MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — September 3, 2022
Prof. Alan Guth

PROBLEM SET 1

DUE DATE: Friday, September 16, 2022, 5:00 pm.

READING ASSIGNMENT: *The First Three Minutes*, Chapters 1 and 2.

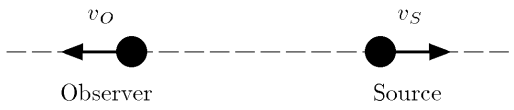
NOTE ABOUT EXTRA CREDIT: This problem set contains 40 points of regular problems and 15 points extra credit, so it is probably worthwhile for me to remind you about the operational definition of "extra credit". (It was also described on the Course Information Sheet.)

We will keep track of the extra credit grades separately, and at the end of the course I will first assign provisional grades based solely on the regular coursework. I will consult with our teaching assistant, Marianne Moore, and we will try to make sure that these grades are fair. Then I will add in the extra credit, allowing the grades to change upwards accordingly. Finally, Marianne and I will look at each student's grades individually, and we might decide to give a higher grade to some students who are slightly below a borderline. Students whose grades have improved significantly during the term, students whose average has been pushed down by a single low grade, and students who have been affected by adverse personal or medical problems will be the ones most likely to be boosted.

The bottom line is that the extra credit problems are OPTIONAL. You should feel free to skip them, and you will still get an A+ for the problem sets if you get nearly perfect scores on the regular problems. However, if you have some time and enjoy an extra challenge, then I hope that you will find the extra credit problems interesting and worthwhile.

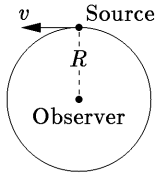
PROBLEM 1: NONRELATIVISTIC DOPPLER SHIFT, SOURCE AND OBSERVER IN MOTION *(15 points)*

Consider the Doppler shift of sound waves, for a case in which both the source and the observer are moving. Suppose the source is moving with a speed v_s relative to the air, while the observer is receding from the source, moving in the opposite direction with speed v_o relative to the air. Calculate the Doppler shift z. (Recall that z is defined by $1 + z \equiv \lambda_o/\lambda_s$, where λ_o and λ_s are the wavelengths as measured by the observer and by the source, respectively.) *Hint:* while this problem can be solved directly, you can save time by finding a way to determine the answer by using the cases that are already calculated in Lecture Notes 1.



PROBLEM 2: THE TRANSVERSE DOPPLER SHIFT *(25 points)*

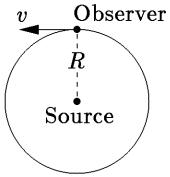
Consider the Doppler shift observed by a stationary observer, from a source that travels in a circular orbit of radius R about the observer. Let the speed of the source be v.



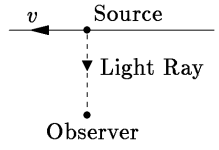
(a) *(5 points)* If the wave in question is sound, and both the source speed v and the wave speed u are very small compared to the speed of light c, what is the Doppler shift z? Assume that the observer is at rest relative to the air.

(b) *(5 points)* If the wave is light, traveling with speed c, and v is not small compared to c, what is the Doppler shift z? This is called the *transverse Doppler shift*, since the velocity of the light ray is perpendicular to the velocity of the source at the time of emission, as seen in the reference frame of the observer.

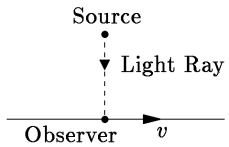
(c) *(5 points)* Still considering light waves and the same pattern of motion as shown in the figure, suppose that the source and the observer were reversed. That is, suppose a light ray is sent from the person at the center of the circle to the person traveling around the circle at speed v. In this case, what would be the Doppler shift z?



(d) *(5 points)* Now suppose that the motion is linear instead of circular. Again we consider light rays, and as in part (b) we assume that the source is moving with a speed v that is not small compared to c. If the light ray is emitted by the source at the moment of its closest approach to the observer, as shown in the diagram, what is the Doppler shift z?



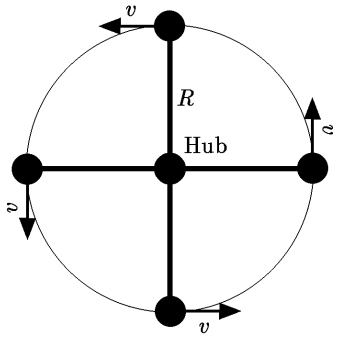
(e) *(5 points)* Again consider linear motion, with light rays. As in part (c), assume that the observer is moving with a speed v that is not small compared to c. If the light ray is received by the observer at the moment of its closest approach to the source, as shown in the diagram, what is the Doppler shift z?



PROBLEM 3: A HIGH-SPEED MERRY-GO-ROUND

(This problem is not required, but can be done for 15 points extra credit.)

Now consider the Doppler shift as it would be observed in a high-speed "merry-go-round." Four evenly-spaced cars travel around a central hub at speed v, all at a distance R from a central hub. Each car is sending waves to all three of the other cars.



(a) If the wave in question is sound, and both the source speed v and the wave speed u are very small compared to the speed of light c, with what Doppler shift z does a given car receive the sound from (i) the car in front of it; (ii) the car behind it; and (iii) the car opposite it?

(b) In the relativistic situation, where the wave is light and the speed v may be comparable to c, what is the answer to the same three parts (i)-(iii) above?

Total points for Problem Set 1: 40, plus 15 points of extra credit.