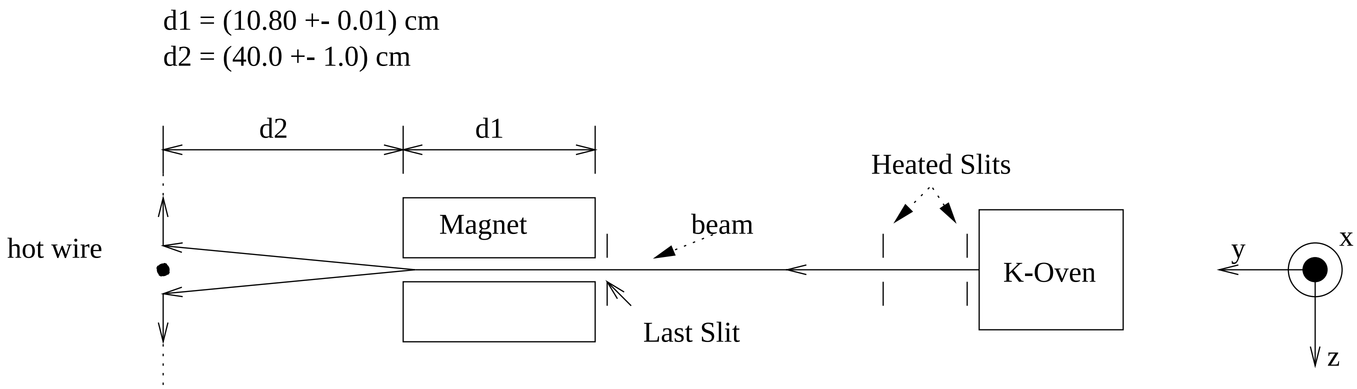

Figure 1: Schematic diagram of the apparatus looking down from above. Note that the indicated coordinate system is not the same as that used in the discussion in Appendix I.

Note that M is the mass of the atom; the force involves the mass of the electron only through μ_B.

2.1.2 Distribution of Velocities

Next we describe the distribution in velocity of the particles in the beam. According to the Maxwell-Boltzmann distribution, the fraction of atoms with velocity V in dV inside the oven is

$$f(V)dV = \frac{4}{\sqrt{\pi}}\left(\frac{V}{V_0}\right)^2 e^{-(\frac{V}{V_0})^2} d\left(\frac{V}{V_0}\right) \tag{5}$$

where $V_0 = \sqrt{\frac{2kT}{M}}$ is the most probable speed of an atom inside the oven, as one can check by differentiating $f(V)$ with respect to V and setting the result equal to zero.

The flux of atoms emerging through the oven slit with velocity V in dV is proportional to the product of the density inside the oven by the velocity with which they emerge. Thus the fraction of atoms with normalized velocities between $\frac{V}{V_0}$ and $\frac{V}{V_0} + d(\frac{V}{V_0})$ that arrive at a target in a given time interval is

$$I\left(\frac{V}{V_0}\right) d\left(\frac{V}{V_0}\right) = 2\left(\frac{V}{V_0}\right)^3 e^{-(\frac{V}{V_0})^2} d\left(\frac{V}{V_0}\right) \tag{6}$$

where the factor 2 has been chosen so that

$$\int_0^\infty I\Big(\frac{V}{V_0}\Big)d\Big(\frac{V}{V_0}\Big) = 1 \tag{7}$$

The distribution in velocity is mapped by the magnetic deflection into a distribution in deflection, which we now calculate for the quantum (true) case of a deflecting force with only two discrete values, and for the classical (false) case of a continuous range of deflecting force.

2.1.3 Distribution of Deflections for a Quantized Deflecting Force

We consider atoms which have been acted on by a particular force and call $I(z)dz$ the fraction which suffer a deflection between z and $z + dz$. To obtain the relation between the differentials of velocity and deflection we take the log of both sides of equation 4 and differentiate:

$\frac{dz}{z} = -2\frac{dV}{V}$

Setting $I(z)dz = -I(\frac{V}{V_0})d(\frac{V}{V_0})$

and carrying out the algebraic manipulations, we find

$$I(z)dz = I_0\Big(\frac{z_0}{|z|}\Big)^3 e^{-\frac{z_0}{|z|}} d\Big(\frac{z}{z_0}\Big) \tag{8}$$

where z_0 is defined as the deflection of an atom with velocity V_0. This function is zero at $z = 0$ and has maxima at $z = \pm\frac{z_0}{3}$.

In the real world of quantum mechanics (and neglecting the unresolvable effects of the nuclear magnetic moment), the z component of force on a potassium atom passing through the apparatus is restricted to the values given by the expression

$$z_0 = \pm\frac{1}{2}g_s\mu_B\frac{\partial B}{\partial z}\Big(\frac{d_1(d_2 + \frac{d_1}{2})}{MV_0^2}\Big) \tag{9}$$

Thus the beam is split in two, with half the atoms deflected toward $+z$ and half toward $-z$. The deflection distribution of each half is described by equation (8) except for a factor of $\frac{1}{2}$ required to normalize the distribution to the total number of atoms. In the actual experiment we will not be concerned with the absolute normalization of the deflection distribution, but only with its shape which yields measures of the source temperature (as reflected in the velocity distribution of the atoms), the multiplicity of magnetic substates, and the magnitude of the magnetic moment of the potassium atom.

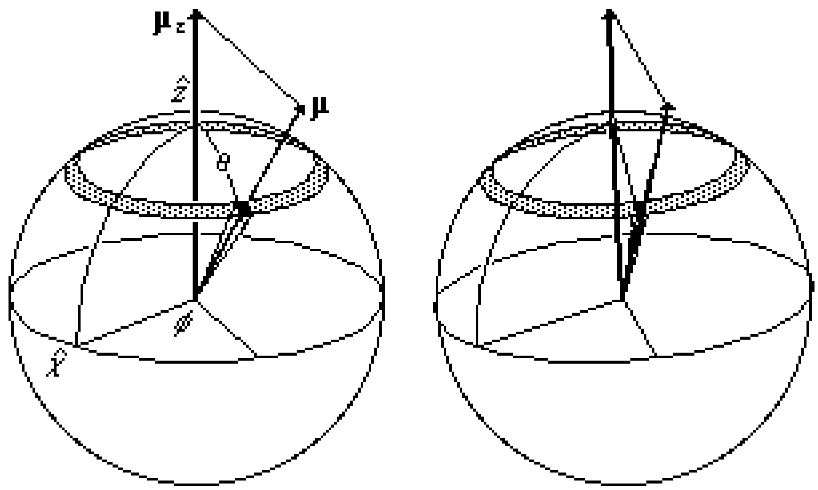

Figure 2: Stereoscopic diagram of the geometry involved in calculating the distribution of directions of randomly oriented magnetic moments and the corresponding deflection distribution. The area of the shaded portion on the unit sphere is $2\pi sin\theta d\theta$ which amounts to a fraction, $\frac{dcos\theta}{2}$, of the entire sphere. (Stare at the diagram with your eyes relaxed until the central one of the multiple images fuses into a 3-D perception.

2.1.4 DEFLECTION DISTRIBUTION FOR RANDOMLY ORIENTED MAGNETIC MOMENTS

To illustrate the difference between the classical and quantum predictions for the deflection distribution, we consider a beam of "classical" atoms with magnetic dipoles distributed uniformly in direction, each dipole making a certain angle θ with the z axis and having a projection of its magnetic moment in the z direction of $\mu_B cos\theta$, as illustrated in Figure 2. The fraction with polar angles between θ and $\theta + d\theta$ is $\frac{2\pi sin\theta d\theta}{4\pi} = -\frac{dcos\theta}{2}$, and this fraction will have a deflection distribution given by equation (8) with z_0 replaced by $z_0 cos\theta$. The fraction of all atoms that suffer deflections with z in dz is then expressed by the integral

$$I(z)dz = -\frac{dz}{2z_0}\int_0^{\frac{\pi}{2}} \left(\frac{z_0 cos\theta}{z}\right)^2 e^{-(\frac{z_0 cos\theta}{z})} d\left(\frac{z_0 cos\theta}{z}\right) \tag{10}$$

$$I(z)dz = +\frac{dz}{2z_0}\int_0^{\frac{z_0}{z}} u^2 e^{-u} du \tag{11}$$

Integrating by parts, we obtain

$$I(z) \propto 1 - \left[1 + \frac{z_0}{z} + \frac{1}{2}\left(\frac{z}{z_0}\right)^2\right] e^{\frac{z_0}{z}} dz \tag{12}$$

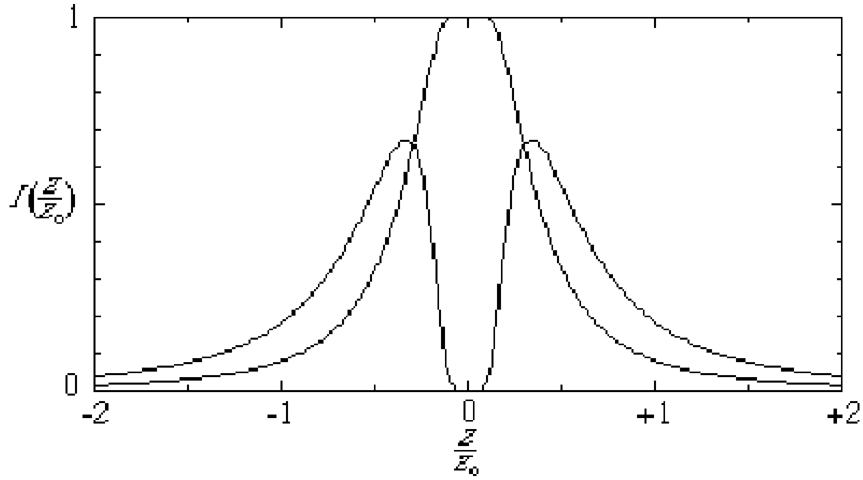

Figure 3: Predicted deflection distributions for a beam of "classical" atoms (no space quantization), and for a beam of real atoms with $j = \frac{1}{2}$.

The deflection distributions from the classical theory and for the quantum theory with $j = \frac{1}{2}$ are plotted in Figure 3 for the case of ideal narrow beams, i.e. atomic beams with negligible initial width and angular divergence.

2.1.5 Beam Width Contribution to the Deflection Distribution

In the above calculation we neglected the width and angular divergence of the beam. In our apparatus the deflection is not much greater than the width of the undeflected beam in the plane of the detector. Thus the measured current at z is a sum of contributions by atoms that have suffered deflections in a range comparable to the width of the beam. We call $g(u-z)dz$ the fraction of atoms which have been deflected by an amount u that arrive at positions in the range from z to $z + dz$. We recall that $I(u)du$ is the fraction of atoms that suffer deflections in the range from u to $u + du$. The fraction of atoms that arrive at positions between z and $z + dz$ per unit interval of z is expressed by the convolution integral

$$I'(z) = \int_{u=-\infty}^{+\infty} I(u)g(z-u)du \tag{13}$$

This simple result is based on the assumption that various parts of the beam do not influence each other by collisions, and that $g(\xi)$ is independent of z, i.e. that the profile of a beam of particles with a given deflection does not depend on the deflection.In Figure 4 we show the results of convolving the distribution function of equation (8) with a beam profile in the form of a Gaussian:

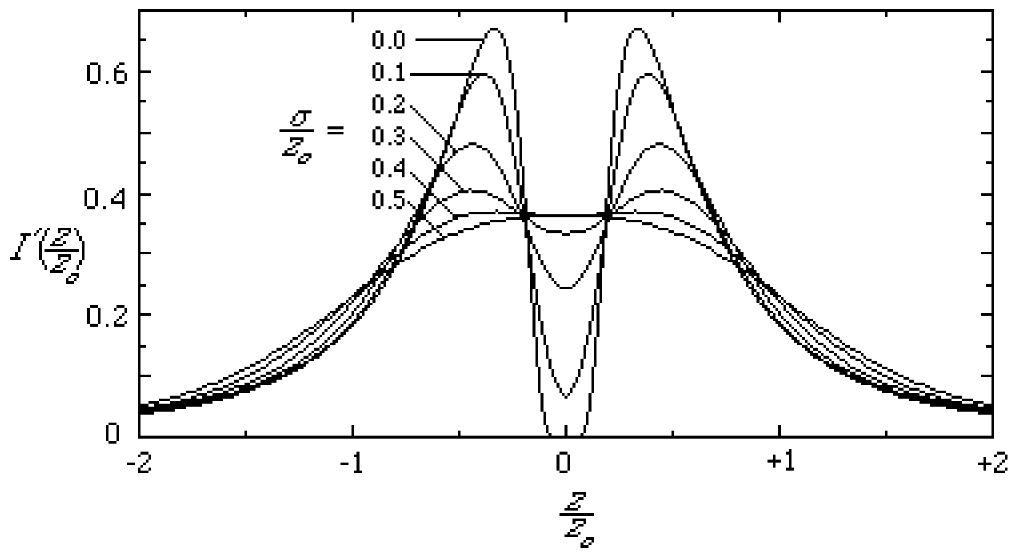

Figure 4: Calculated deflection distributions for a beam of atoms with $j = \frac{1}{2}$, convolved with Gaussian beam-spread functions with various values of σ.

$$g(\xi) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\xi_i^2}{2\sigma^2}} \tag{14}$$

One can compare the plot of data from a measurement with the theoretical curves of Figure 4 and thus estimate what value of the width parameter should be used to generate a fitting curve from the theoretical curve for a line source. Then by comparison, one can determine how much z_{max} is shifted relative to z, i.e. $\frac{1}{3}z_0 = z_{idealmax} = (?)z_{observedmax}$

In your analysis you can use the observed zero-field current distribution to estimate the beam profile function $g(x)$. In principle, you should then be able to fit the convolved function $I'(z)$ to the observed current distribution with the magnetic field turned on by adjustment of the parameters I_0 and z_0.

3 APPARATUS

The four basic parts of the equipment are:

1. **Vacuum System:** A high vacuum is needed to eliminate collisions between the beam atoms and the resident gas, and to allow the hot wire detector to work properly. A

turbomolecular pump, backed by a mechanical forepump, produces a pressure of $< 10^{-6}$ torr.

2. **Oven:** The oven, located at the far end of the apparatus, contains a slug of potassium which is gradually vaporized by the heat. The atoms of the potassium vapor shoot out through a hole in the oven and a tiny fraction of these pass through the slits to form a narrow atomic beam that traverses the inhomogeneous magnetic field between the specially shaped poles of the magnet. For a good beam, the mean free path in the oven should be large compared to the slit width which requires an operating range for temperature from roughly 180° to 250°C. The temperature is monitored with an iron-constantan thermocouple and a digital thermometer. Since the beam flux is proportional to the vapor pressure of potassium in the oven and the vapor pressure is a very steep function of temperature, you should be sure that the temperature has stabilized before taking data.

   CAUTION... $K + moistair = KOH + K_2CO_3 \cdot 2H_2O + ...$ **DO NOT TURN OFF THE VACUUM PUMPS.** In case of a power failure an inert gas (either Ar or N_2) will automatically flood the system through electromagnetically controlled valves. This prevents the oxidation and hydrolysis of the potassium.

3. **Electromagnet:** The electromagnet consists of two coaxial iron cores wound in series on a C-yoke, with iron pole pieces shaped to give a region of very non-uniform magnetic field. There is nevertheless an approximately constant value of the field gradient $\frac{\partial B_z}{\partial z}$ in the gap between the pole pieces. A schematic of the magnet cross-section is shown in the Appendix as Figure 5 The atomic beam passes along the y axis (see Figure 1) and is centered vertically at $x = 0$. The field gradient, which is what concerns us here, is not observed directly, but is calculated in terms of $\vec{B}$ itself. The field strength has been measured previously as a function of magnet current; the results are summarized in Figure 6 of Appendix I. Given a value of the magnet current, you can use Figure 6 to find the corresponding value of $\vec{B}$ from which you can calculate $\frac{\partial B_z}{\partial z}$ as explained in Appendix I.

   The magnet current is furnished by a regulated power supply, and is controlled by a rheostat. The current is read coarsely by a panel meter on the supply and accurately on a large Weston ammeter connected in series with the magnet coils. The procedure for setting the magnetic field is as follows: for measurements of the undeflected beam shape, "degauss" the magnet by bringing the current to -5 amps, then to +5 amps, and then to -0.65 amps. Then turn the current off; the field should now be less than about 20 gauss, not enough to be troublesome. In setting any other magnetic field, run the magnet up to 5 amps and back down, first in one direction and then the other, three or four times before approaching your final current value. This will place you securely on the hysteresis curve plotted in Figure 6. It is essential to follow the same procedure used when B was measured as a function of current, which is to approach the final current value from below (i.e., from lesser magnitude). If you approach it from above, the magnet curve depends much more sensitively on the previous peak value of the current.

4. **Detector:** The detector is a 4 mil diameter platinum wire, heated to about 1300° C, and biased about 15 volts above ground. The potassium atoms are detected only if they hit the wire, whereupon they are ionized and boiled off. (For details of hot wire detectors see Smith (1955) or Ramsey (1956)). The ions are swept to a rectangular collector plate on one side of the wire by the bias voltage, and the resulting positive current goes to ground through a shielded coaxial cable and the electrometer. The detector is mounted on an assembly which may be moved by a micrometer head. To figure the z displacement of the detector, note that the detector to pivot distance is 0.957 ± 0.005 times the pivot to micrometer distance. The micrometer is swept through its range with a reversible motor. There is also a voltage divider attached to the motor drive that is used to advance the x-axis signal of the display; the y-axis signal comes from the $0 - 2$ volt analog output on the back of the Keithly electrometer. The heater current should be in the range from 0.5 to 0.6 amps (0.53 amps has worked well). Be careful not to exceed the posted maximum current.

   When you finish a session switch off the magnet power and go to the central peak-current position to verify that everything if working properly for the next group. Then close the flap door between the magnet and the detector, and turn off the electrometer, the bias voltage on the hot wire, and the computer. Reduce the hot wire heater current to approximately 0.2 amps and leave it on. If you are working in the afternoon session and leave around 5:00 pm, reduce the oven heater current to approximately 3.5 amps and leave it on.

4 PROCEDURE

Set the oven temperature to about 190 ± 5°C. Turn on the digital electrometer and set the sensitivity to 2 picoamp full scale. Turn on the power supply for the positioning motor of the movable hot wire detector. Turn on the magnet power supply and degauss the magnet, leaving it in a demagnetized state. Open the flap door between the magnet and the detector section. Set the detector hot wire collector bias to 15 volts and the heating current to a value between 0.5 and 0.6 amps. (You may find it beneficial to run the heater current at 0.8 amps for a few minutes to clean the wire; in any case, do not exceed 1.0 amps.) Move the detector across its position range and observe the electrometer reading. If you have a beam the electrometer should register a significant current as you approach the center position of the hot wire. If you do not find any signal, get expert help.

1. Properties of the hot wire detector. Study the behavior of the signal strength and the signal-to-noise ratio as a function of the hot wire current in order to determine the optimum current for measuring the beam profile and deflection distribution. To do this, first check that the oven temperature has reached equilibrium and is steady. Then record the undeflected beam profile (a plot of electrometer current versus hot wire position) for several hot wire currents. Analyze the profiles as you go along, and plot both the central beam electrometer current minus background and the ratios of central current to background current as functions of the hot wire current. The final

choice of the operating hot wire current for the magnetic deflection measurements is a compromise. Too high a hot wire current causes a large and unstable background electrometer current. (You can readily find the zero reading by temporarily closing the vacuum gate). Too low a hot-wire current causes a slow detector response. You can check this by moving the detector rapidly between on-peak and off-peak positions. The time constant of the response should be no more than a few seconds.

Data can be recorded either directly by the PC and fit using any analysis package (C, C++, Matlab, LabVIEW, etc.. Be sure to calibrate the X-axis readings against the actual positions of the micrometer.

2. Position of the beam. The deflection of the beam depends critically on its position in the gap of the magnet. The position and orientation of the beam can be adjusted by moving the oven which is mounted on a platform that permits lateral and angular motions. In addition, the position of the aperture at the entrance to the magnet is controlled by a micrometer. With so many degrees of freedom it is easy to lose the beam and get lost in a multi-dimensional parameter space. So any adjustment of the positions of oven and aperture should be made in small steps and carefully noted so that you can return to any previous position. The following procedure is suggested as a way to find the optimum center position for the beam, assuming that you are starting with a significant signal (if there is no evidence of a signal when you scan the detector (zero field) across the path, call for help.):

   Move the hot wire to the point of maximum signal, and note the micrometer reading, the lateral position of the oven (read on the digital display) and the angular position (read on the dial gauge). Then move the oven laterally in a small step, taking care not to go so far as to lose the signal; move the aperture and the wire to maximize the signal. Note all the position readings. Take another lateral step, and repeat the procedure till you reach the point where the beam is blocked by the edge of the magnet. Now rotate the oven slightly one way and the other to see if the signal is restored. If not, then you can assume the beam is parallel to the gap. If you do get back some signal, then move laterally a little further, and repeat the angle motion test. This procedure will establish the parallelism of the beam and the position of one side of the magnet gap. Now go to the other side in the same manner. Knowing the oven positions corresponding to the two sides of the magnet gap, you can place the oven so the beam is in the middle of the gap. (In all of this, take care never to completely lose the signal so that you always have a way to crawl back to safety from a dangerous position of a nearly lost beam.)

3. Measurement of the angular momentum quantum number and magnetic moment of the ground state of the potassium atom. With the hot wire current set at the optimum value, measure the beam profile with zero deflecting field. Then turn on the inhomogeneous magnet field and measure the deflection distribution.

5 ANALYSIS

This experiment is essentially a procedure for measuring the possible values of the projection of the magnetic moment μ_K of the potassium atom in its ground state. It is remarkable that the quantity z_o combines all the experimental parameters: the dimensions of the apparatus, the field gradient $\frac{\partial B_z}{\partial z}$, the length of the magnet, the oven temperature, the mass of the potassium atom and its magnetic moment. On the one hand, this allows one to use the entire curve to extract the one unknown quantity, $\mu_K = mg\mu_B$. On the other hand, it means that we have no way of deducing errors in the other quantities from any combination of our data.

One can express μ_K in terms of z_o which can, in turn, be expressed in terms of the various measured and derived quantities (cf. equation (6 above). The quantity $\frac{\partial B_z}{\partial z}$ is evaluated according to equations 23 and 16 of Appendix I. One can also express the probable error $\Delta\mu_K$ in terms of the probable errors in these parameters and in z_o. You should estimate the uncertainty in each of the measured quantities and deduce the precision with which z_o must be determined so that it will not seriously affect the precision of the measurement of μ_K.

In order to compare our expression for $I'(z)$ with the observed current, we need two other quantities: the position of the undeflected beam and the area under the current versus deflection curve. Both of these are determined from the experimental zero-field curves. The area under the zero-field curve is A, which should remain the same for the deflected current versus position curves provided the oven temperature and the detector sensitivity are constant.

If time permits, determine the dependence of A on oven temperature. Compare your result with the vapor pressures of potassium listed in the CRC Handbook. Estimate the uncertainty in z_o introduced by possible temperature variations.

For our procedure, as outlined below, the position of the undeflected beam can be taken as the centroid of the zero-field curve:

$$z(0) = \frac{1}{A}\int zI_0(z)dz \tag{15}$$

The curve of observed current versus detector position is the convolution of equation (8) for $I'(z)$ with the zero-field curve that we can use for the beam profile $g(x)$. While this seems plausible, it is by no means self-evident. For example: 1. What if the magnet is not degaussed? How would that affect the validity of our assumption? Can you estimate a reasonable value for this effect? 2. Some of the broadening of the resolution curve is due to scattering of the beam. The scattering cross section depends upon the velocity of the atoms. Therefore the effective width will vary with z. Can you devise a procedure to investigate the contribution of beam scattering?

Note that our expression for I'(z) has no adjustable parameters except z_o. If we choose the correct value of z_o and a correct beam profile function, every point on the curve should be predicted correctly without any area normalization or separate determination of the zero position.

5.1 Questions

1. Determine the angular momentum quantum number of the ground state electronic configuration of potassium.

2. Calculate μ_K for potassium, with a careful error estimate.

3. Turn the analysis around: assume the measured value of $\vec{B}$ and the accepted value of μ_K from the CRC Handbook) and calculate the value of the oven temperature. Compare with the measured temperature of the oven.

4. What are the particle densities and mean free paths in the oven and in the beam?

5. About how many cycles of precession do the atoms undergo while passing through the magnet?

6. (Optional) Estimate the variations of F_z and of the other, neglected components of the forces on the magnetic dipoles in a beam of finite divergence. For example, one component of force slows down one half of the beam. Which component of the field interacts this way with which half of the beam, and how much? To do this quantitatively, you need to generalize the equations in Appendix I to get $\vec{B}$ and its derivatives off-axis.

7. Kinetic Theory and Statistical Mechanics 1. What are the particle number densities of a) the residual vacuum; b) the beam at various locations; and c) the average distance between beam particles?

8. Why don't you have to consider the wave nature of the beam - i.e. why does geometric beam optics work?

9. Are the deflection versus intensity plots you generate useful in measuring the Maxwellian velocity distribution?

10. Why can you think of the iron pole faces as scalar magnetic equipotentials?

REFERENCES

[1] Bohm, D.; Quantum Theory (Prentice-Hall, NJ, 1951).

[2] Eisberg, R.H.; Fundamentals of Modern Physics (Wiley, NY, 1961) p. 327-338.

[3] Feynman, R.P.; Lectures on Physics (Addison-Wesley, NY, 1966), Vol. II, III.

[4] Friedrich, B., and Herschbach, D., *Daedalus,* **127**,*1, 165, 1998.*

[5] *Gerlach, W. and Stern, O.; "Concerning the Quantization of Direction in a Magnetic Field" Annalen der Physik, IV folge 74, 1924.*

[6] *Jackson, J.D.; Classical Electrodynamics (Wiley, NY, 1962).*

[7] *Liboff, R. L.; Introductory Quantum Mechanics (Holden-Day, Oakland, 1980), p 461.*

[8] *Platt, D., "A modern analysis of the Stern-Gerlach experiment", Am. J. Phys., 60, 306, 1992.*

[9] *Ramsey, N.F.;Molecular Beams (Oxford University, London, 1956).*

[10] *Smith, K.F.; Molecular Beams (Methuen, London, 1955).*

[11] *American Physical Society Handbook (McGraw-Hill, N.Y., 1972).*

SUGGESTED THEORETICAL TOPICS

1. Quantization of angular momentum.
2. The Maxwell-Boltzman distribution.
3. The magnetic field of the deflecting magnet.
4. The hyperfine structure of the potassium ground state.

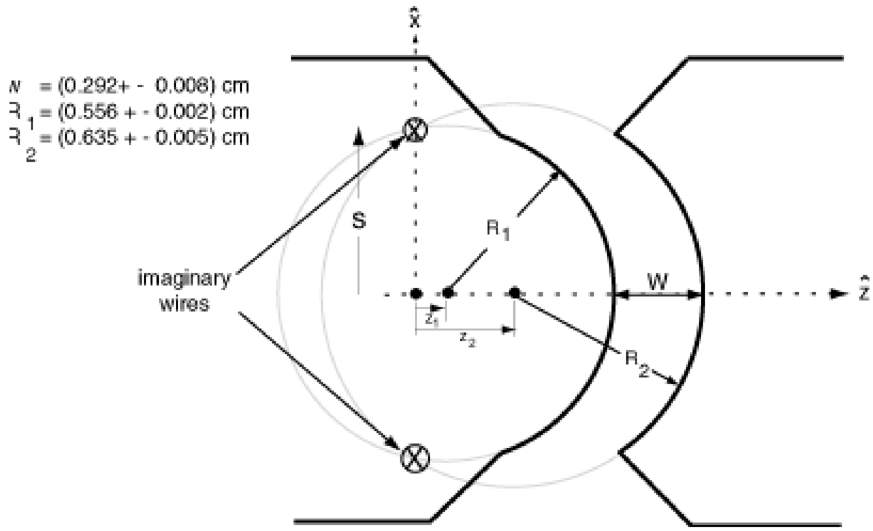

Figure 5: Elevation view of the magnet gap looking from the detector toward the oven. The beam of potassium atoms should be thought of as coming out of the paper at approximately the position of the letter w.

6 APPENDIX I

Calculation of the Magnetic Field The magnetic field in the gap between arcs of two cylindrical surfaces of infinite permeability can be calculated exactly. It is the same as the field from the currents in two long imaginary wires running along two lines parallel to the intersections of the cylindrical surfaces. In the coordinate system of Figures 1 and 5, the wires are located at $x = s, z = 0$, and they carry equal and opposite currents of a magnitude we will call a.

The subscript 1 refers to the "inner" radius, that of the pole piece which has a convex surface; 2 refers to the concave "outer" pole piece. It turns out that one can choose these imaginary currents so as to satisfy the boundary conditions on the magnetic field in the gap, namely that $\vec{B}$ must be perpendicular to the surfaces of the pole pieces because $\frac{\mu_0}{\mu_{iron}} \approx 0$ (In this context μ_0 is the the magnetic permeability of the vacuum). The uniqueness theorem for potential functions then justifies the calculation of the field elsewhere in the gap as the superposition of the fields of the two wires.

The required geometric relations that must be satisfied in the construction of the pole pieces are expressed by the three equations:

$$s^2 + z_1^2 = R_1^2, s^2 + z_2^2 = R_2^2, z_1 + R_1 + W = z_2 + R_2 \tag{16}$$

For $\vec{B}$ to be perpendicular to a surface it is necessary and sufficient that $\frac{B_x}{B_z} = -\frac{\partial z}{\partial x}$ along the surface, where the right side describes the geometry of the cylindrical surface. We will demonstrate that this condition is true on the outer surface only, but then will see that in doing so we have proved our case for the inner surface, too. The equation for the outer surface is:

$$x^2 + (z - z_2)^2 = R_2^2 \tag{17}$$

Thus,

$$-\left(\frac{\partial z}{\partial x}\right)_{outer} = \frac{x}{z - z_2} \tag{18}$$

To evaluate B_x and B_z , we add up the contributions to each from the two wires. In mks units, the contributions from the first wire, carrying a current a, are:

$$B_{1x} = \frac{\mu_0 a}{2\pi} \frac{z}{(x-s)^2 + z^2} \tag{19}$$

and

$$B_{1z} = \frac{\mu_0 a}{2\pi} \frac{-(x-s)}{(x-s)^2 + z^2} \tag{20}$$

Similar expressions give B_{2x} and B_{2z}, but with $(x + s)$ replacing $(x - s)$, and the signs of both components reversed. Dividing the sum of the x components by the sum of the z components we obtain

$$\frac{B_x}{B_z} = \frac{2zx}{z^2 - x^2 + s^2} \tag{21}$$

Does this equal $\frac{x}{z-z_2}$? If we substitute for s^2 from $s^2 = R_2^2 - z_2^2$ and for x^2 on the outer surface from $x^2 + (z - z_2)^2 = R_2^2$ we indeed find that this is so. Note that R_2 cancels out, so that the result is true for the inner surface too. Next we evaluate the magnitude of $\vec{B}$, but for simplicity only on the axis. The x components cancel out, leaving

$$|\vec{B}|_{onaxis} = B_z = \frac{\mu_0 a}{\pi} \frac{s}{s^2 + z^2} \tag{22}$$

We will use this z dependence to correct B_z as a function of z, since it was measured at the "wrong" place. The gradient of the z component of the field along the axis is

$$\frac{\partial B_z}{\partial z} = -\frac{2zB_z}{s^2 + z^2} \tag{23}$$

Thus the measured B in Figure 6 should be corrected to the right z : then you can calculate the gradient from it. The job of finding an expression for s , and hence B and $\frac{\partial B}{\partial z}$ at the center of the gap, is left to the reader, given the input values for R_1, R_2, and W in Figure 5 and the connecting equations above.

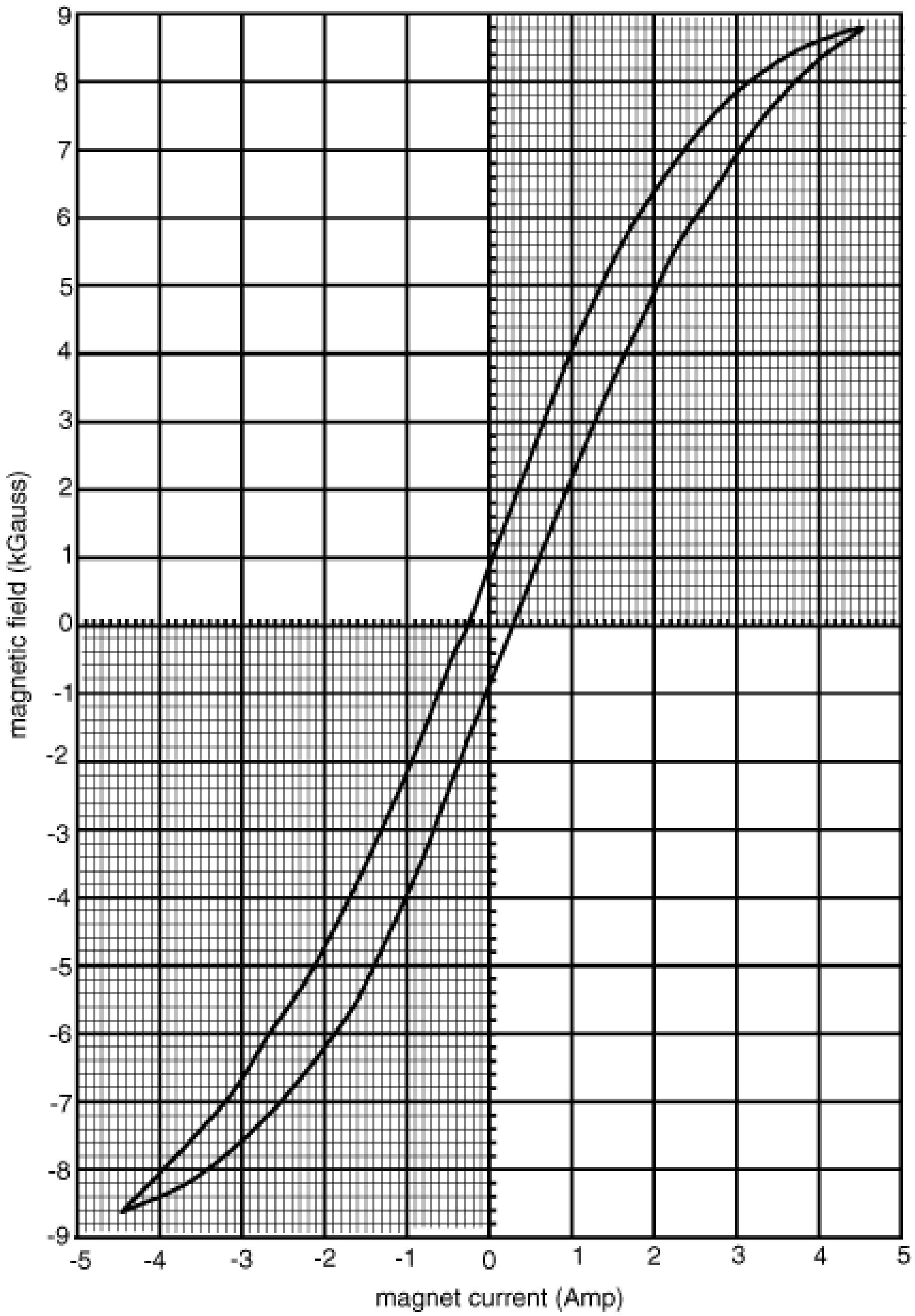

Figure 6: Magnetic Field Measured near the Convex Pole Face. Center of probe was 0.010" from the convex pole piece.