Drifting both sides by the critical density gives
\[
\frac{\rho_{\text{crit}}}{\rho} = \frac{\rho_{\text{crit}}}{\rho} \frac{a^3}{a^3} = \frac{\rho_{\text{crit}}}{\rho} \frac{a^3}{a^3} = \frac{\rho_{\text{crit}}}{\rho} \frac{a^3}{a^3}
\]
Therefore, the statement that the energy density in radiation and matter were equal at \( a = 0 \) can be written

\[
\rho_{\text{rad}}(0) = \rho_{\text{mat}}(0).
\]

As Ryden discusses in chapter 5, the universe today contains not only the photons of the cosmic microwave background (CMB), but also photons that originated as starlight. Including both direct starlight and starlight reflected by dust, we must have

\[
\rho_{\text{rad}}(0) > \rho_{\text{mat}}(0)\text{.}
\]

Since \( 0 < \rho \), we must have

\[
\frac{\rho}{\rho_{\text{crit}}} > \left( \frac{\rho_{\text{crit}}}{\rho} \right) \text{ for } \rho > 0.
\]

Therefore

\[
\frac{\rho_{\text{rad}}}{\rho_{\text{crit}}} > \left( \frac{\rho_{\text{rad}}}{\rho_{\text{crit}}} \right) \text{ for } \rho > 0.
\]

And thus

\[
\frac{\rho_{\text{rad}}}{\rho_{\text{crit}}} > \left( \frac{\rho_{\text{rad}}}{\rho_{\text{crit}}} \right) \text{ for } \rho > 0.
\]

Ryden’s equation (5) is the notation Ryden uses for the scale factor, which in the Lecture Notes refers to the present value of the Hubble parameter. The energy density of radiation at the present time is

\[
\rho_{\text{rad}}(t_0) = \rho_{\text{rad}}(t) = \frac{8 \pi G}{3} \rho_{\text{crit}}
\]

Hence, the energy density of radiation is

\[
\rho_{\text{rad}}(t_0) = \rho_{\text{rad}}(t) = \frac{8 \pi G}{3} \rho_{\text{crit}}
\]

The energy density of matter is given by

\[
\rho_{\text{mat}}(t_0) = \rho_{\text{mat}}(t) = \frac{8 \pi G}{3} \rho_{\text{crit}}
\]

and therefore

\[
\rho_{\text{rad}}(t_0) = \rho_{\text{rad}}(t) = \frac{8 \pi G}{3} \rho_{\text{crit}}
\]

Hence, the energy density of radiation is

\[
\rho_{\text{rad}}(t_0) = \rho_{\text{rad}}(t) = \frac{8 \pi G}{3} \rho_{\text{crit}}
\]

Hence, the energy density of radiation is

\[
\rho_{\text{rad}}(t_0) = \rho_{\text{rad}}(t) = \frac{8 \pi G}{3} \rho_{\text{crit}}
\]

Hence, the energy density of radiation is

\[
\rho_{\text{rad}}(t_0) = \rho_{\text{rad}}(t) = \frac{8 \pi G}{3} \rho_{\text{crit}}
\]

Hence, the energy density of radiation is

\[
\rho_{\text{rad}}(t_0) = \rho_{\text{rad}}(t) = \frac{8 \pi G}{3} \rho_{\text{crit}}
\]

Hence, the energy density of radiation is

\[
\rho_{\text{rad}}(t_0) = \rho_{\text{rad}}(t) = \frac{8 \pi G}{3} \rho_{\text{crit}}
\]

Hence, the energy density of radiation is

\[
\rho_{\text{rad}}(t_0) = \rho_{\text{rad}}(t) = \frac{8 \pi G}{3} \rho_{\text{crit}}
\]
where we have used the time dependence of \( R(t) \) that we found in part (a).

\[
\begin{align*}
\frac{\dot{R}}{R} & = \frac{\ddot{H}}{H} = H
\end{align*}
\]

\( (b) \) The Hubble parameter is, from its definition:

\[
\frac{\dot{R}}{R} \propto H
\]

\( (c) \) Recall that the horizon distance is the physical distance traveled by a light ray since \( t = 0 \).

\[
\ell_{\text{horizon}}(t) = \frac{R(t)}{c} \int_{t'}^t c dt' = \frac{R(t)}{c}
\]

\( (d) \) The Friedman equation is

\[
\frac{\dot{R}}{R} = \frac{\ddot{H}}{H} = H
\]

\( (e) \) The Friedman equation:

\[
\frac{\dot{R}}{R} = \frac{\ddot{H}}{H} = H
\]

\( (f) \) On your forehead.

\( (g) \) Writing the Friedman equation:

\[
\frac{\dot{R}}{R} = \frac{\ddot{H}}{H} = H
\]

\( (h) \) Where does Barbara Ryden suggest writing the Friedman equation?

\( (i) \) Time evolution of a universe with mysterious stuff:

\[
\begin{align*}
\frac{\dot{R}}{R} & = \frac{\ddot{H}}{H} = H
\end{align*}
\]

\( (j) \) Derive Eq. (1) above, and find the values of \( A \) and \( B \).

\[
\begin{align*}
A & = \frac{1}{2} \frac{H}{H_0} \left( 1 + \frac{\alpha}{H_0} \right) \\
B & = -\frac{1}{2} \frac{H}{H_0}
\end{align*}
\]

\( (k) \) Bonus question:

\[
\begin{align*}
\frac{\dot{R}}{R} & = \frac{\ddot{H}}{H} = H
\end{align*}
\]

\( (l) \) This lets us read off the values of \( A \) and \( B \).

\[
\begin{align*}
\frac{\dot{R}}{R} & = \frac{\ddot{H}}{H} = H
\end{align*}
\]

\( (m) \) Taking the square root yields:

\[
\begin{align*}
\dot{R} & = \frac{\ddot{H}}{H} = H
\end{align*}
\]

\( (n) \) Notice that once again we have eliminated the arbitrary integration constant.

\[
\begin{align*}
\dot{R} & = \frac{\ddot{H}}{H} = H
\end{align*}
\]

\( (o) \) Rearranging to a form we can integrate:

\[
\begin{align*}
\dot{R} & = \frac{\ddot{H}}{H} = H
\end{align*}
\]

\( (p) \) Solving for \( R \) yields:

\[
\begin{align*}
\dot{R} & = \frac{\ddot{H}}{H} = H
\end{align*}
\]

\( (q) \) The Friedman equation is a form we can integrate:

\[
\begin{align*}
\dot{R} & = \frac{\ddot{H}}{H} = H
\end{align*}
\]

\( (r) \) Eliminating the arbitrary constant controlling the definition of a notch.

\[
\begin{align*}
\dot{R} & = \frac{\ddot{H}}{H} = H
\end{align*}
\]
\[ \rho \int_0^{2\pi} \int_0^\theta r^2 \sin \theta \, dr \, d\theta = V \]

so

\[ \rho \int_0^{2\pi} \int_0^\theta r^2 \sin \theta \, dr \, d\theta = V \]

Thus the area is then

\[ \rho \int_0^{2\pi} \int_0^\theta r^2 \sin \theta \, dr \, d\theta = \theta^2 \rho = V \]

Since the metric does not contain a term in \( dr \), the \( r \) and \( \theta \) directions are

\[ \cdot \rho \int_0^\theta r^2 \, dr + \int_0^{2\pi} \cot \theta \, d\theta = V \]

and so

\[ \cdot \rho \int_0^\theta r^2 \, dr + \int_0^{2\pi} \cot \theta \, d\theta = V \]

As a check on our algebra, since we found in (a) that

\( \frac{\partial x}{\partial \theta} = \sin \theta \]

the 2-dimensional integral for which

\[ \rho \int_0^{2\pi} \int_0^\theta r^2 \sin \theta \, dr \, d\theta = V \]

and

\[ \rho \int_0^\theta r^2 \, dr + \int_0^{2\pi} \cot \theta \, d\theta = V \]

Since the metric does not vary along this path

\[ \rho \int_0^\theta r^2 \sin \theta + \int_0^{2\pi} \cot \theta \, d\theta = V \]

Since \( r \) runs from \( 0 \) to \( r \), the curve is swept out

\[ \rho \int_0^\theta r^2 \sin \theta + \int_0^{2\pi} \cot \theta \, d\theta = V \]

So

\[ \rho \int_0^\theta r^2 \sin \theta + \int_0^{2\pi} \cot \theta \, d\theta = V \]

PROBLEM 3: AN EXERCISE IN TWO-DIMENSIONAL METRICS

Let's begin our work on the problem.

First, let's calculate the area of the region where \( r \) and \( \theta \) are constant. The area is then

\[ \int_0^\theta r^2 \sin \theta \, dr \]

The area is then

\[ \int_0^\theta r^2 \sin \theta \, dr = \theta^2 \]

and

\[ \int_0^\theta r^2 \sin \theta \, dr = \theta^2 \]

Since \( \rho \) does not vary along this path

\[ \rho \int_0^\theta r^2 \sin \theta + \int_0^{2\pi} \cot \theta \, d\theta = V \]

Since \( r \) runs from \( 0 \) to \( r \), the curve is swept out

\[ \rho \int_0^\theta r^2 \sin \theta + \int_0^{2\pi} \cot \theta \, d\theta = V \]

So

\[ \rho \int_0^\theta r^2 \sin \theta + \int_0^{2\pi} \cot \theta \, d\theta = V \]

Using the result from part (b), we find

\[ \frac{\rho \sin \theta \, dx}{\rho H} = (1) \]

From the Hubble equation, we know the Hubble parameter, we can find the mass density easily

\[ \int_0^\theta r^2 \sin \theta \, dr = (1) \]

Since we know the Hubble parameter, we can find the mass density easily

\[ \omega = \left( \frac{\rho}{\rho H} \right) = (1) \]

We find

\[ \int_0^\theta r^2 \sin \theta \, dr = (1) \]
\[ \left. \frac{\partial}{\partial \theta} \right|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial^2 \mathbf{r}}{\partial \psi \partial \theta} I - 1 \right| = \frac{dP}{d\Psi} \]

so

\[ \frac{\partial}{\partial \theta} \left|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \psi} I - 1 \right| = \frac{dP}{d\Psi} \]

Taking a square root,

\[ \frac{\partial}{\partial \theta} \left|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \psi} I - 1 \right| = \frac{\partial}{\partial \theta} \left|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \psi} \right) \]

from which it follows that

\[ \frac{\partial}{\partial \theta} \left|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \psi} \right) = \frac{dP}{d\Psi} \cdot \frac{\partial^2 \mathbf{r}}{\partial \psi \partial \theta} \]

Since \( \tau \) and \( \phi \) are not changing,

\[ \text{(a) Simplifying slightly,} \]

\[ \left( \frac{\partial}{\partial \theta} \right|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \psi} I - 1 \right| = \left( \frac{\partial}{\partial \theta} \right|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \psi} \right) \]

\( \text{PROBLEM 4: TRAJECTORIES IN AN OPEN UNIVERSE} \)

\[ 0 = \frac{\partial}{\partial \psi} \left|_{\tau} \left( \frac{\partial}{\partial \theta} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \theta} \right) \]

left-hand side of the boundary equation vanishes. One can simplify

\[ \left( I - \left( \frac{\partial}{\partial \psi} \right|_{\tau} \left( \frac{\partial}{\partial \theta} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \theta} \right) \right) = \mathbf{V} \]

The nonzero metric coefficients are given by

\[ \frac{\partial}{\partial \theta} \left|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \psi} \right) = \frac{dP}{d\Psi} \]

From the substitution \( n = \frac{\partial}{\partial \psi} \), \( n \cdot \mathbf{V} = \mathbf{V} \).

The result is

\[ \left[ I - \left( \frac{\partial}{\partial \psi} \right|_{\tau} \left( \frac{\partial}{\partial \theta} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \theta} \right) \right] \frac{\partial}{\partial \psi} \frac{\partial}{\partial \theta} \mathbf{r} = \mathbf{V} \]

You were not asked to carry out the integration, but it can be done by using

\[ \text{The left-hand side of the above equation above, the first term can be} \]

\[ \text{Inserting this expansion into the boxed equation above, the first term can be} \]

\[ \frac{\partial}{\partial \psi} \left|_{\tau} \left( \frac{\partial}{\partial \theta} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \theta} \right) \]

The answer above is perfectly acceptable. But one might want to expand the

\[ \left( \frac{\partial}{\partial \theta} \right|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \psi} \right) \]

So if we substitute the values from above, we have

\[ \frac{\partial}{\partial \psi} \left|_{\tau} \left( \frac{\partial}{\partial \theta} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \theta} \right) \]

The nonzero metric coefficients are given by

\[ \left. \frac{\partial}{\partial \theta} \right|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \psi} \right) \]

The result is

\[ \frac{\partial}{\partial \psi} \left|_{\tau} \left( \frac{\partial}{\partial \theta} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \theta} \right) \]

From the substitution \( n = \frac{\partial}{\partial \psi} \), \( n \cdot \mathbf{V} = \mathbf{V} \).

The result is

\[ \left[ I - \left( \frac{\partial}{\partial \psi} \right|_{\tau} \left( \frac{\partial}{\partial \theta} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \theta} \right) \right] \frac{\partial}{\partial \psi} \frac{\partial}{\partial \theta} \mathbf{r} = \mathbf{V} \]

You were not asked to carry out the integration, but it can be done by using

\[ \text{The left-hand side of the above equation above, the first term can be} \]

\[ \frac{\partial}{\partial \psi} \left|_{\tau} \left( \frac{\partial}{\partial \theta} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \theta} \right) \]

The answer above is perfectly acceptable. But one might want to expand the

\[ \left( \frac{\partial}{\partial \theta} \right|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \psi} \right) \]

So if we substitute the values from above, we have

\[ \frac{\partial}{\partial \psi} \left|_{\tau} \left( \frac{\partial}{\partial \theta} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \theta} \right) \]

The nonzero metric coefficients are given by

\[ \left. \frac{\partial}{\partial \theta} \right|_{\tau} \left( \frac{\partial}{\partial \psi} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \psi} \right) \]

From the substitution \( n = \frac{\partial}{\partial \psi} \), \( n \cdot \mathbf{V} = \mathbf{V} \).

The result is

\[ \left[ I - \left( \frac{\partial}{\partial \psi} \right|_{\tau} \left( \frac{\partial}{\partial \theta} \right|_{\tau} \frac{\partial \mathbf{r}}{\partial \theta} \right) \right] \frac{\partial}{\partial \psi} \frac{\partial}{\partial \theta} \mathbf{r} = \mathbf{V} \]
This equation does not allow \( \tau^2 = 0 \). Thus, the equation becomes:

\[
0 = \\left\{ \frac{2p}{\partial p} \varepsilon^\alpha(i) \varepsilon^\beta \right\} \frac{2p}{p} = \text{SHI}
\]

Finally, then, the geodesic equation is:

\[
\left( \frac{2p}{\partial p} \varepsilon^\alpha(i) \right) \varepsilon^\beta = \left\{ \frac{2p}{\partial p} \varepsilon^\alpha(i) \varepsilon^\beta \right\} \frac{2p}{p} = \text{SHI}
\]

Note that the subscript \( \alpha \) is necessary. As indices that we are using in the geodesic equation are taken from the formula sheet:

\[
\varepsilon^\alpha(i) \varepsilon^\beta = \delta^\alpha_\beta
\]

Note that the subscript \( \alpha \) is necessary. As indices that we are using in the geodesic equation are taken from the formula sheet:

\[
\varepsilon^\alpha(i) \varepsilon^\beta = \delta^\alpha_\beta
\]

Notice that the subject of \( \alpha \) is necessary. As indices that we are using in the geodesic equation are taken from the formula sheet:

\[
\varepsilon^\alpha(i) \varepsilon^\beta = \delta^\alpha_\beta
\]