

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Physics Department

Physics 8.286: The Early Universe  
Prof. Alan Guth

September 25, 2022

**PROBLEM SET 2\***

**DUE DATE:** Monday, September 26, 2022, 11:00 pm.

SEPTEMBER/OCTOBER					
MON	TUES	WED	THURS	FRI	SAT
September 19 Class 4	20	21 Class 5	22	23 Student Holiday	24
26 Class 6 PS2 due	27	28 Class 7	29	30	October 1 PS3 due
3 Class 8	4	5 Class 9 Quiz 1	6	7 ADD Date	8

**READING ASSIGNMENT:** Barbara Ryden, *Introduction to Cosmology*, Chapters 1-3.

**PLANNING AHEAD:** If you want to read ahead, the reading assignment with Problem Set 3 will be Weinberg, *The First Three Minutes*, Chapter 3. Problem Sets 1 through 3, including the reading assignments, will be included in the material covered on Quiz 1, on Wednesday, October 5. Contrary to previous announcements, Problem Set 3 (and all other problem sets previously scheduled to be due at 5:00 pm on Fridays) will be due on Saturday at noon. In this way you will be able to make better use of Marianne's Friday 1:00 pm office hour.

**INTRODUCTION TO THE PROBLEM SET**

In this problem set we will consider a universe in which the scale factor is given by

$$a(t) = bt^{2/3} ,$$

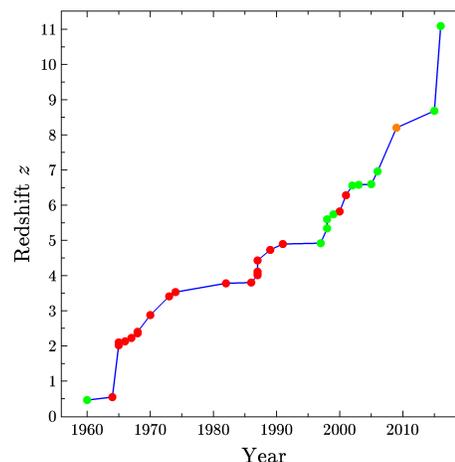
where  $b$  is an arbitrary constant of proportionality which should not appear in the answers to any of the questions below. (We will see in Lecture Notes 3 that this is the behavior of a flat universe with a mass density that is dominated by nonrelativistic matter.) We will

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\* Due time changed from 5:00 pm to 11:00 pm.

suppose that a distant galaxy is observed with a redshift  $z$ . As a concrete example we will consider the most distant known object with a well-determined redshift, the galaxy GN-z11, which has a redshift  $z = 11.1$ . The discovery of this galaxy was announced in March 2016 by an international group of astronomers, using the Hubble Space Telescope\*

The rate at which the highest measured redshift has been growing has been dramatic. In 1960 the highest measured redshift was only  $z=0.461$ . The diagram at the right shows a graph of the highest confirmed redshift by year of discovery, using the listing in the Wikipedia.† The red circles represent quasars, the green circles represent galaxies, and the one orange circle at 2009 is a gamma ray burst. The search for high redshift objects continues to be an exciting area of research, as astronomers try to sort out the conditions in the universe when the first galaxies began to form.



With the spectacular imaging capabilities of the James Webb Space Telescope, which was launched in December 2021 and released its first images on July 12, 2022, the record for the highest measured redshift is poised for another big jump. In July a group of astronomers\* announced the discovery of a galaxy candidate at  $z = 16.7$ . The galaxy candidate has been named CEERS-93316, where CEERS is an acronym for the Cosmic Evolution Early Release Science Survey in which the galaxy was found. Since the galaxy and its redshift have not yet been confirmed, we will continue to use GN-z11 for the problem set.

### PROBLEM 1: DISTANCE TO THE GALAXY (10 points)

Let  $t_0$  denote the present time, and let  $t_e$  denote the time at which the light that we are currently receiving was emitted by the galaxy. In terms of these quantities, find the present value of the physical distance  $\ell_p$  between this distant galaxy and us.

\* P. A. Oesh et al., “A Remarkably Luminous Galaxy at  $z = 11.1$  Measured with Hubble Space Telescope Grism Spectroscopy,” *The Astrophysical Journal* **819**, 129 (2016), <https://arxiv.org/abs/1603.00461>.

† “List of the most distant astronomical objects.” In *Wikipedia, The Free Encyclopedia*. Retrieved 16:35, September 12, 2020, from [https://en.wikipedia.org/wiki/List\\_of\\_the\\_most\\_distant\\_astronomical\\_objects](https://en.wikipedia.org/wiki/List_of_the_most_distant_astronomical_objects).

\* C. T. Donnan et al., “The evolution of the galaxy UV luminosity function at redshifts  $z \sim 8-15$  from deep JWST and ground-based near-infrared imaging,” <https://arxiv.org/abs/2207.12356>.

**PROBLEM 2: TIME OF EMISSION** (10 points)

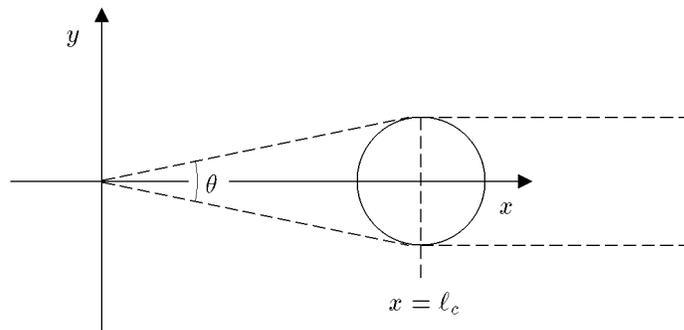
Express the redshift  $z$  in terms of  $t_0$  and  $t_e$ . Find the ratio  $t_e/t_0$  for the  $z = 11.1$  galaxy.

**PROBLEM 3: DISTANCE IN TERMS OF REDSHIFT  $z$**  (10 points)

Express the present value of the physical distance in terms of the present value of the Hubble expansion rate  $H_0$  and the redshift  $z$ . Taking  $H_0 \approx 67 \text{ km-sec}^{-1}\text{-Mpc}^{-1}$ , how far away is the galaxy? Express your answer both in light-years and in Mpc.

**PROBLEM 4: SPEED OF RECESSION** (10 points)

Find the present rate at which the physical distance  $\ell_p$  between the distant galaxy and us is changing. Express your answer in terms of the redshift  $z$  and the speed of light  $c$ , and evaluate it numerically for the case  $z = 11.1$ . Express your answer as a fraction of the speed of light. [If you get it right, this “fraction” is greater than one! Our expanding universe violates special relativity, but is consistent with general relativity.]

**PROBLEM 5: APPARENT ANGULAR SIZES** (20 points)

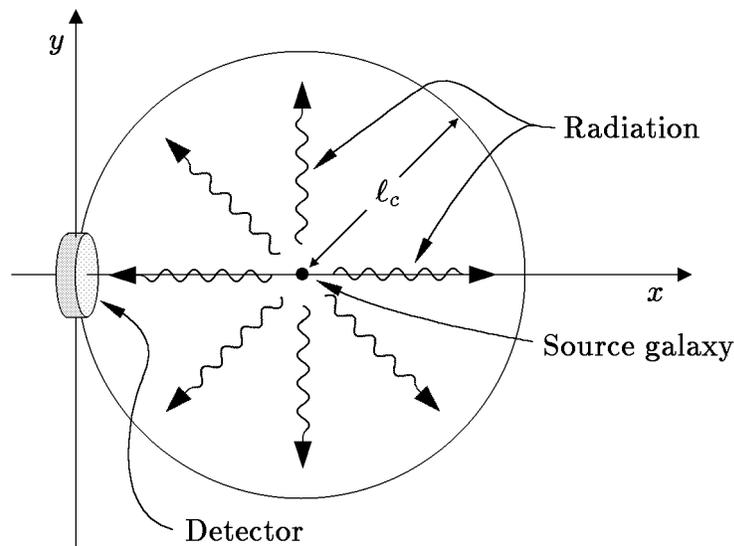
Now suppose for simplicity that the galaxy is spherical, and that its physical diameter was  $w$  at the time it emitted the light. (The actual galaxy is seen as an unresolved point source, so we don't know its actual size and shape.) Find the apparent angular size  $\theta$  (measured from one edge to the other) of the galaxy as it would be observed from Earth today. Express your answer in terms of  $w$ ,  $z$ ,  $H_0$ , and  $c$ . You may assume that  $\theta \ll 1$ . Compare your answer to the apparent angular size of a circle of diameter  $w$  in a static Euclidean space, at a distance equal to the present value of the physical distance to the galaxy, as found in Problem 1. [Hint: draw diagrams which trace the light rays in the

**comoving** coordinate system. If you have it right, you will find that  $\theta$  has a minimum value for  $z = 1.25$ , and that  $\theta$  increases for larger  $z$ . This phenomenon makes sense if you think about the distance to the galaxy at the time of emission. If the galaxy is **very** far away today, then the light that we now see must have left the object very early, when it was rather close to us!]

### PROBLEM 6: RECEIVED RADIATION FLUX

*(This problem is not required, but can be done for 15 points extra credit.)*

At the time of emission, the galaxy had a power output  $P$  (measured, say, in ergs/sec) which was radiated uniformly in all directions. This power was emitted in the form of photons. What is the radiation energy flux  $J$  from this galaxy at the earth today? Energy flux (which might be measured in ergs-cm<sup>-2</sup>-sec<sup>-1</sup>) is defined as the energy per unit area per unit time striking a surface that is orthogonal to the direction of energy flow. The easiest way to solve this problem is to consider the trajectories of the photons, as viewed in comoving coordinates. You must calculate the rate at which photons arrive at the detector, and you must also use the fact that the energy of each photon is proportional to its frequency, and is therefore decreased by the redshift. You may find it useful to think of the detector as a small part of a sphere that is centered on the source, as shown in the following diagram:



**Total points for Problem Set 2: 60, plus 15 points of extra credit.**