MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — March 12, 2002
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QUIZ 1

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USEFUL INFORMATION:

DOPPLER SHIFT:

$$z = v/u \quad \text{(nonrelativistic, source moving)}$$

$$z = \frac{v/u}{1 - v/u} \quad \text{(nonrelativistic, observer moving)}$$

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \quad \text{(special relativity, with } \beta = v/c\text{)}$$

COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_{\text{observed}})}{R(t_{\text{emitted}})}$$

EVOLUTION OF A MATTER-DOMINATED UNIVERSE:

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$

$$\ddot{R} = -\frac{4\pi}{3}G\rho R$$

$$\rho(t) = \frac{R^3(t_i)}{R^3(t)}\,\rho(t_i)$$

Flat ($\Omega \equiv \rho/\rho_c = 1$): $\quad R(t) \propto t^{2/3}$

Closed ($\Omega > 1$):

$$ct = \alpha(\theta - \sin\theta) \ ,$$

$$\frac{R}{\sqrt{k}} = \alpha(1 - \cos\theta) \ ,$$

$$\text{where } \alpha \equiv \frac{4\pi}{3}\frac{G\rho R^3}{k^{3/2}c^2}$$

Open ($\Omega < 1$):

$$ct = \alpha\,(\sinh\theta - \theta)$$

$$\frac{R}{\sqrt{\kappa}} = \alpha\,(\cosh\theta - 1) \ ,$$

$$\text{where } \alpha \equiv \frac{4\pi}{3}\frac{G\rho R^3}{\kappa^{3/2}c^2} \ ,$$

$$\kappa \equiv -k \ .$$

PROBLEM 1: DID YOU DO THE READING? *(25 points)*

Part (a) is worth 4 points, and all the other parts are worth 3 points each.

(a) The radio emission from the Milky Way is primarily produced by which one of the following processes?

(i) cosmic ray electrons spiralling in the Galaxy's magnetic field
(ii) nuclear reactions in stars
(iii) dust emission
(iv) spectral line radiation from neutral hydrogen clouds
(v) proton decay

(b) The ***Hertzsprung-Russell*** diagram, on which stars burning hydrogen like our sun lie on a line called the ***main sequence***, is a plot of the luminosity of a star against what other property of the star?

(c) Do the oldest stars in our galaxy lie in the galactic nucleus, in globular clusters in the halo, or in star-forming gas clouds?

(d) Newton proposed a method for testing whether or not stars are distributed with uniform number density throughout the universe. Which one of the following is the method that Newton actually proposed?

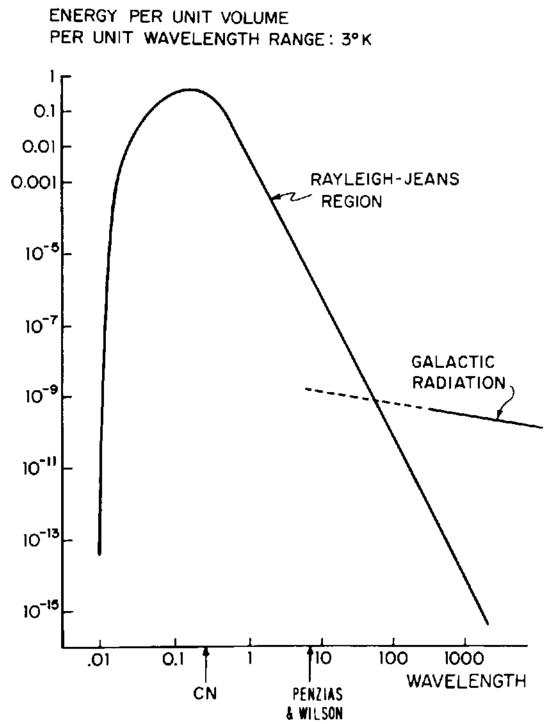
(i) measuring the change in direction of light from the stars as the earth goes round the sun
(ii) looking for a shift in the spectral lines of the stars
(iii) counting the number of stars as a function of their observed flux
(iv) finding a standard candle against which to compare the luminosity of the stars

(e) Cosmologists now believe that the universe today is dominated by which one of the following forms of energy?

(i) photons and neutrinos
(ii) dark matter
(iii) baryonic matter
(iv) magnetic fields
(v) "dark energy" (i.e., energy density of the vacuum, or some form of peculiar matter that behaves very similarly)

— Problem 1 Continues —

The following three questions all refer to the diagram at the right, which shows the distribution of energy density per unit wavelength range as a function of wavelength for the cosmic background radiation. This graph, often called the Planck distribution, was reproduced from Weinberg's *The First Three Minutes*, except that the units have been removed from both axes.



(f) Which one of the following units of wavelength gives the correct scale on the horizontal axis: angstroms, microns, millimeters, centimeters, meters, or kilometers?

(g) Which one of the following physical arguments best explains why the curve falls off at long wavelengths?

(i) It is hard for atoms to emit radiation with wavelengths much larger than the size of the atoms.

(ii) Wavelengths are strongly suppressed if they are longer than the size of the universe at the time when the radiation was emitted.

(iii) It is hard to fit radiation into any volume whose dimensions are smaller than the wavelength.

(iv) It is a direct consequence of the fact that photons are bosons and therefore obey Bose-Einstein statistics.

(h) Which one of the following physical arguments best explains why the curve falls off at short wavelengths?

(i) Short wavelength photons decay rapidly into electron-positron pairs.

(ii) Photons obey Fermi-Dirac statistics and therefore it is hard to pack many short wavelength photons in a box.

(iii) The energy of any photon is inversely proportional to the wavelength, so at a given temperature there will not be enough energy to produce many photons of very short wavelength.

(iv) It is hard for atoms to emit radiation with wavelengths much shorter than the size of the atoms.

(v) Wavelengths are strongly suppressed if they are smaller than the size of the universe at the time when the radiation was emitted.

PROBLEM 2: ANOTHER FLAT UNIVERSE WITH AN UNUSUAL TIME EVOLUTION *(40 points)*

The following problem was Problem 7 on the Review Problems, and it was also Problem 3, Quiz 1, 2000.

Consider a ***flat*** universe which is filled with some peculiar form of matter, so that the Robertson–Walker scale factor behaves as

$$R(t) = bt^\gamma ,$$

where b and γ are constants. *[This universe differs from the matter-dominated universe described in the lecture notes in that ρ is not proportional to $1/R^3(t)$. Such behavior is possible when pressures are large, because a gas expanding under pressure can lose energy (and hence mass) during the expansion.]* For the following questions, any of the answers may depend on γ, whether it is mentioned explicitly or not. Any answer may be expressed in terms of symbols representing the answers to previous parts of this question, whether or not the previous part was answered correctly.

a) *(5 points)* Let t_0 denote the present time, and let t_e denote the time at which the light that we are currently receiving was emitted by a distant object. In terms of these quantities, find the value of the redshift parameter z with which the light is received.

b) *(5 points)* Find the "look-back" time as a function of z and t_0. The look-back time is defined as the length of the interval in cosmic time between the emission and observation of the light.

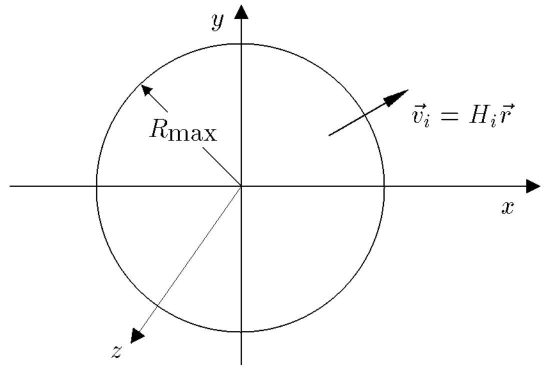
c) *(10 points)* Express the present value of the physical distance to the object as a function of H_0, z, and γ, where H_0 is the present value of the Hubble constant.

d) *(10 points)* At the time of emission, the distant object had a power output P (measured, say, in ergs/sec) which was radiated uniformly in all directions, in the form of photons. What is the radiation energy flux J from this object at the earth today? Express your answer in terms of P, H_0, z, and γ. *[Energy flux (which might be measured in erg-cm^{-2}-sec^{-1}) is defined as the energy per unit area per unit time striking a surface that is orthogonal to the direction of energy flow.]*

e) *(10 points)* Suppose that the distant object is a galaxy, moving with the Hubble expansion. Within the galaxy a supernova explosion has hurled a jet of material directly towards Earth with a speed v, measured relative to the galaxy, which is comparable to the speed of light c. Assume, however, that the distance the jet has traveled from the galaxy is so small that it can be neglected. With what redshift z_J would we observe the light coming from this jet? Express your answer in terms of all or some of the variables v, z (the redshift of the galaxy), t_0, H_0, and γ, and the constant c.

PROBLEM 3: PARTICLE TRAJECTORIES IN NEWTONIAN COSMOLOGY *(35 points)*

In lecture we developed a Newtonian model of cosmology, in which we started with a uniform sphere of matter, as illustrated on the right, with an initial density ρ_i and with an initial velocity pattern given by Hubble's law, $\vec{v}_i = H_i \vec{r}$. We then calculated how such a model would evolve, showing that it would remain uniform in density. All of the motion is radial, and we found that the radial coordinate of a particle that began at radius r_i is given at any time by



$$r(r_i, t) = R(t) r_i \ , \tag{1}$$

where $R(t)$ is called the scale factor. The equations of motion obeyed by $R(t)$ are shown on the front of this exam.

In this problem we will consider the motion of a test particle A that moves ***through*** this model universe, moving at a velocity that is ***different*** from the Hubble velocity. The mass of the test particle is negligible, so it will not affect the motion of the other particles. The only force acting on the test particle is the gravitational force of the other particles. All velocities and accelerations are expressed in the ***inertial*** reference frame shown in the diagram above (i.e., not in comoving coordinates), so we can directly apply Newton's laws. All velocities are nonrelativistic.

(a) *(10 points)* Suppose that at some time t, the test particle is located at position $\vec{r}_A$ and moves with velocity $\vec{v}_A \equiv \mathrm{d}\vec{r}_A/\mathrm{d}t$. What is the acceleration $\vec{a}_A$ of the test particle? Your answer should be expressed in terms of some or all of the following quantities: $\vec{r}_A$, $\vec{v}_A$, Newton's constant G, the speed of light c, the mass density $\rho(t)$, the scale factor $R(t)$, and the Hubble parameter (also called the Hubble "constant") $H(t)$. If your answer is not written explicitly as a vector, you should describe the direction of the acceleration.

(b) *(8 points)* The matter that makes up the expanding universe is undergoing Hubble expansion. At the location of the test particle, what is the Hubble velocity $\vec{v}_H$ of this matter? Your answer should again be expressed in terms of some or all of the quantities on the list from part (a). Again, if your answer is not written explicitly as a vector, you should describe its direction.

— *Problem 3 Continues* —

(c) *(9 points)* The velocity of the test particle relative to the Hubble velocity at the same location is called the ***proper velocity,***

$$\vec{v}_p \equiv \vec{v}_A - \vec{v}_H \ . \tag{2}$$

Find an expression for the time derivative of the proper velocity. The correct answer can be written as

$$\frac{\mathrm{d}\vec{v}_p}{\mathrm{d}t} = -\lambda H(t)\vec{v}_p \ , \tag{3}$$

where λ is a dimensionless constant that you must determine.

(d) *(8 points)* Whether or not you answered part (c), assume that Eq. (3) holds with an arbitrary value of λ. By considering the x-component of the equation, show that

$$v_{p,x}(t) = R^n(t)\, v_0 \ , \tag{4}$$

where v_0 is an undetermined constant, and n is a constant that you should find in terms of λ.