MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — October 2, 2007
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QUIZ 1 FORMULA SHEET

DOPPLER SHIFT (For motion along a line):

$$z = v/u \quad \text{(nonrelativistic, source moving)}$$

$$z = \frac{v/u}{1 - v/u} \quad \text{(nonrelativistic, observer moving)}$$

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \quad \text{(special relativity, with } \beta = v/c\text{)}$$

COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_{\text{observed}})}{R(t_{\text{emitted}})}$$

SPECIAL RELATIVITY:

Time Dilation Factor:

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \ , \qquad \beta \equiv v/c$$

Lorentz-Fitzgerald Contraction Factor: γ

Relativity of Simultaneity:
Trailing clock reads later by an amount $\beta \ell_0 / c$.

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EVOLUTION OF A MATTER-DOMINATED UNIVERSE:

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2} \ , \quad \ddot{R} = -\frac{4\pi}{3}G\rho R \ ,$$

$$\rho(t) = \frac{R^3(t_i)}{R^3(t)}\,\rho(t_i) \ ,$$

$$\Omega \equiv \rho/\rho_c \ , \quad \text{where} \quad \rho_c = \frac{3H^2}{8\pi G} \ .$$

Flat ($k = 0$): $R(t) \propto t^{2/3}$,

$\Omega = 1$.

Closed ($k > 0$): $ct = \alpha(\theta - \sin\theta)$, $\dfrac{R}{\sqrt{k}} = \alpha(1 - \cos\theta)$,

$\Omega = \dfrac{2}{1 + \cos\theta} > 1$,

where $\alpha \equiv \dfrac{4\pi}{3}\dfrac{G\rho}{c^2}\left(\dfrac{R}{\sqrt{k}}\right)^3$.

Open ($k < 0$): $ct = \alpha\,(\sinh\theta - \theta)$, $\dfrac{R}{\sqrt{\kappa}} = \alpha\,(\cosh\theta - 1)$,

$\Omega = \dfrac{2}{1 + \cosh\theta} < 1$,

where $\alpha \equiv \dfrac{4\pi}{3}\dfrac{G\rho}{c^2}\left(\dfrac{R}{\sqrt{\kappa}}\right)^3$,

$\kappa \equiv -k > 0$.