Pressure of Light

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PREPARATORY PROBLEMS

1. Derive the pressure exerted on a surface by a light beam of power W striking at normal incidence. Do it two ways:

   a. Consider an electromagnetic wave with an intensity (energy/area/time) impinging on the surface. Assume the material has a conductivity σ. The electric field causes a current in the plane of the surface thereby generating a certain amount of heat by ohmic dissipation. The magnetic field exerts a force on the current which is transmitted to the material, thereby exerting pressure (force/area). Show how this pressure is related to the intensity.

   b. Use the relativistic equation for the relation between the rest mass, momentum, and total energy of a particle.

2. Solve the equation of angular motion of a torsion pendulum under the action of a harmonic restoring torque -$\kappa\Theta$ and a constant torque τ (e.g., due to light pressure). Assume the pendulum is at rest at the equilibrium position when the torque is applied. Derive expressions for the amplitudes of the oscillations that results from applying the torque for exactly half a period, and for a full period.

3. Add a term to the pendulum equation to represent a possible steady increase of a radiometer-type torque that may result from a gradual heating of the mirror due to partial absorption of the laser beam. Solve the new equation if you can, perhaps with the aid of a Laplace transform.

INTRODUCTION

Imagine two separated charges, each tethered by a spring and at rest. A kick given to one charge will cause the other to move after a delay due to the finite propagation time of the disturbance in the electromagnetic field. Thus both energy and momentum, conveyed by the field, must exist in the field during the delay. According to the Maxwell theory, a plane electromagnetic wave delivering energy to an absorbing surface at a rate P exerts on the surface a force P/c, where c is the speed of light. The large value of c implies that the force exerted on a body by a light beam of ordinary intensity is very small indeed. The pressure of bright sunlight on a black surface with an area of one square centimeter is only about 5×10^{-4} dynes. Yet, the tiny force of sunlight on the dust particles shed by a comet can produce a spectacular cometary tail. And the huge pressure of radiation from a fission bomb compresses the fuel of a hydrogen bomb and thereby triggers the fusion reaction.

The pressure of light is measured in this Junior Lab experiment in a way similar to that of Nichols and Hull who did the classic experiment at Dartmouth College in 1903. They measured the force exerted on a mirror mounted on a sensitive torsion pendulum suspended by a quartz fiber in a low vacuum. Others had previously failed in similar attempts due to the masking effects of gas kinetics and convection in the relatively poor vacuums attainable at the time. It is those effects that cause the vanes of the common radiometer to spin in the presence of a bright light. The report of Nichols and Hull in the Physical Review describes the elaborate procedures they devised to surmount the complex problem of "radiometer" effects.

The present experiment employs a much higher vacuum than was attainable in 1903. This greatly reduces the radiometer effects of the 0.2 watt light beam provided by a green YAG laser for this experiment. Other critical components of the experimental setup are a commercial laser power meter, and a highly reflective mirror mounted on a torsion pendulum suspended by a long 1-mil tungsten wire, all unavailable in 1903.

EXPERIMENT

CAUTION: TAKE EXTREME CARE NOT TO ALLOW THE LASER BEAM OR ANY OF ITS SPECULAR, i.e., NON-DIFFUSE, REFLECTIONS TO ENTER YOUR EYE. THE DIRECT UNFOCUSED BEAM IS TOO WEAK TO BURN YOUR SKIN. HOWEVER, BECAUSE IT IS HIGHLY COHERENT, IT IT ENTERS YOUR EYE IT WILL BE FOCUSED TO A SPOT SO TINY THAT THE INTENSITY WILL APPROACH $I = P/\lambda^2$ WHERE P IS THE POWER AND λ IS THE WAVELENGTH. THE YAG LASER EMITS A BEAM WITH $P \approx 0.2 watts$ AND $\lambda \approx 5 \times 10^{-5}$ and $I \approx 8 \times 10^7 watts/cm^2$. SO, MUCH MORE THAN ENOUGH TO BURN A PERMANENT HOLE IN YOUR RETINA!

You should find the torsion pendulum properly suspended, i.e. freely swinging, and under a vacuum of better than 10^{-5} torr. If it isn't, please ask your instructor for help. For practical reasons the weights and dimensions of the pendulum were measured before the apparatus was assembled and put under vacuum. The results you need to compute the moment of inertia are provided in Figure 1.

Your tasks are:

1. Determine the pendulum period.
2. Stop the pendulum at its equilibrium point with the aid of the electromagnet acting on the damper vane mounted on the pendulum opposite to the mirror.
3. Measure the power of the laser beam emerging from the laser and after reflection from the pendulum mirror and transmission back and forth through the front glass plate of the vacuum chamber.
4. Measure the amplitude of the pendulum motion after reflecting the laser beam from the mirror for half a period.
5. Measure the amplitude of the pendulum motion after a full-period exposure.
6. Repeat the half period and full period measurements at least three times.
7. Compute the moment of inertia of the pendulum from the data given below.
8. Evaluate the radiometer effect from the full period swings. Compute the force exerted by the laser beam on the mirror during the half period exposures with correction for the radiometer effect.
9. Compare the result with the theoretical prediction.

You will measure the angular position of the pendulum by viewing through the reflection of a millimeter scale in the mirror mounted on one arm of the pendulum. A brass vane is attached to the other arm so that as the pendulum turns about its equilibrium position, the vane passes between the poles of an electromagnetic connected through a vacuum feedthrough to an external power supply. By adjusting the control on the power supply you can control the strength of the magnetic field in a manner that can bring the pendulum to rest at its equilibrium position.

Make sure the pendulum is swinging freely, i.e. that the fiber hangs freely without touching the brass tube except at the top point of suspension. Check that the lateral swing frequency agrees with what you expect for an ordinary pendulum with its length of approximately 1 m. The torsion period should be somewhat more than 4 minutes. Practice controlling the pendulum swings with the internal electromagnet. To damp the torsion oscillations if they are large and to bring the pendulum to rest at its neutral position you can use a combination of Eddy current braking and paramagnetic force. If the torsion amplitude is large, apply full current to the electromagnet when the damper vane is passing between the jaws of the magnet. When the amplitude is reduced to the degree at which the damper vane is always between the jaws, you will find that applying the current pulls the pendulum one way due to the unsymmetrical attractive force of the non-uniform field on the paramagnetism of the brass plane. Judicious application of this force can be used to bring the pendulum to rest at its zero current equilibrium position. Avoid leaving the power supply on for more that about 20get destructively hot with no convection cooling in the vacuum).

Adjust the orientation of the YAG laser so its beam reflects from the center of the mirror when it is near its equilibrium position. Measure the powers of the direct and the reflected beam (be sure the reflected beam is the one reflected from the mirror on the pendulum and not one of the beams reflected from the outside or inside surfaces of the vacuum chamber window.

When the vacuum is 10^{-5} torr or better, and you have stopped the pendulum at or very near its equilibrium position (amplitude not more than 1 or 2 mm on the reflected scale), open the laser shutter, start the stopwatch, and allow the beam to push on the mirror for exactly one-half the pendulum period. (You will not be able to follow the motion of the pendulum because the scattered laser light makes viewing the scale impossible.) Commence recording the scale reading immediately after the laser is shuttered, marking time from the start of the exposure. Do the experiment again, but with an exposure of one full period. Repeat the half period and full period experiments at least three times, starting each exposure with the pendulum at rest at the equilibrium position.

ANALYSIS

Call Θ the angular displacement of the torsion pendulum with a moment of inertia I from its torque-free position of equilibrium under the action of a constant torque τ and a harmonic restoring torque -$\kappa\Theta$. Assume the initial conditions are $\Theta = 0$, $\dot{\Theta} = 0$, and $t = 0$ at The equation that describes the motion is

$$\Theta(t) = \frac{\tau}{\kappa}(1 - cos\omega t) \ ,$$

where the angular frequency ω is related to the torsion constant κ and the period P by

$$\omega = \sqrt{\frac{\kappa}{I}} = \frac{2\pi}{P} \ ,$$

When $t = P/2$ i. e. one half period, $\Theta_{1/2} = 2\tau/\kappa$. With a little algebra one can express the torque τ in terms of measured quantities $I, \Theta_{1/2}$, and P. The torque is related to the force F of the laser beam on the mirror by

$$F = \tau/l \ ,$$

where l is the moment arm of the force applied to the pendulum by the laser beam. Finally, the force of the beam is (taking account of the kick back exerted by the portion of the beam that is reflected)

$$F = \frac{WT}{c}(1+R) \ ,$$

where W is the power of the beam as it emerges from the laser, T is the transmission coefficient of the vacuum chamber window, and R is the reflection coefficient of the mirror. The power of the reflected beam after passing back through the chamber window is

$$W' = WRT^2 \ .$$

Thus

$$F = \sqrt{WW'}\tfrac{1+R}{\sqrt{R}} \ ,$$

As you can verify, the quotient $(1+R)/\sqrt{R}$ does not vary much from 2 for $R > 0.9$. Since the reflectivity of this special mirror is certainly greater than 0.9, one can assume the quotient is 2 without suffering a significant error.

Suppose that in addition to a harmonic restoring torque and a constant torque the pendulum is subject to a "radiometric" torque increasing linearly with time, such as might be caused by a steady heating of the mirror due to absorption of a portion of the laser power. Call such a torque. then, with the same initial conditions, the motion is described by the equation:

$$\Theta(t) = \frac{\tau}{\kappa}(1 - cos\omega t) + \frac{\alpha}{\kappa}(t - \frac{1}{\omega}sin\omega t)$$

There are two possible approaches to deriving the force, F, of the the laser beam on the mirror. One approach is to compute the moment of inertia, I, of the pendulum from the dimensions and weights of the pendulum and its components. Then, the force can be computed according to the equation:

$$F = \tfrac{I}{8L}(\tfrac{2\pi}{T}\tfrac{\delta x}{D}$$

where L is the distance from the point of suspension to the center of the mirror.

The second approach is to determine the torsion constant by measurements of the period of a torsion pendulum comprised of a bob of known moment of inertia suspended by a tungsten wire from the same spool. To facilitate this approach there is available such a pendulum with a spherical bob of uniform composition whose weight can be measured by raising a balance over which it is suspended so that the bob is supported on the balance table.

The relevant weights and dimensions of the pendulum components are given in Figure 2. The problem of computing the moment of inertia is left to you.

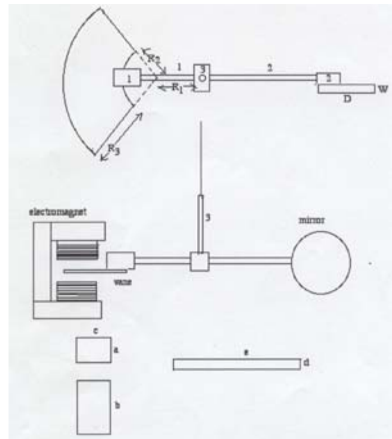

FIG. 1: Schematic diagram of the torsion pendulum. The labelled dimensions are tabulated in Table 1.

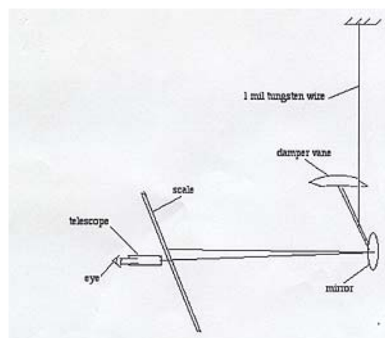

FIG. 2: Schematic diagram of the setup for measuring the angular position of the torsion pendulum.

SUGGESTED THEORETICAL TOPICS

1.