**PROBLEM SET 3**

DUE DATE: Thursday, October 1, 2009


FIRST QUIZ: The first of three quizzes for the term will be given on Tuesday, October 6, 2009.

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**PROBLEM 1: A CYLINDRICAL UNIVERSE**

10 points

The following problem originated on Quiz 2 of 1994, where it counted 30 points.

(a) For this universe, find the value of the Hubble expansion rate $H$.

(b) The Robertson–Walker scale factor becomes as

$$r(t) = (t)^{1/2}.$$  

(c) Show that $r(t)$ is in fact independent of $r_i$, implying that the cylinder will undergo uniform expansion, just as the sphere did in the case discussed in the lecture notes.

(d) Express the mass density $\rho(t)$ in terms of the initial mass density $\rho_i$ and the scale factor $a(t)$.

(e) Find an expression for a conserved quantity of the form $\frac{d}{dt} \left( \frac{r_i}{r} \right) = 0$.

**PROBLEM 2: A FLAT UNIVERSE WITH UNUSUAL TIME EVOLUTION**

5 points

Consider a flat universe which is filled with some peculiar form of matter, so that the Robertson–Walker scale factor behaves as

$$a(t) = (bt)^{3/4},$$

where $b$ is a constant.

(a) Find an expression for the Hubble expansion rate $H(t)$.

(b) What is the mass density of the universe, $\rho(t)$? (In answering this question, you will need to know that the equation for $\dot{a}/a$, Eq. (4.30) in Lecture Notes 4, holds for all forms of matter, while the equation for $\ddot{a}$, Eq. (4.23), requires modification if the matter has a significant pressure. By (4.30) and any relevant constants, $H^2$ is therefore not a constant, and $\rho$ is not the usual mass density of the universe, $\rho$.)

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The following problems originated on Quiz 2 of 1994, where it counted 50 points.

**PROBLEM 1: A CYLINDRICAL UNIVERSE**

(10 points)

October 6, 2009.

The first of three quizzes for the term will be given on Thursday, October 6.

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**PROBLEM SET 3**

Due Date: Thursday, October 1, 2009

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**PROBLEM 1: A CYLINDRICAL UNIVERSE**

(10 points)

(a) Find an expression for a conserved quantity of the form $\frac{d}{dt} \left( \frac{r_i}{r} \right) = 0$.

(b) Show that $r(t)$ is in fact independent of $r_i$, implying that the cylinder will undergo uniform expansion, just as the sphere did in the case discussed in the lecture notes.

(c) Express the mass density $\rho(t)$ in terms of the initial mass density $\rho_i$ and the scale factor $a(t)$.

(d) Find an expression for a conserved quantity of the form $\frac{d}{dt} \left( \frac{r_i}{r} \right) = 0$.

(e) Show that $r(t)$ is in fact independent of $r_i$, implying that the cylinder will undergo uniform expansion, just as the sphere did in the case discussed in the lecture notes.

(f) Express the mass density $\rho(t)$ in terms of the initial mass density $\rho_i$ and the scale factor $a(t)$.

(g) Find an expression for a conserved quantity of the form $\frac{d}{dt} \left( \frac{r_i}{r} \right) = 0$.

(h) Show that $r(t)$ is in fact independent of $r_i$, implying that the cylinder will undergo uniform expansion, just as the sphere did in the case discussed in the lecture notes.

(i) Express the mass density $\rho(t)$ in terms of the initial mass density $\rho_i$ and the scale factor $a(t)$.

(j) Find an expression for a conserved quantity of the form $\frac{d}{dt} \left( \frac{r_i}{r} \right) = 0$.

(k) Show that $r(t)$ is in fact independent of $r_i$, implying that the cylinder will undergo uniform expansion, just as the sphere did in the case discussed in the lecture notes.

(l) Express the mass density $\rho(t)$ in terms of the initial mass density $\rho_i$ and the scale factor $a(t)$.

(m) Find an expression for a conserved quantity of the form $\frac{d}{dt} \left( \frac{r_i}{r} \right) = 0$.

(n) Show that $r(t)$ is in fact independent of $r_i$, implying that the cylinder will undergo uniform expansion, just as the sphere did in the case discussed in the lecture notes.

(o) Express the mass density $\rho(t)$ in terms of the initial mass density $\rho_i$ and the scale factor $a(t)$.

(p) Find an expression for a conserved quantity of the form $\frac{d}{dt} \left( \frac{r_i}{r} \right) = 0$.

(q) Show that $r(t)$ is in fact independent of $r_i$, implying that the cylinder will undergo uniform expansion, just as the sphere did in the case discussed in the lecture notes.

(r) Express the mass density $\rho(t)$ in terms of the initial mass density $\rho_i$ and the scale factor $a(t)$.

(s) Find an expression for a conserved quantity of the form $\frac{d}{dt} \left( \frac{r_i}{r} \right) = 0$.

(t) Show that $r(t)$ is in fact independent of $r_i$, implying that the cylinder will undergo uniform expansion, just as the sphere did in the case discussed in the lecture notes.

(u) Express the mass density $\rho(t)$ in terms of the initial mass density $\rho_i$ and the scale factor $a(t)$.

(v) Find an expression for a conserved quantity of the form $\frac{d}{dt} \left( \frac{r_i}{r} \right) = 0$.

(w) Show that $r(t)$ is in fact independent of $r_i$, implying that the cylinder will undergo uniform expansion, just as the sphere did in the case discussed in the lecture notes.

(x) Express the mass density $\rho(t)$ in terms of the initial mass density $\rho_i$ and the scale factor $a(t)$.

(y) Find an expression for a conserved quantity of the form $\frac{d}{dt} \left( \frac{r_i}{r} \right) = 0$.

(z) Show that $r(t)$ is in fact independent of $r_i$, implying that the cylinder will undergo uniform expansion, just as the sphere did in the case discussed in the lecture notes.

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**PROBLEM 2: A FLAT UNIVERSE WITH UNUSUAL TIME EVOLUTION**

5 points

Consider a flat universe which is filled with some peculiar form of matter, so that the Robertson–Walker scale factor behaves as

$$a(t) = (bt)^{3/4},$$

where $b$ is a constant.

(a) Find an expression for the Hubble expansion rate $H(t)$.

(b) What is the mass density of the universe, $\rho(t)$? (In answering this question, you will need to know that the equation for $\dot{a}/a$, Eq. (4.30) in Lecture Notes 4, holds for all forms of matter, while the equation for $\ddot{a}$, Eq. (4.23), requires modification if the matter has a significant pressure. Eq. (4.23) is therefore not applicable to this problem.)

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The following problems originated on Quiz 2 of 1994, where it counted 50 points.
Total points for Problem Set 3: 25.

(10)

E_{\text{grav}} = \frac{GMm}{r}

This is called the gravitational potential energy, and it is negative because it is energy associated with the gravitational force.

(9)

K = \frac{1}{2}mr^2 \dot{r}^2

This is the kinetic energy of the test particle, and it is positive because it is energy associated with the motion of the test particle.

(8)

V = \frac{GM}{r} + \frac{1}{2}mr^2 \dot{r}^2

This is the total energy of the test particle, and it is the sum of the gravitational potential energy and the kinetic energy.

(7)

\dot{V} = dV/dt = 0

This is the Hubble expansion constraint equation, which states that the expansion rate of the universe is constant. It is equivalent to the conservation of energy equation, which states that the total energy of a closed system is constant.

(6)

\dot{V} = \frac{dV}{dt} = 0

This is the conservation of energy equation, which is derived from the Lagrangian of the system. It is equivalent to the Hubble expansion constraint equation.

(5)

E = \frac{1}{2}mr^2 \dot{r}^2 - \frac{GMm}{r}

This is the Lagrangian of the system, which is the difference between the kinetic energy and the gravitational potential energy.

(4)

\dot{E} = 0

This is the conservation of energy equation, which is derived from the Lagrangian of the system. It states that the total energy of the system is constant.

(3)

E = \frac{1}{2}mr^2 \dot{r}^2 - \frac{GMm}{r}

This is the Lagrangian of the system, which is the difference between the kinetic energy and the gravitational potential energy.

(2)

E = \frac{1}{2}mr^2 \dot{r}^2 - \frac{GMm}{r}

This is the Lagrangian of the system, which is the difference between the kinetic energy and the gravitational potential energy.

(1)

E = \frac{1}{2}mr^2 \dot{r}^2 - \frac{GMm}{r}

This is the Lagrangian of the system, which is the difference between the kinetic energy and the gravitational potential energy.