

Physics 8.286: The Early Universe
 Prof. Alan Guth

PROBLEM SET 1

DUE DATE: Tuesday, September 15, 2009

READING ASSIGNMENT: *The First Three Minutes*, Chapters 1 and 2.

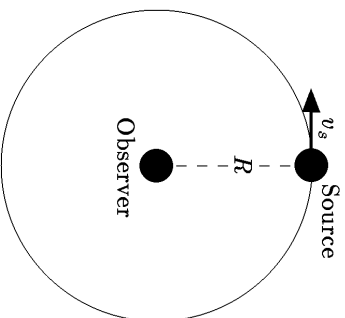
PROBLEM 1: NONRELATIVISTIC DOPPLER SHIFT, SOURCE AND OBSERVER IN MOTION

Consider the Doppler shift of sound waves, for a case in which both the source and the observer are moving. Suppose the source is moving with a speed v_s relative to the air, while the observer is receding from the source, moving in the opposite direction with speed v_o relative to the air. Calculate the Doppler shift z .



PROBLEM 2: THE TRANSVERSE DOPPLER SHIFT

Consider the Doppler shift observed by a stationary observer, from a source that travels in a circular orbit of radius R about the observer. Let the speed of the source be v_s .



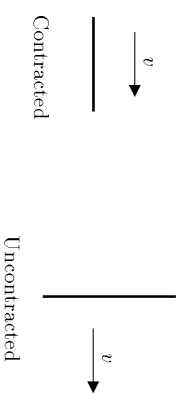
- (a) If the wave in question is sound, and both the source speed v_s and the wave speed u are very small compared to the speed of light c , what is the Doppler shift z ?

- (b) If the wave is light, traveling with speed c , and v_s is not necessarily small compared to c , what is the Doppler shift z ? In answering this part of the question, you will want to keep in mind the following facts from special relativity:

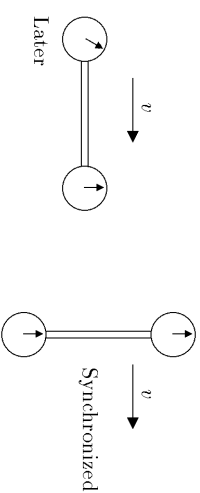
- (1) **TIME DILATION:** Any clock which is moving at speed v relative to a given reference frame will appear (to an observer using that reference frame) to run slower than normal by a factor denoted by the Greek letter γ (gamma), and given by

$$\gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta \equiv v/c.$$

- (2) **LORENTZ-FITZGERALD CONTRACTION:** Any rod which is moving at a speed v along its length relative to a given reference frame will appear (to an observer using that reference frame) to be shorter than its normal length by the same factor γ . A rod which is moving perpendicular to its length does not undergo a change in apparent length.

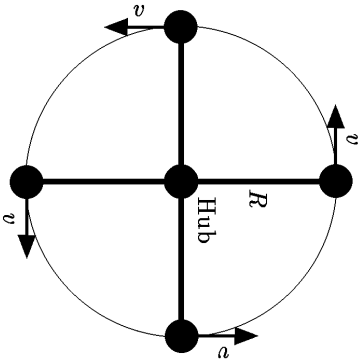


- (3) **RELATIVITY OF SIMULTANEITY:** Suppose a rod which has rest length ℓ_0 is equipped with a clock at each end. The clocks can be synchronized in the rest frame of the system by using light pulses. If the system moves at speed v along its length, then the trailing clock will appear to read a time which is later than the leading clock by an amount $\beta\ell_0/c$. If, on the other hand, the system moves perpendicular to its length, then the synchronization of the clocks is not disturbed.



PROBLEM 3: A HIGH-SPEED MERRY-GO-ROUND

Now consider the Doppler shift as it would be observed in a high-speed “merry-go-round.” Four evenly-spaced cars travel around a central hub at speed v , all at a distance R from a central hub. Each car is sending waves to all three of the other cars.



- (a) If the wave in question is sound, and both the source speed v and the wave speed u are very small compared to the speed of light c , with what Doppler shift z does a given car receive the sound from (i) the car in front of it; (ii) the car behind it; and (iii) the car opposite it?
- (b) In the relativistic situation, where the wave is light and the speed v may be comparable to c , what is the answer to the same three parts (i)-(iii) above?