PROBLEM 1: DID YOU DO THE READING? (25 points)

(a) In the 1940’s, three astrophysicists proposed a “steady state” theory of cosmology, in which the universe has always looked about the same as it does now. State the last name of at least one of these authors. (Bonus points: you can earn 1 point each for naming the other two authors, and hence up to 2 additional points, but 1 point will be taken off for each incorrect answer.)

(b) In 1917, a Dutch astronomer named Willem de Sitter presented a model for the universe, based on general relativity, which appeared to be static but which produced a redshift proportional to the distance.

(c) In 1964–65, Arno A. Penzias and Robert W. Wilson observed a flux of microwave radiation coming from all directions in the sky, which was interpreted as the cosmic background radiation left over from the big bang.

(d) A summary of useful information is at the end of the exam.
has the time-dependence expected for rotation about the Sun) and by the behavior of the Foucault pendulum (which showed that the Earth rotates). These discoveries were made

(i) during Copernicus’ lifetime.

(ii) approximately two and three decades after Copernicus’ death, respectively.

(iii) about one hundred years after Copernicus’ death.

(iv) approximately two and three centuries after Copernicus’ death, respectively.

(e) If one averages over sufficiently large scales, the universe appears to be homogeneous and isotropic. How large must the averaging scale be before this homogeneity and isotropy set in?

(i) 1 AU (1 AU = 1.496 \times 10^{11} m).

(ii) 100 kpc (1 kpc = 1000 pc, 1 pc = 3.086 \times 10^{16} m = 3.262 light-year).

(iii) 1 Mpc (1 Mpc = 10^{6} pc).

(iv) 10 Mpc.

(v) 100 Mpc.

(vi) 1000 Mpc.

PROBLEM 2: AN EXPONENTIALLY EXPANDING UNIVERSE

The following problem was Problem 4 on the Review Problems for Quiz 1.

Consider a flat (i.e., a \( k = 0 \), or a Euclidean) universe with scale factor given by

\[ R(t) = R_0 e^{\chi t} , \]

where \( R_0 \) and \( \chi \) are constants.

(a) Find the Hubble constant \( H \) at an arbitrary time \( t \).

(b) Let \((x, y, z, t)\) be the coordinates of a comoving coordinate system. Suppose that at \( t = 0 \) a galaxy located at the origin of this system emits a light pulse along the positive \( x \)-axis. Find the trajectory \( x(t) \) which the light pulse follows.

(c) Suppose that we are living on a galaxy along the positive \( x \)-axis, and that we receive this light pulse at some later time. We analyze the spectrum of the light pulse and determine the redshift \( z \). Express the time \( t \) at which we receive the pulse in terms of \( z \), \( \chi \), and any relevant physical constants.

(d) At the time of reception, what is the physical distance between our galaxy and the galaxy which emitted the pulse? Express your answer in terms of \( z \), \( \chi \), and any relevant physical constants.

The previous problem was Problem 4 on the Review Problems for Quiz 1. For Quiz 2.

PROBLEM 3: A TWO-LEVEL HIGH-SPEED MERRY-GO-ROUND

Consider a high-speed merry-go-round which is similar to the one discussed in Problem 3 of Problem Set 1, but which has two levels. That is, there are four evenly-spaced cars which travel around a central hub at speed \( v \) at a distance \( R \) from a central hub, and also another four cars that are attached to extensions of the four radial arms, each moving at a speed \( 2v \) at a distance \( 2R \) from the center. In this problem we will consider only light waves, not sound waves, and we will assume that a high-speed merry-go-round which is similar to the one discussed in Problem 2 of Problem Set 1, but which has two levels, is similar to the one discussed in Problem 3.

(15 points)

(a) Suppose that cars 5–8 are all emitting light waves in all directions. If an observer in car 1 receives light waves from each of these cars, what redshift does she observe for each of the four signals?

(b) Suppose that a spaceship is receding to the right at a relativistic speed \( u \) along a line through the hub, as shown in the diagram. Suppose that an observer in car 6 receives a radio signal from the spaceship, at the time when the car is in the position shown in the diagram. What redshift is observed?
Your answer may depend upon any of the variables held in part (d).

(6) (a) Which of the variables α and/or β appear in your answer above? Explain your reasoning.
(b) Which of the variables α and/or β appear in your answer above? Explain your reasoning.
(c) Which of the variables α and/or β appear in your answer above? Explain your reasoning.

(5) (a) What is the physical parameter Φ? Why is it essential for the model universe to have a finite, non-zero horizon?
(b) This model universe has neither a big bang nor a big crunch, but
(c) The Friedmann equation can be rewritten as

\[
0 < \frac{\dot{\rho}}{\rho^2} \equiv \frac{\ddot{a}}{a^2} \equiv \frac{\dot{\rho}}{ho} = \frac{\dot{a}}{a} \equiv \dot{\theta}
\]

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\]

and

\[
1 - \frac{\dot{\rho}}{\rho} \equiv (\rho)^{1/3} \quad \text{where} \quad (\rho)^{1/3} \equiv (1)H
\]

We consider a closed universe so \( H > 0 \), and we define

\[
H \equiv \frac{\dot{\rho}}{\rho}
\]

The Friedmann equation describes the total mass density.
USEFUL INFORMATION:

DOPPLER SHIFT (For motion along a line):
\[
\begin{align*}
\text{nonrelativistic, source moving:} & \quad z = \frac{v}{u} \\
\text{nonrelativistic, observer moving:} & \quad z = \frac{v}{u} \left(1 - \frac{v}{u}\right) \\
\text{special relativity, with } \beta = \frac{v}{c} & \quad z = \sqrt{1 + \beta^2 - \beta - 1}
\end{align*}
\]

COSMOLOGICAL REDSHIFT:
\[
\begin{align*}
1 + z & \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_{\text{observed}})}{R(t_{\text{emitted}})}
\end{align*}
\]

SPECIAL RELATIVITY:

Time Dilation Factor:
\[
\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \beta \equiv \frac{v}{c}
\]

Lorentz-Fitzgerald Contraction Factor:
\[
\gamma
\]

Relativity of Simultaneity:

Trailing clock reads later by an amount \( \beta \ell / c \).

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EVOLUTION OF A MATTER-DOMINATED UNIVERSE:

\[
\begin{align*}
H^2 = \frac{\ddot{R} \cdot R}{2} & = \frac{8\pi G \rho - \frac{k}{R^2}}{2} \\
\ddot{R} & = -\frac{4\pi G \rho R}{3}
\end{align*}
\]

\[
\begin{align*}
\rho(t) & = \rho(t_i) \left(\frac{R(t_i)}{R(t)}\right)^{3/2} = \rho_c(t) \\
\Omega & \equiv \frac{\rho}{\rho_c} = \frac{1}{8\pi G H^2}, \quad \rho_c \equiv \frac{3}{8\pi G H^2}
\end{align*}
\]

Flat \((k = 0)\):
\[
R(t) \propto t^{2/3}
\]

\[
\Omega = 1
\]

Closed \((k > 0)\):
\[
ct = \alpha (\theta - \sin \theta), \quad R = \alpha (1 - \cos \theta)
\]

\[
\Omega = 2 \left(1 + \cos \theta > 1\right)
\]

\[
\alpha \equiv \frac{4\pi}{3} G \rho_c^2 \left(\frac{R}{\sqrt{k}}\right)^3
\]

Open \((k < 0)\):
\[
ct = \alpha (\sinh \theta - \theta), \quad R = \alpha (\cosh \theta - 1)
\]

\[
\Omega = 2 \left(1 + \cosh \theta < 1\right)
\]

\[
\alpha \equiv \frac{4\pi}{3} G \rho_c^2 \left(\frac{R}{\sqrt{\kappa}}\right)^3
\]

κ \equiv -\frac{k}{0}.

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