In this problem set we will consider a universe in which the scale factor is given by
\[ a(t) = \frac{bt^2}{3}, \]
where \( b \) is an arbitrary constant of proportionality which should not appear in the answers to any of the questions below. (We will see in Lecture Notes 3 that this is the behavior of a flat universe with a mass density that is dominated by nonrelativistic matter.) We will suppose that a distant galaxy is observed with a redshift \( z \).

**Problem 1: Distance to the Galaxy** (10 points)

Let \( t_0 \) denote the present time, and let \( t_e \) denote the time at which the light that we are currently receiving was emitted by the galaxy. In terms of these quantities, find the present value of the physical distance \( \ell_p \) between this distant galaxy and us.

**Problem Set 2**

**Due Date:** Due to the Student Holiday this Friday, the problem set will not be due until Monday — Monday, September 23, 2013, 5:00 pm. The first quiz will be on Thursday, October 3, which I think makes this problem set due the day after the quiz. My policy is that quizzes should only include material that has been reinforced by a problem set that was due before the quiz.

**Planning Ahead:** If you want to read ahead, the reading assignment with Problem Set 3 will be Weinberg, *The First Three Minutes*, Chapter 3. Problem Sets 1 through 3 will be included in the material covered on Quiz 1. On Thursday, October 3.

**Reading Assignment:** Barbara Ryden, *Introduction to Cosmology*, Chapters 1-3.

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PROBLEM 2: TIME OF EMISSION

Express the redshift \( z \) in terms of \( t_0 \) and \( t_e \). Find the ratio \( t_e/t_0 \) for the \( z = 8.55 \) galaxy.

PROBLEM 3: DISTANCE IN TERMS OF REDSHIFT

Express the present value of the physical distance in terms of the present value of the Hubble expansion rate \( H_0 \) and the redshift \( z \). Taking \( H_0 \approx 67 \text{ km-sec}^{-1}\text{-Mpc}^{-1} \), how far away is the galaxy? Express your answer both in light-years and in Mpc.

PROBLEM 4: SPEED OF RECESSION

Find the present rate at which the physical distance \( \ell_p \) between the distant galaxy and us is changing. Express your answer in terms of the redshift \( z \) and the speed of light \( c \), and evaluate it numerically for the case \( z = 8.55 \). Express your answer as a fraction of the speed of light. If you get it right, this "fraction" is greater than one! Our expanding universe violates special relativity, but is consistent with general relativity.

PROBLEM 5: APPARENT ANGULAR SIZES

Now suppose for simplicity that the galaxy is spherical, and that its physical diameter was \( w \) at the time it emitted the light. (The actual galaxy is seen as an unresolved point source, so we don't know its actual size and shape.) Find the apparent angular size \( \theta \) of the galaxy on Earth today. Express your answer in terms of \( w, z, H_0, \) and \( c \). Compare your answer to the apparent angular size \( \theta \) measured from one edge to the other of the galaxy at the time it emitted the light. (The apparent angular size is seen as an image of a circle of diameter \( w \) in a static Euclidean space, at a distance equal to the present value of the physical distance between the distant galaxy and us.)

PROBLEM 6: RECEIVED RADIATION FLUX

At the time of emission, the galaxy had a power output \( P \) (measured, say, in ergs/sec) which was radiated uniformly in all directions. This power was emitted in photons. What is the radiation energy flux \( J \) from this galaxy at the earth today? Energy flux (which might be measured in ergs-cm\(^{-2}\)-sec\(^{-1}\)) is defined as the energy per unit area per unit time striking a surface that is orthogonal to the direction of energy flow. The easiest way to solve this problem is to consider the direction of energy flow. The easiest way to solve this problem is to consider the direction of energy flow. The easiest way to solve this problem is to consider the direction of energy flow.

General relativity.

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Find the present rate at which the physical distance between the distant galaxy and us is changing. Express your answer in terms of the redshift \( z \) and the speed of light \( c \) and evaluate it numerically for the case \( z = 8.55 \). Express your answer in light-years and in Mpc.

Express the present value of the physical distance between the distant galaxy and us in terms of the redshift \( z \) and the speed of light \( c \). Find the ratio \( c/t_0 \) for the \( z = 8.55 \) galaxy.

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Total points for Problem Set 2: 80.