MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — March 16, 2004
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QUIZ 1

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USEFUL INFORMATION:

DOPPLER SHIFT:

$$z = v/u \quad \text{(nonrelativistic, source moving)}$$

$$z = \frac{v/u}{1 - v/u} \quad \text{(nonrelativistic, observer moving)}$$

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \quad \text{(special relativity, with } \beta = v/c\text{)}$$

COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_{\text{observed}})}{R(t_{\text{emitted}})}$$

EVOLUTION OF A MATTER-DOMINATED UNIVERSE:

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{R^2}$$

$$\ddot{R} = -\frac{4\pi}{3} G\rho R$$

$$\rho(t) = \frac{R^3(t_i)}{R^3(t)}\, \rho(t_i)$$

Flat ($\Omega \equiv \rho/\rho_c = 1$): $\quad R(t) \propto t^{2/3}$

Closed ($\Omega > 1$):

$$ct = \alpha(\theta - \sin\theta) \ ,$$

$$\frac{R}{\sqrt{k}} = \alpha(1 - \cos\theta) \ ,$$

$$\text{where } \alpha \equiv \frac{4\pi}{3} \frac{G\rho R^3}{k^{3/2} c^2}$$

Open ($\Omega < 1$):

$$ct = \alpha\,(\sinh\theta - \theta)$$

$$\frac{R}{\sqrt{\kappa}} = \alpha\,(\cosh\theta - 1) \ ,$$

$$\text{where } \alpha \equiv \frac{4\pi}{3} \frac{G\rho R^3}{\kappa^{3/2} c^2} \ ,$$

$$\kappa \equiv -k \ .$$

PROBLEM 1: DID YOU DO THE READING? *(25 points)*

Each of the following parts is worth 5 points. There are blank pages at the back if you need more room for your answers.

(a) In 1826, the astronomer Heinrich Olber wrote a paper on a paradox regarding the night sky. What is Olber's paradox? What is the primary resolution of it?

(b) What is the value of the Newtonian gravitational constant G in Planck units? The Planck length is of the order of 10^{-35} m, 10^{-15} m, 10^{15} m, or 10^{35} m?

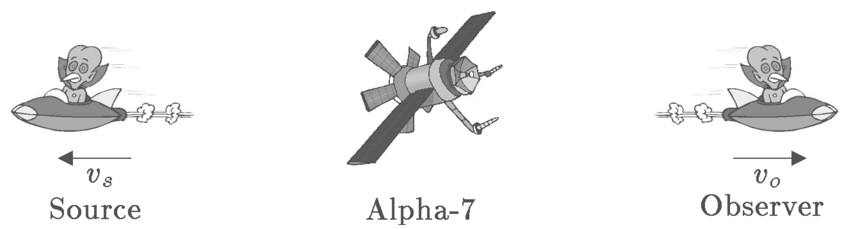
(c) What is the Cosmological Principle? Is the Hubble expansion of the universe consistent with it? (For the latter question, a simple "yes" or "no" will suffice.)

(d) In the "Standard Model" of the universe, when the universe cooled to about 3×10^a K, it became transparent to photons, and today we observe these as the Cosmic Microwave Background (CMB) at a temperature of about 3×10^b K. What are the integers a and b?

(e) What did the universe primarily consist of at about 1/100th of a second after the Big Bang? Include any constituent that is believed to have made up more than 1% of the mass density of the universe.

PROBLEM 2: SPECIAL RELATIVITY DOPPLER SHIFT *(20 points)*

Consider the Doppler shift of radio waves, for a case in which both the source and the observer are moving. Suppose the source is a spaceship moving with a speed v_s relative to the space station Alpha-7, while the observer is on another spaceship, moving in the opposite direction from Alpha-7 with speed v_o relative to Alpha-7.



(a) *(10 points)* Calculate the Doppler shift z of the radio wave as received by the observer. (Recall that radio waves are electromagnetic waves, just like light except that the wavelength is longer.)

(b) *(10 points)* Use the results of part (a) to determine v_{tot}, the velocity of the source spaceship as it would be measured by the observer spaceship. *(8 points will be given for the basic idea, whether or not you have the right answer for part (a), and 2 points will be given for the algebra.)*

PROBLEM 3: EVOLUTION OF A FLAT UNIVERSE WITH $R(t) = bt^{1/4}$ *(35 points)*

The following questions all pertain to a flat universe, with a scale factor given by

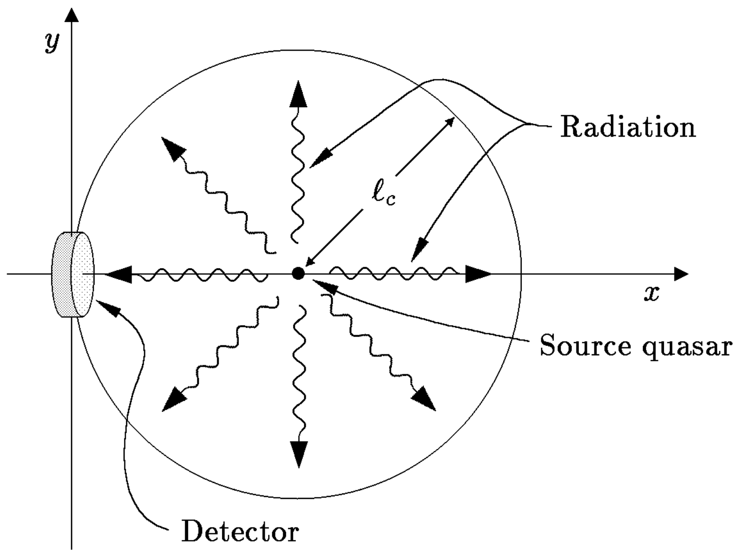
$$R(t) = bt^{1/4} ,$$

where b is a constant and t denotes cosmic time.

(a) *(10 points)* If a light pulse is emitted by a quasar at time t_e and observed here on Earth at time t_o, find the physical separation $\ell_p(t_e)$ between the quasar and the Earth, at the time of ***emission.***

(b) *(10 points)* At what time is the light pulse equidistant from the quasar and the Earth?

(c) *(15 points)* At the time of emission, the quasar had a power output P (measured, say, in ergs/sec), which was radiated uniformly in all directions. What is the radiation energy flux J from this quasar at the Earth today? Energy flux (which might be measured in ergs-cm^{-2}-sec^{-1}) is defined as the energy per unit area per unit time striking a surface that is orthogonal to the direction of energy flow. You may find it useful to think of the detector as a small part of a sphere that is centered on the source, as shown in the following diagram:



PROBLEM 4: EVOLUTION OF A CLOSED, MATTER-DOMINATED UNIVERSE *(20 points)*

The following problem was Problem Set 2, Problem 4.

It was shown in Lecture Notes 5 that the evolution of a closed, matter-dominated universe can be described by introducing the time-parameter θ, with

$$ct = \alpha(\theta - \sin\theta) \ ,$$

$$\frac{R}{\sqrt{k}} = \alpha(1 - \cos\theta) \ ,$$

where α is a constant with the units of length.

(a) *(10 points)* Use these expressions to find H, the Hubble "constant," as a function of α and θ. *(Hint: You can use the first of the equations above to calculate $d\theta/dt$.)*

(b) *(5 points)* Find ρ, the mass density, as a function of α and θ. *(4 points will be given for a correct answer, with 1 additional point if the answer is algebraically simplified.)*

(c) *(5 points)* Find Ω, where $\Omega \equiv \rho/\rho_c$, as a function of α and θ.