PROBLEM SET 1

DUE DATE: Thursday, September 15, 2011

READING ASSIGNMENT: The First Three Minutes, Chapters 1 and 2.

PROBLEM 1: NONRELATIVISTIC DOPPLER SHIFT, SOURCE AND OBSERVER IN MOTION (5 points)

Consider the Doppler shift of sound waves, for a case in which both the source and the observer are moving. Suppose the source is moving with a speed $v_s$ relative to the air, while the observer is receding from the source, moving in the opposite direction with speed $v_o$ relative to the air. Calculate the Doppler shift $z$.

![Diagram of observer moving away from source]

PROBLEM 2: THE TRANSVERSE DOPPLER SHIFT (5 points)

Consider the Doppler shift observed by a stationary observer, from a source that travels in a circular orbit of radius $R$ about the observer. Let the speed of the source be $v_s$.

(a) If the wave in question is sound, and both the source speed $v_s$ and the wave speed $u$ are very small compared to the speed of light $c$, what is the Doppler shift $z$?
(b) If the wave is light, traveling with speed \( c \), and \( v_s \) is not necessarily small compared to \( c \), what is the Doppler shift \( z \)? In answering this part of the question, you will want to keep in mind the following facts from special relativity:

1. **TIME DILATION:** Any clock which is moving at speed \( v \) relative to a given reference frame will appear (to an observer using that reference frame) to run slower than normal by a factor denoted by the Greek letter \( \gamma \) (gamma), and given by
   \[
   \gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta \equiv v/c .
   \]

2. **LORENTZ-FITZGERALD CONTRACTION:** Any rod which is moving at a speed \( v \) along its length relative to a given reference frame will appear (to an observer using that reference frame) to be shorter than its normal length by the same factor \( \gamma \). A rod which is moving perpendicular to its length does not undergo a change in apparent length.

   ![Diagram of contraction](image)

3. **RELATIVITY OF SIMULTANEITY:** Suppose a rod which has rest length \( \ell_0 \) is equipped with a clock at each end. The clocks can be synchronized in the rest frame of the system by using light pulses. (That is, a light pulse can be sent out from the center, and the clocks at both ends can be started when they receive the pulses.) If the system moves at speed \( v \) along its length, then the trailing clock will appear to read a time which is later than the leading clock by an amount \( \beta \ell_0/c \). If, on the other hand, the system moves perpendicular to its length, then the synchronization of the clocks is not disturbed.

   ![Diagram of simultaneity](image)
PROBLEM 3: A HIGH-SPEED MERRY-GO-ROUND (5 points)

Now consider the Doppler shift as it would be observed in a high-speed “merry-go-round.” Four evenly-spaced cars travel around a central hub at speed $v$, all at a distance $R$ from a central hub. Each car is sending waves to all three of the other cars.

(a) If the wave in question is sound, and both the source speed $v$ and the wave speed $u$ are very small compared to the speed of light $c$, with what Doppler shift $z$ does a given car receive the sound from (i) the car in front of it; (ii) the car behind it; and (iii) the car opposite it?

(b) In the relativistic situation, where the wave is light and the speed $v$ may be comparable to $c$, what is the answer to the same three parts (i)-(iii) above?

Total points for Problem Set 1: 15.