MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

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# QUIZ 1 FORMULA SHEET

## DOPPLER SHIFT (For motion along a line):

$$z = v/u \quad \text{(nonrelativistic, source moving)}$$

$$z = \frac{v/u}{1 - v/u} \quad \text{(nonrelativistic, observer moving)}$$

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \quad \text{(special relativity, with } \beta = v/c\text{)}$$

## COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_{\text{observed}})}{R(t_{\text{emitted}})}$$

## SPECIAL RELATIVITY:

Time Dilation Factor:

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \ , \qquad \beta \equiv v/c$$

Lorentz-Fitzgerald Contraction Factor: $\gamma$

Relativity of Simultaneity:

Trailing clock reads later by an amount $\beta \ell_0 / c$ .

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**EVOLUTION OF A MATTER-DOMINATED UNIVERSE:**

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2} \ , \quad \ddot{R} = -\frac{4\pi}{3}G\rho R \ ,$$

$$\rho(t) = \frac{R^3(t_i)}{R^3(t)}\,\rho(t_i) \ ,$$

$$\Omega \equiv \rho/\rho_c \ , \quad \text{where} \quad \rho_c = \frac{3H^2}{8\pi G} \ .$$

Flat ($k = 0$):

$$R(t) \propto t^{2/3} \ ,$$

$$\Omega = 1 \ .$$

Closed ($k > 0$):

$$ct = \alpha(\theta - \sin\theta) \ , \quad \frac{R}{\sqrt{k}} = \alpha(1 - \cos\theta) \ ,$$

$$\Omega = \frac{2}{1 + \cos\theta} > 1 \ ,$$

$$\text{where } \alpha \equiv \frac{4\pi}{3}\frac{G\rho}{c^2}\left(\frac{R}{\sqrt{k}}\right)^3 \ .$$

Open ($k < 0$):

$$ct = \alpha\left(\sinh\theta - \theta\right) \ , \quad \frac{R}{\sqrt{\kappa}} = \alpha\left(\cosh\theta - 1\right) \ ,$$

$$\Omega = \frac{2}{1 + \cosh\theta} < 1 \ ,$$

$$\text{where } \alpha \equiv \frac{4\pi}{3}\frac{G\rho}{c^2}\left(\frac{R}{\sqrt{\kappa}}\right)^3 \ ,$$

$$\kappa \equiv -k > 0 \ .$$