PROBLEM 1: NONRELATIVISTIC DOPPLER SHIFT, SOURCE AND OBSERVER IN MOTION

Consider the Doppler shift of sound waves, for a case in which both the source and the observer are moving. Suppose the source is moving with a speed \( v_s \) relative to the air, while the observer is receding from the source, moving in the opposite direction with a speed \( v_o \) relative to the air. Calculate the Doppler shift \( z \).

PROBLEM 2: THE TRANSVERSE DOPPLER SHIFT

Consider the Doppler shift observed by a stationary observer, from a source that travels in a circular orbit of radius \( R \) about the observer. Let the speed of the source be \( v_s \).

(a) If the wave in question is sound, and both the source speed \( v_s \) and the wave speed \( u \) are very small compared to the speed of light \( c \), what is the Doppler shift \( z \)?

(b) If the wave is light, traveling with speed \( c \), and \( v_s \) is not necessarily small compared to \( c \), what is the Doppler shift \( z \)? In answering this part of the question, you will want to keep the following facts from special relativity in mind:

1. TIME DILATION: Any clock which is moving relative to a given reference frame will appear to run slower than normal by a factor denoted by the Greek letter \( \gamma \) (gamma), and given by
   \[
   \gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta = \frac{v}{c},
   \]
   where \( \beta \) is the speed of the moving clock.

2. LORENTZ-FITZGERALD CONTRACTION: Any rod which is moving with a speed \( v \) relative to a given reference frame will appear to be shorter than its normal length by the same factor \( \gamma \). A rod which is moving parallel to the direction of motion of the moving frame will appear at a speed \( v' \) relative to the motion of the moving frame to have a length equal to its normal length divided by \( \gamma \).

3. RELATIVITY OF SIMULTANEITY: Suppose a rod which has rest length \( \ell_0 \) is equipped with a clock at each end. The clocks can be synchronized in the rest frame of the system by using light pulses. If the system moves at speed \( v \) along its length, then the trailing clock will appear to read a time which is later than the leading clock by an amount \( \beta \ell_0 / c \). If, on the other hand, the system moves perpendicular to its length, the clocks do not undergo a change in apparent length.

Consider the Doppler shift observed by a stationary observer, from a source that travels in a circular orbit of radius \( R \).

Problem 2.4. The Transverse Doppler Shift
In the rest of this problem set we will consider a universe in which the scale factor is given by
\[ R(t) = \frac{b}{t^2/3}, \]
where \( b \) is an arbitrary constant of proportionality which should not appear in the answers to any of the questions below. (We will see in Lecture Notes 4 that this is the behavior of a flat universe with a mass density that is dominated by nonrelativistic matter.) We will suppose that a distant quasar is observed with a redshift \( z \).

As a concrete example we will consider the most distant known object with a well-determined redshift, the galaxy J1324+2714, which has a redshift \( z = 6.58 \). The discovery of this galaxy was announced in January 2003 by Kodaira et al. of the Subaru Deep Field project.* The ability of astronomers to observe objects at high redshift has been increasing rapidly. In 1986 the object of highest known redshift was only 3.78. It was 4.01 in 1988, 4.73 in 1992, 4.897 in 1994, and 4.92 in 1998, 5.34 in 2000, and 6.28 in 2002. The discovery of a \( z = 6.58 \) galaxy was announced in 1999, but the measurement of the redshift was later found to be unreliable.

**Problem 3: Distance to the Galaxy**

Let \( t_0 \) denote the present time, and let \( t_e \) denote the time at which the light that we are currently receiving was emitted by the galaxy. In terms of these quantities, find the present value of the physical distance \( \ell_p \) between this distant galaxy and us.

**Problem 4: Time of Emission**

Express the redshift \( z \) in terms of \( t_0 \) and \( t_e \). Find the ratio \( t_e/t_0 \) for the \( z = 6.58 \) galaxy.

**Problem 5: Distance in Terms of Redshift**

Express the present value of the physical distance in terms of the present value of the Hubble constant \( H_0 \) and the redshift \( z \). Taking \( H_0 \approx 72 \text{ km}-\text{sec}^{-1}\text{-Mpc}^{-1} \), how far away is the galaxy? Express your answer both in light-years and in Mpc.

**Problem 6: Speed of Recession**

Find the rate at which the physical distance \( \ell_p \) between the distant galaxy and us is changing. Express your answer in terms of the redshift \( z \) and the speed of light \( c \), and evaluate it numerically for the case \( z = 6.58 \). Express your answer as a fraction of the speed of light. [If you get it right, this "fraction" is greater than one! Our expanding universe violates special relativity, but is consistent with general relativity.]

**Problem 7: Apparent Angular Sizes**

Now suppose for simplicity that the galaxy is spherical, and that its physical diameter was \( w \) at the time it emitted the light. (The actual galaxy is seen as an unresolved point source, so we don't know its actual size and shape.) Find the apparent angular size \( \theta \) (measured from one edge to the other) of the galaxy as it would be observed from Earth today. Express your answer in terms of \( c, \theta, w, H_0, \) and \( c \). You may assume that \( \theta \ll 1 \). Compare your answer to the apparent angular size of a circle of diameter \( w \) in a static Euclidean space, at a distance equal to the present value of the physical distance to the galaxy, as found in Problem 3. [Hint: draw diagrams which trace the light rays in the comoving coordinate system. If you have it right, you will find that \( \theta \) has a minimum value for \( z = 1.25 \) and then increases for larger \( z \).]

We now assume that the Earth is at rest and that the light is coming from a distance \( d \) light-years away.

**Problem 2: Distance in Terms of Redshift**

Express the physical distance \( \ell_p \) between the distant galaxy and us.

**Problem 4: Time of Emission**

Find the redshift \( z \) in terms of \( t_0 \) and \( t_e \). Find the ratio \( t_e/t_0 \) for the \( z = 6.58 \) galaxy.
PROBLEM 8: RECEIVED RADIATION FLUX

At the time of emission, the galaxy had a power output $P$ (measured in ergs/sec) which was radiated uniformly in all directions. This power was emitted in the form of photons. What is the radiation energy flux $J$ from this galaxy at the earth today? Energy flux (which might be measured in ergs-cm$^{-2}$-sec$^{-1}$) is defined as the energy per unit area per unit time striking a surface that is orthogonal to the direction of energy flow. The easiest way to solve this problem is to consider the trajectories of the photons, as viewed in comoving coordinates. You must calculate the rate at which photons arrive at the detector, and you must also use the fact that the energy of each photon is proportional to its frequency, and is therefore decreased by the redshift. The easiest way to solve this problem is to consider the direction of energy flow. The easiest way to solve this problem is to consider the direction of energy flow. The easiest way to solve this problem is to consider the direction of energy flow. The easiest way to solve this problem is to consider the direction of energy flow. The easiest way to solve this problem is to consider the direction of energy flow. The easiest way to solve this problem is to consider the direction of energy flow.

PROBLEM 8: RECEIVED RADIATION FLUX