MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — October 18, 2005
Prof. Alan Guth

QUIZ 1

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Problem	Maximum	Score
1	25	
2	30	
3	25	
4	20	
TOTAL	100	

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Your Name

USEFUL INFORMATION:

DOPPLER SHIFT:

$$z = v/u \quad \text{(nonrelativistic, source moving)}$$

$$z = \frac{v/u}{1 - v/u} \quad \text{(nonrelativistic, observer moving)}$$

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \quad \text{(special relativity, with } \beta = v/c\text{)}$$

COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_{\text{observed}})}{R(t_{\text{emitted}})}$$

EVOLUTION OF A MATTER-DOMINATED UNIVERSE:

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$

$$\ddot{R} = -\frac{4\pi}{3}G\rho R$$

$$\rho(t) = \frac{R^3(t_i)}{R^3(t)}\,\rho(t_i)$$

Flat ($\Omega \equiv \rho/\rho_c = 1$): $\quad R(t) \propto t^{2/3}$

Closed ($\Omega > 1$):

$$ct = \alpha(\theta - \sin\theta)\ ,$$
$$\frac{R}{\sqrt{k}} = \alpha(1 - \cos\theta)\ ,$$
$$\text{where } \alpha \equiv \frac{4\pi}{3}\frac{G\rho R^3}{k^{3/2}c^2}$$

Open ($\Omega < 1$):

$$ct = \alpha\,(\sinh\theta - \theta)$$
$$\frac{R}{\sqrt{\kappa}} = \alpha\,(\cosh\theta - 1)\ ,$$
$$\text{where } \alpha \equiv \frac{4\pi}{3}\frac{G\rho R^3}{\kappa^{3/2}c^2}\ ,$$
$$\kappa \equiv -k\ .$$

PHYSICAL CONSTANTS:

$$G = 6.673 \times 10^{-8}\ \text{cm}^3 \cdot \text{g}^{-1} \cdot \text{s}^{-2}$$
$$k = \text{Boltzmann's constant} = 1.381 \times 10^{-16}\,\text{erg/K}$$
$$= 8.617 \times 10^{-5}\,\text{eV/K}\ ,$$
$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-27}\ \text{erg-sec}$$
$$= 6.582 \times 10^{-16}\ \text{eV-sec}\ ,$$
$$c = 2.998 \times 10^{10}\ \text{cm/sec}$$
$$1\ \text{yr} = 3.156 \times 10^{7}\ \text{s}$$
$$1\ \text{eV} = 1.602 \times 10^{-12}\ \text{erg}\ .$$

PROBLEM 1: DID YOU DO THE READING? *(25 points)*

(a) *(4 points)* What was the first external galaxy that was shown to be at a distance significantly greater than the most distant known objects in our galaxy? How was the distance estimated?

(b) *(5 points)* What is recombination? Did galaxies begin to form before or after recombination? Why?

(c) *(4 points)* In Chapter IV of his book, Weinberg develops a "recipe for a hot universe," in which the matter of the universe is described as a gas in thermal equilbrium at a very high temperature, in the vicinity of 10^9 K (several thousand million degrees Kelvin). Such a thermal equilibrium gas is completely described by specifying its temperature and the density of the conserved quantities. Which of the following is on this list of conserved quantities? Circle as many as apply.

(i) baryon number (ii) energy per particle (iii) proton number

(iv) electric charge (v) pressure

(d) *(4 points)* The wavelength corresponding to the mean energy of a CMB (cosmic microwave background) photon today is approximately equal to which of the following quantities? (You may wish to look up the values of various physical constants at the end of the quiz.)

(i) 2 fm (2×10^{-15} m)

(ii) 2 microns (2×10^{-6} m)

(iii) 2 mm (2×10^{-3} m)

(iv) 2 m.

(e) *(4 points)* What is the equivalence principle?

(f) *(4 points)* Why is it difficult for Earth-based experiments to look at the small wavelength portion of the graph of CMB energy density per wavelength vs. wavelength?

PROBLEM 2: EVOLUTION OF AN OPEN, MATTER-DOMINATED UNIVERSE *(30 points)*

The following question was Problem 5 on Problem Set 2.

The equations describing the evolution of an open, matter-dominated universe are shown on the formula sheets for this quiz, at the back of the quiz. The following mathematical identities, which you should know, may also prove useful on parts (e) and (f):

$$\sinh\theta = \frac{e^\theta - e^{-\theta}}{2} \quad , \quad \cosh\theta = \frac{e^\theta + e^{-\theta}}{2}$$

$$e^\theta = 1 + \frac{\theta}{1!} + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \ldots \ .$$

(a) *(4 points)* Find the Hubble "constant" H as a function of α and θ.

(b) *(4 points)* Find the mass density ρ as a function of α and θ.

(c) *(4 points)* Find the mass density parameter Ω as a function of α and θ.

(d) *(6 points)* Find the physical value of the horizon distance, $\ell_{p,\mathrm{horizon}}$, as a function of α and θ.

(e) *(6 points)* For very small values of t, it is possible to use the first nonzero term of a power-series expansion to express θ as a function of t, and then R as a function of t. Give the expression for $R(t)$ in this approximation. The approximation will be valid for $t \ll t^*$. Estimate the value of t^*.

(f) *(6 points)* Even though these equations describe an open universe, one still finds that Ω approaches one for very early times. For $t \ll t^*$ (where t^* is defined in part (e)), the quantity $1 - \Omega$ behaves as a power of t. Find the expression for $1 - \Omega$ in this approximation.

PROBLEM 3: TRACING A LIGHT PULSE THROUGH A RADIATION-DOMINATED UNIVERSE *(25 points)*

Consider a flat universe that expands with a scale factor

$$R(t) = bt^{1/2} ,$$

where b is a constant. We will learn later that this is the behavior of the scale factor for a radiation-dominated universe.

(a) *(5 points)* At an arbitrary time $t = t_f$, what is the physical horizon distance? (By "physical," I mean as usual the distance in physical units, such as meters or centimeters, as measured by a sequence of rulers, each of which is at rest relative to the comoving matter in its vicinity.)

(b) *(3 points)* Suppose that a photon arrives at the origin, at time t_f, from a distant piece of matter that is precisely at the horizon distance at time t_f. What is the time t_e at which the photon was emitted?

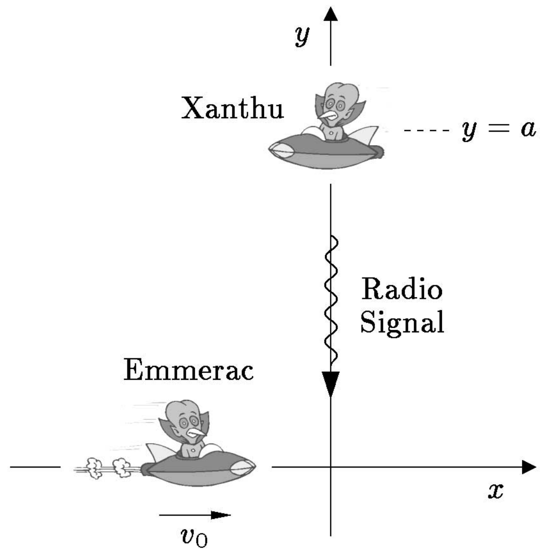
(c) *(2 points)* What is the **coordinate** distance from the origin to the point from which the photon was emitted?

(d) *(10 points)* For an arbitrary time t in the interval $t_e \leq t \leq t_f$, while the photon is traveling, what is the **physical** distance $\ell_p(t)$ from the origin to the location of the photon?

(e) *(5 points)* At what time $t_{\max}$ is the physical distance of the photon from the origin at its largest value?

PROBLEM 4: TRANSVERSE DOPPLER SHIFTS *(20 points)*

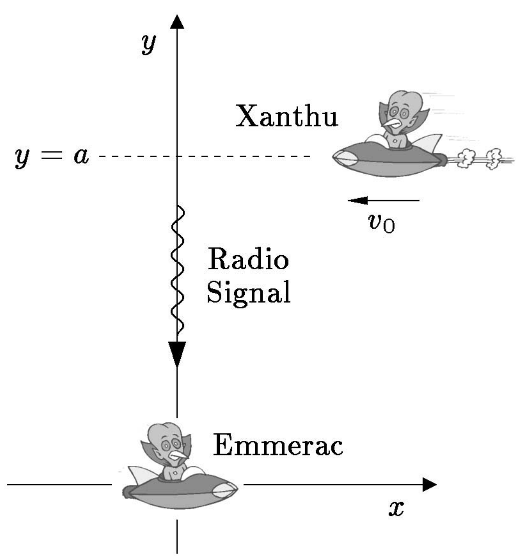
(a) *(8 points)* Suppose the spaceship Xanthu is at rest at location $(x=0, y=a, z=0)$ in a Cartesian coordinate system. (We assume that the space is Euclidean, and that the distance scales in the problem are small enough so that the expansion of the universe can be neglected.) The spaceship Emmerac is moving at speed v_0 along the x-axis in the positive direction, as shown in the diagram, where v_0 is comparable to the speed of light. As the Emmerac crosses the origin, it receives a radio signal that had been sent some time earlier from the Xanthu. Is the radiation received redshifted or blueshifted? What is the redshift z (where negative values of z can be used to describe blueshifts)?



(b) *(7 points)* Now suppose that the Emmerac is at rest at the origin, while the Xanthu is moving in the negative x-direction, at $y = a$ and $z = 0$, as shown in the diagram. That is, the trajectory of the Xanthu can be taken as

$$(x=-v_0 t, y=a, z=0) \ .$$

At $t = 0$ the Xanthu crosses the y-axis, and at that instant it emits a radio signal along the y-axis, directed at the origin. The radiation is received some time later by the Emmerac. In this case, is the radiation received redshifted or blueshifted? What is the redshift z (where again negative values of z can be used to describe blueshifts)?



(c) *(5 points)* Is the sequence of events described in (b) physically distinct from the sequence described in (a), or is it really the same sequence of events described in a reference frame that is moving relative to the reference frame used in part (a)? Explain your reasoning in a sentence or two. *(Hint: note that there are three objects in the problem: Xanthu, Emmerac, and the photons of the radio signal.)*