6 Radio-Telescope Antennas

6-1 Introduction An antenna may be defined as the region of transition between a free-space wave and a guided wave (receiving case) or vice versa (transmitting case.)† The antenna of a radio telescope acts as a collector of radio waves. The antenna is analogous to the lens or mirror of an optical telescope.

The response of an antenna as a function of direction is given by the antenna *pattern*. By reciprocity this pattern is the same for both receiving and transmitting conditions.

The pattern commonly consists of a number of lobes, as suggested in Fig. 6-1*a*. The lobe with the largest maximum is called the *main lobe*, while the smaller lobes are referred to as the minor lobes or side and back lobes.

If the pattern is measured at a sufficient distance from the antenna so that an increase in the distance causes no change in the pattern, the pattern is the *far-field pattern*. Measurements at lesser distances yield *near-field patterns*, which are a function of both angle and distance. The pattern may be expressed in terms of the field intensity (*field pattern*) or in terms of the Poynting vector or radiation intensity (*power patterns*). Figure 6-1*a* is a power pattern in polar coordinates. To show the minor-lobe structure in more detail the pattern can be plotted on a logarithmic or decibel scale (decibels below main-lobe maximum). Figure 6-1*b* is an example of a pattern on a decibel scale in rectangular coordinates. The pattern in Fig. 6-1*b* is the same as the one in Fig. 6-1*a*.

A single pattern, as in Fig. 6-1, would be sufficient to completely specify the variation of radiation with angle provided the pattern is symmetrical. This would mean, in the case of Fig. 6-1*a*, that the three-dimensional pattern is a figure of revolution of the one shown around the pattern axis. If the pattern is not symmetrical, a three-dimensional diagram or a contour map is required to show the pattern in its entirety. However, in practice two patterns, one like that in Fig. 6-1*a* through the narrowest part of the lobe and another perpendicular to it through the widest part of the lobe, may suffice. These mutually perpendicular patterns through the main-lobe axis are called the *principal-plane patterns*. The above statement assumes that the antenna is linearly polarized in one of the principal planes. If this is not the case, more patterns may be required.

† For a more general discussion of antennas and their basic properties see, for example, Kraus (1950).

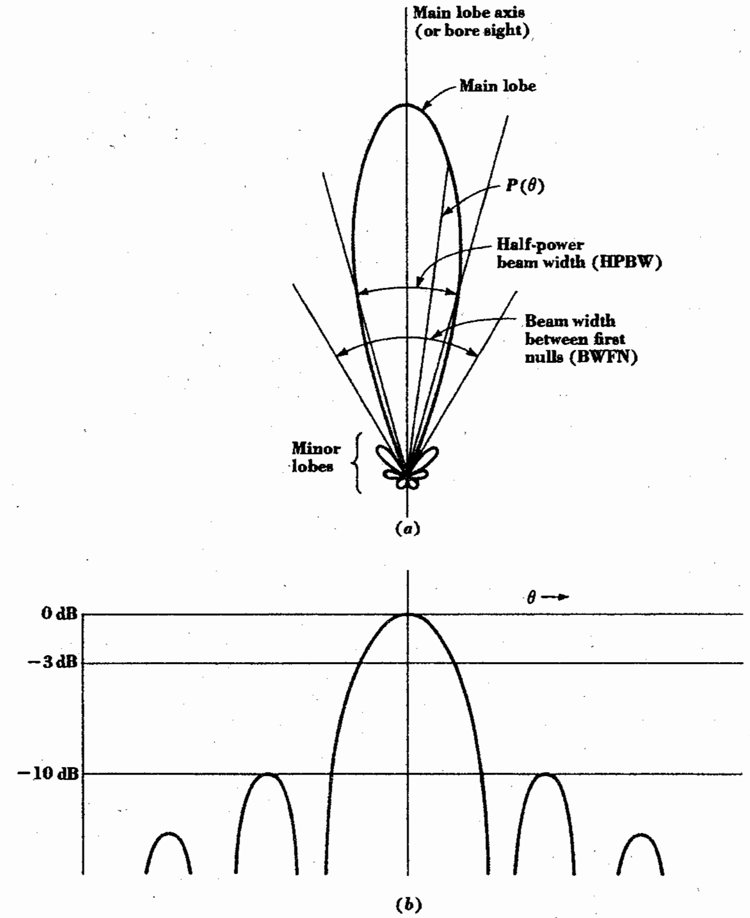

Fig. 6-1. (*a*) Antenna pattern in polar coordinates and linear power scale; (*b*) antenna pattern in rectangular coordinates and decibel power scale.

As an example, the dominant radiation from an antenna might be linearly polarized in one principal plane, but the radiation from some minor lobes might be cross-polarized, i.e., linearly polarized in the principal plane at right angles. Or the antenna might be elliptically polarized. A discussion of such patterns and their measurement is given by Kraus (1950, Chap. 15).

6-2 Beam Width, Beam Solid Angle, Directivity, and Effective Aperture A useful numerical specification of the pattern can be made in terms of the angular width of the main lobe at a particular level. The angle at the half-power level or *half-power beam width* (HPBW) is the one most commonly used. The *beam width between first nulls* (BWFN) or the beam widths at −10 or −20 dB below the pattern maximum are also useful.

Another significant way of describing the pattern is in terms of *solid angle*. Let the relative antenna power pattern as a function of angle be given by $P(\theta,\phi)$ [$= E(\theta,\phi)E^*(\theta,\phi)$, where $E(\theta,\phi)$ is the far-field pattern] and its maximum value by $P(\theta,\phi)_{max}$†. Then (Kraus, 1950),

$$\Omega_A = \iint_{4\pi} P_n(\theta,\phi)\, d\Omega \qquad (6\text{-}1)$$

where Ω_A = beam solid angle, rad²
$P_n(\theta,\phi) = P(\theta,\phi)/P(\theta,\phi)_{max}$ = normalized antenna power pattern, dimensionless
$d\Omega$ = elemental solid angle ($= \sin\theta\, d\theta\, d\phi$), rad²

The *beam solid angle* Ω_A is the angle through which all the power from a transmitting antenna would stream if the power (per unit solid angle) were constant over this angle and equal to the maximum value. This is suggested in Fig. 6-2.

In (6-1) the integration is carried out over a solid angle of 4π. If the integration is restricted to the main lobe, as bounded by the first minimum, the *main-beam solid angle* is obtained.‡ Thus,

$$\Omega_M = \iint_{\substack{\text{main}\\ \text{lobe}}} P_n(\theta,\phi)\, d\Omega \qquad (6\text{-}2)$$

where Ω_M = main-beam or main-lobe solid angle, rad²

It follows that the *minor-lobe solid angle* Ω_m is given by the difference of the (total) beam solid angle and the main-beam solid angle. That is,

$$\Omega_m = \Omega_A - \Omega_M \qquad (6\text{-}3)$$

If the antenna has no minor lobes ($\Omega_m = 0$), we have $\Omega_A = \Omega_M$.

† $E^*(\theta,\phi)$ is the complex conjugate of $E(\theta,\phi)$. $P(\theta,\phi)$ is proportional to the Poynting vector $S(\theta,\phi) = E(\theta,\phi)E^*(\theta,\phi)/Z$, where Z = intrinsic impedance of the medium.

‡ Ω_A, the beam solid angle, should not be confused with Ω_M, the solid angle of the main lobe or main beam. Ω_A is the solid angle of the entire antenna pattern, so that *pattern solid angle* might be a more appropriate name for Ω_A. However, beam solid angle has been in wide use for many years for Ω_A.

In patterns for which no clearly defined minimum exists the extent of the main lobe may be somewhat indefinite, and an arbitrary level such as −20 dB can be used to delineate it.

Another important antenna parameter is the *directivity*, which may be defined as the ratio of the maximum radiation intensity (antenna transmitting) to the average radiation intensity, or

$$D = \frac{U(\theta,\phi)_{max}}{U_{avg}} \qquad (6\text{-}4)$$

where $U(\theta,\phi)_{max}$ = maximum radiation intensity, watts sr⁻¹
U_{avg} = average radiation intensity, watts sr⁻¹

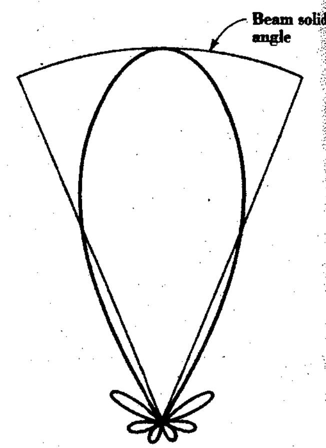

Fig. 6-2. Relation of beam solid angle to antenna pattern.

The average radiation intensity is given by the total power W radiated divided by 4π, and the total power is equal to the radiation intensity $U(\theta,\phi)$ integrated over 4π. Hence,

$$D = \frac{U(\theta,\phi)_{max}}{W/4\pi} = \frac{4\pi U(\theta,\phi)_{max}}{\iint_{4\pi} U(\theta,\phi)\, d\Omega} \qquad (6\text{-}5)$$

or

$$D = \frac{4\pi}{\iint_{4\pi} \frac{U(\theta,\phi)}{U(\theta,\phi)_{max}}\, d\Omega} \qquad (6\text{-}6)$$

Since the radiation intensity is proportional to the Poynting vector, we note from (6-1) that (6-6) can be expressed as

$$D = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_A} \qquad (6\text{-}7)$$

Thus, the directivity of an antenna is equal to the solid angle of a sphere (4π) divided by the antenna beam solid angle.

The directivity of an antenna is a fixed numerical (dimensionless) quantity. Multiplying the directivity by the normalized power pattern yields the *directive gain*, a quantity which is a function of angle. Thus,

$$DP_n(\theta,\phi) = D(\theta,\phi) \tag{6-8}$$

where $D(\theta,\phi)$ = directive gain, dimensionless

Since $P_n(\theta,\phi)_{max} = 1$, it follows that

$$D = D(\theta,\phi)_{max} \tag{6-9}$$

From (6-7) and (6-8) it is also clear that

$$\iint_{4\pi} D(\theta,\phi)\, d\Omega = 4\pi \tag{6-10}$$

Antenna patterns may be plotted in terms of directive gain, as in Fig. 6-3. For a nondirectional antenna the pattern would be everywhere equal to the

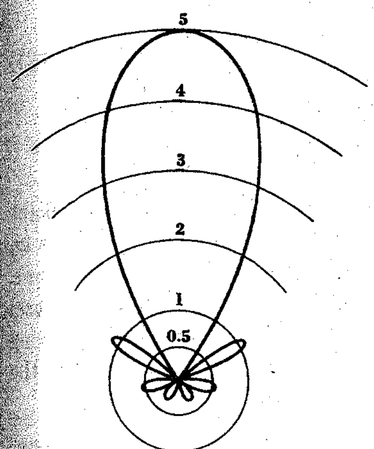

Fig. 6-3. Directive-gain pattern.

level $D(\theta,\phi) = 1$. This is called the *isotropic level*. In specifying the minor-lobe structure of an antenna the isotropic level is often a convenient reference.

In the foregoing discussion the directivity has been expressed entirely as a function of the antenna pattern with no reference to the size or geometry of the antenna. To show that the directivity is a function of the antenna size consider the far electric-field intensity E_r at a distance r in a direction broadside to a radiating aperture, as in Fig. 6-4. If the field intensity in the aperture is constant and equal to E_a (volts per meter), the power W radiated is given by

$$W = \frac{|E_a|^2}{Z} A \tag{6-11}$$

where A = antenna aperture, m²
Z = intrinsic impedance of the medium, ohms square^{-1}

The power radiated may also be expressed in terms of the field intensity E_r (volts m^{-1}) at a distance r by

$$W = \frac{|E_r|^2}{Z} r^2\Omega_A \tag{6-12}$$

where Ω_A = beam solid angle of antenna, rad²

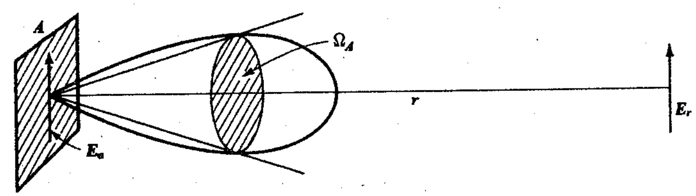

Fig. 6-4. Radiation from aperture A with uniform field E_a.

It is shown in Sec. 6-7 that the field intensities E_r and E_a are related by

$$|E_r| = \frac{|E_a|A}{r\lambda} \tag{6-13}$$

where λ = wavelength, m

Substituting (6-13) in (6-12) and equating (6-11) and (6-12) yields

$$\lambda^2 = A\Omega_A \tag{6-14}$$

where λ = wavelength, m
A = antenna aperture, m²
Ω_A = beam solid angle, sr

In (6-14) the aperture A is the physical aperture A_p if the field is uniform over the aperture, as assumed, but in general A is the *effective aperture* A_e. Thus, more generally,

$$\lambda^2 = A_e\Omega_A \tag{6-15}$$

where A_e = effective aperture, m²

According to this important relation, the product of the effective aperture of the antenna and the antenna beam solid angle is equal to the wavelength squared. From (6-15) and (6-7) we have that

$$D = \frac{4\pi}{\lambda^2} A_e \tag{6-16}$$