MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — September 24, 2009
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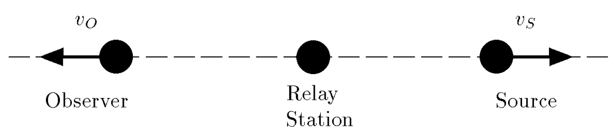
PROBLEM SET 1 SOLUTIONS

PROBLEM SET GRADING:

The total number of points will vary some from problem set to problem set. Your final problem set grade will be the sum of the individual grades, so the problem sets with more points will count a little more than the others.

PROBLEM 1: NONRELATIVISTIC DOPPLER SHIFT, SOURCE AND OBSERVER IN MOTION *(5 points)*

The easiest way to do this problem, given the results described in the lecture notes, is to introduce an intermediate relay station which is at rest relative to the air:



The relay station simply rebroadcasts the signal it receives from the source, at exactly the instant that it receives it. The relay station therefore has no effect on the signal received by the observer, but it serves as a crutch for our thinking— it allows us to divide the problem into two parts, each already solved. Let Δt denote the time interval between wave crests, with subscripts to indicate which clock is making the measurement. Then

$$\Delta t_{\text{relay}} = \left(1 + \frac{v_s}{u}\right) \Delta t_{\text{source}}$$

$$\Delta t_{\text{observer}} = \frac{1}{1 - v_o/u} \Delta t_{\text{relay}} \ ,$$

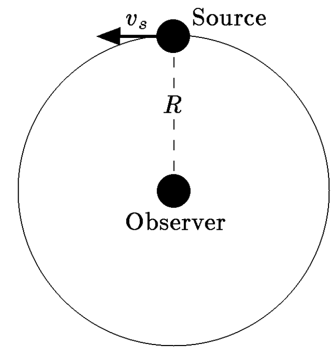
so

$$\Delta t_{\text{observer}} = \frac{1 + v_s/u}{1 - v_o/u} \Delta t_{\text{source}} \ .$$

Thus

$$\boxed{z = \frac{1 + v_s/u}{1 - v_o/u} - 1 \ .}$$

PROBLEM 2: THE TRANSVERSE DOPPLER SHIFT *(5 points)*



(a) Since this part of the problem is nonrelativistic, time can be considered universal, so all clocks run at the same speed. If wave crests are emitted by the source at a time interval Δt_S as measured by the source, the time interval between the emissions of the crests will also appear to the observer to be Δt_S. Furthermore, since the distance between the source and observer does not change with time, each crest must travel the same distance between the source and the observer, and hence the separation between their arrivals will also be Δt_S. Thus the time interval between crests as measured by the observer is $\Delta t_O = \Delta t_S$, and the redshift is

$$\boxed{z = \frac{\Delta t_O}{\Delta t_S} - 1 = 0 \ .}$$

There is no Doppler shift.

(b) Again the distance between the source and the observer remains constant, but in this case there is nonetheless a Doppler shift caused by the relativistic effects of motion on the speed of the clocks. From the point of view of the observer, the clock on the source is a moving clock which runs slowly by a factor of

$$\gamma \equiv \frac{1}{\sqrt{1 - \beta^2}} \ , \qquad \beta \equiv v/c \ .$$

If the time interval between crests is measured on the source clock as Δt_S, then to the observer it will appear to be $\gamma \, \Delta t_S$. The redshift is then given by

$$\boxed{z = \frac{\Delta t_O}{\Delta t_S} - 1 = \gamma - 1 \ .}$$

This phenomenon is known as the transverse Doppler effect.

PROBLEM 3: A HIGH-SPEED MERRY-GO-ROUND *(5 points)**

The following comments apply both for part (a) and for part (b). Number the cars from 1 to 4. Consider the signals emitted by car i and detected by car j, where $j \neq i$ and i and j are any integers from one to four.

As seen in the stationary non-rotating laboratory frame the crests of the waves emitted by the source at i are always separated by the same time interval $(\Delta t_S)_L$; here the subscript S means that we are defining the period at the source, and the subscript L indicates that it is the value measured in the laboratory frame. Each crest emitted by i always takes the same time to reach j, because the relative positions of i and j on the merry-go-round never change. It follows that in the laboratory frame the crests received by the observer at j are always separated by the time interval $(\Delta t_O)_L$ that is exactly equal to $(\Delta t_S)_L$:

$$(\Delta t_O)_L = (\Delta t_S)_L \, . \tag{1}$$

This is true for any pair of cars on the merry-go-round. The angles that separate the various cars need not be multiples of $\pi/2$, as long as the relative angular positions are preserved in time.

(a) In the nonrelativistic approximation the time differences measured by the laboratory coincide with the time differences measured by either sources or receivers. We thus have that (1) gives

$$\Delta t_O = \Delta t_S \, , \tag{2}$$

where the absence of the L subscript indicates that Δt_O is time measured by the observer and Δt_S is time measured by the source. It follows from the definition of z that

$$1 + z \equiv \frac{\Delta t_O}{\Delta t_S} = 1 \, , \qquad \Longrightarrow \qquad \boxed{z = 0 \, .} \tag{3}$$

(b) Because the sources and receivers are moving their clocks lag — times measured at the laboratory are longer by the relativistic factor γ (this is true even though the motion is circular and only the speed is constant). We thus have

$$(\Delta t_O)_L = \gamma_O \Delta t_O \, , \qquad (\Delta t_S)_L = \gamma_S \Delta t_S \, , \tag{4}$$

* Solution by Barton Zwiebach.

where Δt_O and Δt_S are the times measured by the clocks carried by the observer and the source, respectively. Moreover,

$$\gamma_O = \frac{1}{\sqrt{1-(v_0/c)^2}} \, , \quad \gamma_S = \frac{1}{\sqrt{1-(v_S/c)^2}} \, , \tag{5}$$

with v_O and v_S the speed of the observer and source, respectively. Since $v_0 = v_S$, we have $\gamma_O = \gamma_S \equiv \gamma$ and thus (4) becomes

$$(\Delta t_O)_L = \gamma \Delta t_O \, , \qquad (\Delta t_S)_L = \gamma \Delta t_S \, , \tag{4'}$$

Substituting this information in (1) we find

$$\Delta t_O = \Delta t_S \, . \tag{5'}$$

Therefore, even relativistically there is no Doppler shift: $\boxed{z = 0.}$ Intuitively, the coincidence of the lab intervals for emission and observation implies the coincidence of intervals at the level of the emitter and the observer because their γ factors are the same.