MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Physics

8.14 Analysis Exercise: Gaussian Or Lorentzian ?

In the experiments of Dopplerfree, Moessbauer, QIP, and Zeeman you will have to fit line-shapes (or dips).

From the Uncertainty Principle we know: $\Delta E\, \Delta t \geq h/2$, for a wavepacket, which translates to: $\Gamma\, \tau \geq h$ for a resonant line width Γ (FWHM) and lifetime τ. There is an interesting relation between Γ and τ, by the fact, that the energy Fourier transform of an exponential decay in time results into a Lorentzian (non-relativistically as in 8.14). An easy derivation follows:

Emission of a Spectral line is a damped vibration of an oscillator ω_0:

:Time dep amplitude:

$$f(t) = C \cdot e^{i\omega t} e^{\gamma t} \quad \text{with} \quad \int_{-\infty}^{\infty} |f(t)|\, dt = C^2 \int_{-\infty}^{\infty} e^{2\gamma t} dt = \frac{C^2}{2\gamma} = 1 \quad \text{hence} \quad C = \sqrt{2\gamma}$$

Complete Amplitude: $$f(t) = \sqrt{2\gamma}\, e^{i\omega_0 t} e^{-\gamma t}$$

Fourier transform: $$F(\omega) = \sqrt{\frac{\gamma}{\pi}} \cdot \int_{-\infty}^{\infty} e^{-i\omega t} e^{i\omega_0 t} e^{-\gamma t} dt = \sqrt{\frac{\gamma}{\pi}} \frac{i}{\omega_0 - \omega + i\gamma}$$

Intensity(ω) $$|F(\omega)|^2 = \frac{\gamma}{\pi} \frac{1}{(\omega_0 - \omega)^2 + \gamma^2} \quad \text{with} \quad \int |F(\omega)|^2\, dt = 1$$

Lorentzian

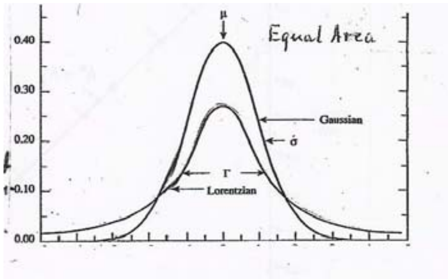
$$\boxed{I(\omega) = I_0 \frac{\Gamma / 2\pi}{(E_0 - E)^2 + \Gamma^2 / 4} \quad \text{with } \Gamma = 2\hbar\gamma,\ \mathrm{E} = \hbar\omega}$$

Compare this line shape to a Gaussian

you will see a substantial difference.

$$I(E, E_{0,} \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{E - E_0}{\sigma}\right)^2\right]$$

with FWHM $2.354 \cdot \sigma = \Gamma$



What is the point? Well the physically interesting quantity is the “Natural Line Width Γ”, whereas resolution effects, the Doppler broadening, and stochastic errors will make the line-shape more Gaussian – recall the Central Limit Theorem. So, if you want to claim that you measured a Natural Line better prove that the shape is right.