MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — September 6, 2007
Prof. Alan Guth

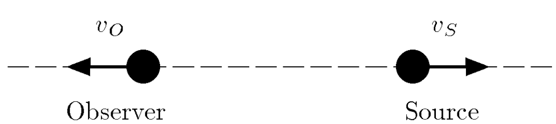
PROBLEM SET 1

DUE DATE: Tuesday, September 11, 2007

READING ASSIGNMENT: *The First Three Minutes*, Chapters 1 and 2.

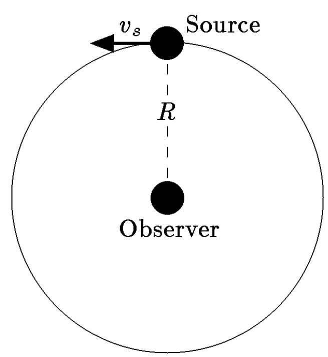
PROBLEM 1: NONRELATIVISTIC DOPPLER SHIFT, SOURCE AND OBSERVER IN MOTION

Consider the Doppler shift of sound waves, for a case in which both the source and the observer are moving. Suppose the source is moving with a speed v_s relative to the air, while the observer is receding from the source, moving in the opposite direction with speed v_o relative to the air. Calculate the Doppler shift z.



PROBLEM 2: THE TRANSVERSE DOPPLER SHIFT

Consider the Doppler shift observed by a stationary observer, from a source that travels in a circular orbit of radius R about the observer. Let the speed of the source be v_s.



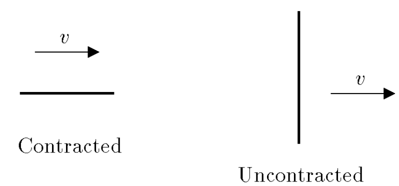
(a) If the wave in question is sound, and both the source speed v_s and the wave speed u are very small compared to the speed of light c, what is the Doppler shift z?

(b) If the wave is light, traveling with speed c, and v_s is not necessarily small compared to c, what is the Doppler shift z? In answering this part of the question, you will want to keep in mind the following facts from special relativity:

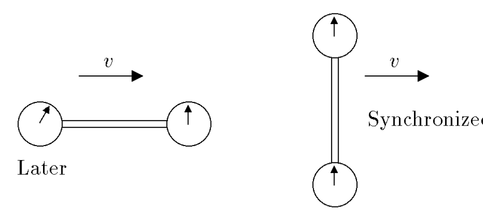
(1) TIME DILATION: Any clock which is moving at speed v relative to a given reference frame will appear (to an observer using that reference frame) to run slower than normal by a factor denoted by the Greek letter γ (gamma), and given by

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \, , \qquad \beta \equiv v/c \, .$$

(2) LORENTZ-FITZGERALD CONTRACTION: Any rod which is moving at a speed v along its length relative to a given reference frame will appear (to an observer using that reference frame) to be shorter than its normal length by the same factor γ. A rod which is moving perpendicular to its length does not undergo a change in apparent length.

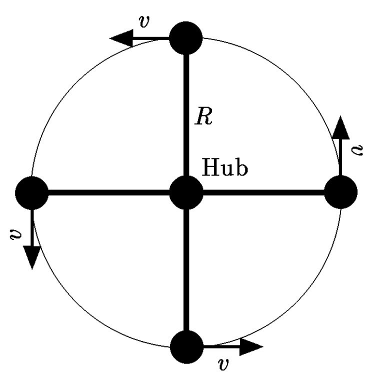


(3) RELATIVITY OF SIMULTANEITY: Suppose a rod which has rest length ℓ_0 is equipped with a clock at each end. The clocks can be synchronized in the rest frame of the system by using light pulses. If the system moves at speed v along its length, then the trailing clock will appear to read a time which is later than the leading clock by an amount $\beta\ell_0/c$. If, on the other hand, the system moves perpendicular to its length, then the synchronization of the clocks is not disturbed.



PROBLEM 3: A HIGH-SPEED MERRY-GO-ROUND

Now consider the Doppler shift as it would be observed in a high-speed "merry-go-round." Four evenly-spaced cars travel around a central hub at speed v, all at a distance R from a central hub. Each car is sending waves to all three of the other cars.



(a) If the wave in question is sound, and both the source speed v and the wave speed u are very small compared to the speed of light c, with what Doppler shift z does a given car receive the sound from (i) the car in front of it; (ii) the car behind it; and (iii) the car opposite it?

(b) In the relativistic situation, where the wave is light and the speed v may be comparable to c, what is the answer to the same three parts (i)-(iii) above?