The VLSR Correction

Three excellent references are from Wikipedia:
1. `http://en.wikipedia.org/wiki/Celestial_coordinate_system`
2. `http://en.wikipedia.org/wiki/Equatorial_coordinate_system`
3. `http://en.wikipedia.org/wiki/Ecliptic_coordinate_system`

Others you might find useful are:
4. `http://www.phy.duke.edu/courses/055/syllabus/lecture7.pdf`
5. *Galactic Astronomy*, J. Binney, M. Merrifield, Princeton University Press (1998).
6. *Astrophysical Formulae*, K. R. Lang, Springer-Verlag (1980), Section 5.3.4.

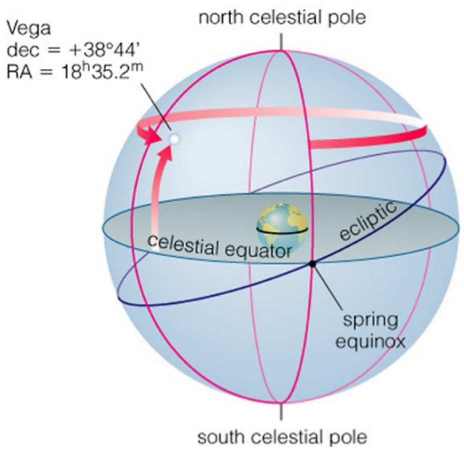
When you measure a shift in the frequency of the 21cm line it is the result of a velocity difference between you (the observer) and the target (source) along the line of sight. Usually both the source and the observer will be moving, and you probably want to correct for the observer's motion in order to get the source motion.

In astronomers' language, the source has two types of motion: "proper" motion, which is transverse to the line of sight and "radial" motion, which is along the line of sight. In Junior Lab, we can measure only the radial motion. The question that arises, is what should we measure the motion relative to? Conventionally, it is with respect to the *Local Standard of Rest* or LSR.

Local Standard of Reference (LSR):
This is the average for a group of stars that are in the vicinity of our solar system; for details see the Binney/Merrifield reference above. The motion of the LSR itself is not terribly well known, but the current consensus is that it follows a circular path about the center of our galaxy, in the galactic plane, with a radius of about 8 kpc ($8 \times 3.0857 \times 10^{16}$ m) and a tangential speed of about 220 m/s.

In Junior Lab, you should usually correct for the telescope's radial motion relative to the LSR. This has two significant parts: (i) the motion of the Sun relative to the LSR, and (ii) the orbital motion of the Earth relative to the Sun.

Motion of the Sun:
To find the radial motion of the Sun, the Celestial coordinate system is commonly used. The relevant coordinates are the right ascension α, expressed as an angle, and the declination δ, always an angle. The figure to the right shows the celestial coordinates of the bright star Vega, and is taken from the Duke slides (reference 4, above).



Sometimes it is useful to work in a Cartesian celestial coordinate system where the x axis is along the vernal equinox and the z axis points to the north pole. In that case, the direction to Vega would be

$$\cos\delta\,\cos\alpha\,\hat{\mathbf{x}}_c + \cos\delta\,\sin\alpha\,\hat{\mathbf{y}}_c + \sin\delta\,\hat{\mathbf{z}}_c$$

The generally accepted motion of the Sun in the LSR is that it is moving at a speed of about 20 km/s towards an "apex" that is rather close to the direction of Vega, with coordinates $\alpha = 18\,\mathrm{hr}$, or $270°$ and $\delta = 30°$. For a target or source specified by α and δ, the radial component of the Sun's velocity relative to the LSR is the dot product

$$V_{\mathrm{r}\odot} = 20.0\big[\cos 270°\ \cos 30°\ \cos\alpha\ \cos\delta + \sin 270°\ \cos 30°\ \sin\alpha\ \cos\delta + \sin 30°\ \sin\delta\big]\ \mathrm{km/s}.$$

Earth's Orbital Motion:

This calculation is better done in geocentric Ecliptic coordinates. In this system, the Sun always has latitude $\beta = 0$ and longitude $\lambda_\odot$ that varies by $360°$ over the course of the year. The zero of λ is the vernal equinox. The ecliptic north pole makes an angle $23.5°$ with the celestial (or equatorial) north pole direction. In a Cartesian ecliptic system $\hat{\mathbf{x}}_{\mathrm{ec}}$ coincides with $\hat{\mathbf{x}}_{\mathrm{c}}$ and the vernal equinox, and $\hat{\mathbf{z}}_{\mathrm{ec}}$ points towards the ecliptic north pole. In this system the vector to the Sun is

$$\cos\lambda_\odot\,\hat{\mathbf{x}}_{\mathrm{ec}} + \sin\lambda_\odot\,\hat{\mathbf{y}}_{\mathrm{ec}}\,.$$

Assuming the Earth's orbit is circular (the eccentricity is actually about 1.6%) its orbital speed is 30.0 km/s and the Sun will appear to have a velocity

$$30.0\big[-\sin\lambda_\odot\,\hat{\mathbf{x}}_{\mathrm{ec}} + \cos\lambda_\odot\,\hat{\mathbf{y}}_{\mathrm{ec}}\big]\,.$$

In the geocentric system the Earth is at rest and the Sun moves around it, but of course the Earth has a velocity with respect to the Sun that is the negative of the above vector. Thus for a target or source whose ecliptic coordinates are λ and β the radial component of the velocity *relative to the Sun* will be the dot product

$$V_{\mathrm{rE}} = 30.0\big[\cos\beta\ \sin\lambda_\odot\ \cos\lambda - \cos\beta\ \cos\lambda_\odot\ \sin\lambda\big] = 30.0\ \cos\beta\ \sin(\lambda_\odot - \lambda)\,.$$

The radial velocity of an observer on the Earth towards a target or source will be the sum of $V_{\mathrm{r}\odot}$ and V_{rE}. As positive values of each of these represent motion towards the source, they cause a blue shift in the 21cm line and they should be subtracted from the radial velocity you measure with the telescope in order to get the radial velocity of the source relative to the LSR. If you want the motion relative to the center of the galaxy, you then need to include the motion of the LSR.

Stated another way, if you measure a red shift $\Delta\nu$ in frequency, it should be increased by $(V_{\mathrm{r}\odot} + V_{\mathrm{rE}})/c$ to get the red shift you would measure at the LSR.

The practice for the SRT has been to provide a quantity called `vlsr` for each measurement; it is equal to $-(V_{\mathrm{r}\odot} + V_{\mathrm{rE}})$ as defined here, so the VLSR output on the Antenna Tab and in the data files is also $-(V_{\mathrm{r}\odot} + V_{\mathrm{rE}})$.

Note to SRT users: while writing these notes, I looked at the *java* code in the file `geom.java` and found that the Haystack staff had made a mistake in calculating `vlsr`. It has, I suppose, been there for nearly 20 years. It has been corrected in the file `srt.jar` for version 2.05s, which can be obtained from the course web site.