MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — February 29, 2000
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QUIZ 1

USEFUL INFORMATION:

DOPPLER SHIFT:

$$z = v/u \quad \text{(nonrelativistic, source moving)}$$

$$z = \frac{v/u}{1 - v/u} \quad \text{(nonrelativistic, observer moving)}$$

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \quad \text{(relativistic)}$$

COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_{\text{observed}})}{R(t_{\text{emitted}})}$$

NOTE: Any answer may be expressed in terms of symbols representing the answers to previous parts of the same question, whether or not the previous part was answered correctly. When I say that an answer should be expressed in terms of a specified set of variables, I mean that the variables used in your answer should be drawn from that set; this does not imply that all the variables in the set need be used.

PROBLEM 1: DID YOU DO THE READING? *(35 points)*

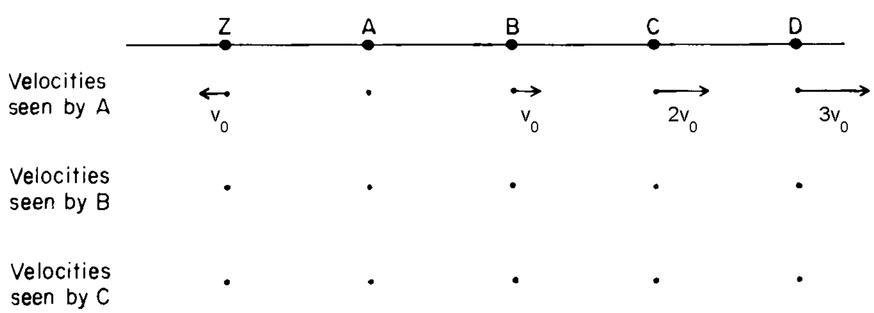
The following questions are worth 5 points each.

a) The Doppler effect for both sound and light waves is named for Johann Christian Doppler, a professor of mathematics at the Realschule in Prague. He predicted the effect for both types of waves in xx42. What are the two digits xx?

b) When the sky is very clear (as it almost never is in Boston), one can see a band of light across the night sky that has been known since ancient times as the Milky Way. Explain in a sentence or two how this band of light is related to the shape of the galaxy in which we live, which is also called the Milky Way.

c) The statement that the distant galaxies are on average receding from us with a speed proportional to their distance was first published by Edwin Hubble in 1929, and has become known as Hubble's law. Was Hubble's original paper based on the study of 2, 18, 180, or 1,800 galaxies?

d) The following diagram, labeled *Homogeneity and the Hubble Law,* was used by Weinberg to explain how Hubble's law is consistent with the homogeneity of the universe:



The arrows and labels from the "Velocities seen by B" and the "Velocities seen by C" rows have been deleted from this copy of the figure, and it is your job to sketch the figure in your exam book with these arrows and labels included. (Actually, in Weinberg's diagram these arrows were not labeled, but the labels are required here so that the grader does not have to judge the precise length of hand-drawn arrows.)

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e) The horizon is the present distance of the most distant objects from which light has had time to reach us since the beginning of the universe. The horizon changes with time, but of course so does the size of the universe as a whole. During a time interval in which the linear size of the universe grows by 1%, does the horizon distance

(i) grow by more than 1%, or

(ii) grow by less than 1%, or

(iii) grow by the same 1%?

f) Name the two men who in 1964 discovered the cosmic background radiation. With what institution were they affiliated?

g) At a temperature of 3000 K, the nuclei and electrons that filled the universe combined to form neutral atoms, which interact very weakly with the photons of the background radiation. After this process, known as "recombination," the background radiation expanded freely. Since recombination, how have each of the following quantities varied as the size of the universe has changed? (Your answers should resemble statements such as "proportional to the size of the universe," or "inversely proportional to the square of the size of the universe". The word "size" will be interpreted to mean linear size, not volume.)

(i) the average distance between photons

(ii) the typical wavelength of the radiation

(iii) the number density of photons in the radiation

(iv) the energy density of the radiation

(v) the temperature of the radiation

PROBLEM 2: THE STEADY-STATE UNIVERSE THEORY *(25 points)*

The steady-state theory of the universe was proposed in the late 1940s by Hermann Bondi, Thomas Gold, and Fred Hoyle, and was considered a viable model for the universe until the cosmic background radiation was discovered and its properties were confirmed. As the name suggests, this theory is based on the hypothesis that the large-scale properties of the universe do not change with time. The expansion of the universe was an established fact when the steady-state theory was invented, but the steady-state theory reconciles the expansion with a steady-state density of matter by proposing that new matter is created as the universe expands, so that the matter density does not fall. Like the conventional theory, the steady-state theory describes a homogeneous, isotropic, expanding universe, so the same comoving coordinate formulation can be used.

a) *(10 points)* The steady-state theory proposes that the Hubble constant, like other cosmological parameters, does not change with time, so $H(t) = H_0$. Find the most general form for the scale factor function $R(t)$ which is consistent with this hypothesis.

b) *(15 points)* Suppose that the mass density of the universe is ρ_0, which of course does not change with time. In terms of the general form for $R(t)$ that you found in part (a), calculate the rate at which new matter must be created for ρ_0 to remain constant as the universe expands. Your answer should have the units of mass per unit volume per unit time. *[If you failed to answer part (a), you will still receive full credit here if you correctly answer the question for an arbitrary scale factor function $R(t)$.]*

PROBLEM 3: ANOTHER FLAT UNIVERSE WITH AN UNUSUAL TIME EVOLUTION *(40 points)*

The following problem is based on Problem 5 of Review Problems for Quiz 1 (2000). The original part (d) has been deleted, the original part (e) has been renamed part (d), and a new part (e) has been added.

Consider a ***flat*** universe which is filled with some peculiar form of matter, so that the Robertson–Walker scale factor behaves as

$$R(t) = bt^{\gamma} ,$$

where b and γ are constants. *[This universe differs from the matter-dominated universe described in the lecture notes in that ρ is not proportional to $1/R^3(t)$. Such behavior is possible when pressures are large, because a gas expanding under pressure can lose energy (and hence mass) during the expansion.]* For the following questions, any of the answers may depend on γ, whether it is mentioned explicitly or not.

a) *(5 points)* Let t_0 denote the present time, and let t_e denote the time at which the light that we are currently receiving was emitted by a distant object. In terms of these quantities, find the value of the redshift parameter z with which the light is received.

b) *(5 points)* Find the "look-back" time as a function of z and t_0. The look-back time is defined as the length of the interval in cosmic time between the emission and observation of the light.

c) *(10 points)* Express the present value of the physical distance to the object as a function of H_0, z, and γ.

d) *(10 points)* At the time of emission, the distant object had a power output P (measured, say, in ergs/sec) which was radiated uniformly in all directions, in the form of photons. What is the radiation energy flux J from this object at the earth today? Express your answer in terms of P, H_0, z, and γ. *[Energy flux (which might be measured in erg-cm^{-2}-sec^{-1}) is defined as the energy per unit area per unit time striking a surface that is orthogonal to the direction of energy flow.]*

e) *(10 points)* Suppose that the distant object is a galaxy, moving with the Hubble expansion. Within the galaxy a supernova explosion has hurled a jet of material directly towards Earth with a speed v, measured relative to the galaxy, which is comparable to the speed of light c. Assume, however, that the distance the jet has traveled from the galaxy is so small that it can be neglected. With what redshift z_J would we observe the light coming from this jet? Express your answer in terms of all or some of the variables v, z (the redshift of the galaxy), t_0, H_0, and γ, and the constant c.