Problem 1: Did you do the reading?

(a) (6 points)

In 1948 Ralph A. Alpher and Robert Herman wrote a paper predicting a cosmic microwave background with a temperature of 5 K. The paper was based on a cosmological model that they had developed with George Gamow, in which the early universe was assumed to have been filled with hot neutrons.

As the universe expanded and cooled, the neutrons underwent beta decay into protons, electrons, and antineutrinos, until at some point the universe cooled enough for light elements to be produced. According to the predictions of the standard model of particle physics, the ratio of photons to nuclear particles must have been about $10^9$. Although the predicted temperature was very close to the actual value of 2.7 K, the theory differed from our present theory in two ways. Circle the two correct statements in the following list. (3 points for each right answer; circle at most 2.)

(i) Gamow, Alpher, and Herman assumed that the neutron could decay, but now the neutron is thought to be absolutely stable.

(ii) In the current theory, the universe started with nearly equal densities of protons and neutrons, not all neutrons as Gamow, Alpher, and Herman assumed.

(iii) In the current theory, the universe started with mainly alpha particles, not all neutrons as Gamow, Alpher, and Herman assumed.

(iv) In the current theory, the conversion of neutrons into protons (and vice versa) took place mainly through collisions with electrons, positrons, neutrinos, and antineutrinos, not through the decay of the neutrons.

(v) The ratio of photons to nuclear particles in the early universe is now believed to have been about $10^9$, not $10^6$ as Alpher and Herman concluded.

(b) (6 points)

In Weinberg's "Recipe for a Hot Universe," he described the primordial composition of the universe in terms of three conserved quantities: electric charge, baryon number, and lepton number. If electric charge is measured in units of the electron charge, then all three quantities are integers for which the number density can be compared with the number density of photons. For each quantity, which choice most accurately describes the initial ratio of the number density of the quantity to the number density of photons? For each choice, write the ratio of number density of the quantity to the number density of photons. (Note: an alpha particle is the nucleus of a helium atom, composed of two protons and two neutrons.)

Electric Charge:

(i) $\sim 10^9$

(ii) $\sim 1000$

(iii) $\sim 1$

(iv) $\sim 10^{-6}$

(v) either zero or negligible

Baryon Number:

(i) $\sim 10^{-20}$

(ii) $\sim 10^{-9}$

(iii) $\sim 10^{-6}$

(iv) $\sim 1$ (v) anywhere from $10^{-5}$ to 1

Lepton Number:

(i) $\sim 10^9$

(ii) $\sim 1000$

(iii) $\sim 1$

(iv) $\sim 10^{-6}$

(v) could be as high as $\sim 1$, but is assumed to be very small.
The figure below comes from Weinberg's Chapter 5, and is labeled "The Shifting Neutron-Proton Balance.

(i) (3 points) During the period labeled "neutron decay," the neutron fraction is changing because (choose one):

(A) The neutron is unstable, and decays into a proton, electron, and antineutrino with a lifetime of about 1 second.

(B) The neutron is unstable, and decays into a proton, electron, and antineutrino with a lifetime of about 15 seconds.

(C) The neutron is unstable, and decays into a proton, electron, and antineutrino with a lifetime of about 15 minutes.

(D) Neutrons and protons can be converted from one into the other through reactions such as:

\[ \text{antineutrino} + \text{proton} \rightarrow \text{electron} + \text{neutron} \]
\[ \text{neutrino} + \text{neutron} \rightarrow \text{positron} + \text{proton} \]

(E) Neutrons and protons can be converted from one into the other through reactions such as:

\[ \text{antineutrino} + \text{proton} \rightarrow \text{positron} + \text{neutron} \]
\[ \text{neutrino} + \text{neutron} \rightarrow \text{electron} + \text{proton} \]

(F) Neutrons and protons can be created and destroyed by reactions such as:

\[ \text{proton} + \text{neutrino} \rightarrow \text{positron} + \text{antineutrino} \]
\[ \text{neutron} + \text{antineutrino} \rightarrow \text{electron} + \text{positron} \]

(ii) (3 points) During the period labeled "neutron decay," the neutron fraction is changing because (choose one):

(A) The neutron is unstable, and decays into a proton, electron, and antineutrino with a lifetime of about 1 second.

(B) The neutron is unstable, and decays into a proton, electron, and antineutrino with a lifetime of about 15 seconds.

(C) The neutron is unstable, and decays into a proton, electron, and antineutrino with a lifetime of about 15 minutes.

(D) Neutrons and protons can be converted from one into the other through reactions such as:

\[ \text{antineutrino} + \text{proton} \rightarrow \text{electron} + \text{neutron} \]
\[ \text{neutrino} + \text{neutron} \rightarrow \text{positron} + \text{proton} \]

(E) Neutrons and protons can be converted from one into the other through reactions such as:

\[ \text{antineutrino} + \text{proton} \rightarrow \text{positron} + \text{neutron} \]
\[ \text{neutrino} + \text{neutron} \rightarrow \text{electron} + \text{proton} \]

(F) Neutrons and protons can be created and destroyed by reactions such as:

\[ \text{proton} + \text{neutrino} \rightarrow \text{positron} + \text{antineutrino} \]
\[ \text{neutron} + \text{antineutrino} \rightarrow \text{electron} + \text{positron} \]

(iii) (3 points) The masses of the neutron and proton are not exactly equal, but one is more massive than the other with a rest energy difference of 1.293 GeV (1 GeV = 10^9 eV).

(A) The neutron is more massive with a rest energy difference of 1.293 GeV.

(B) The neutron is more massive with a rest energy difference of 1.293 MeV (1 MeV = 10^6 eV).

(C) The neutron is more massive with a rest energy difference of 1.293 KeV (1 KeV = 10^3 eV).

(D) The proton is more massive with a rest energy difference of 1.293 GeV.

(E) The proton is more massive with a rest energy difference of 1.293 MeV.

(F) The proton is more massive with a rest energy difference of 1.293 KeV.
During the period labeled "era of nucleosynthesis," (choose one:)

(A) Essentially all the neutrons present combine with protons to form helium nuclei, which mostly survive until the present time.

(B) Essentially all the neutrons present combine with protons to form deuterium nuclei, which mostly survive until the present time.

(C) About half the neutrons present combine with protons to form helium nuclei, which mostly survive until the present time, and the other half of the neutrons remain free.

(D) About half the neutrons present combine with protons to form deuterium nuclei, which mostly survive until the present time, and the other half of the neutrons remain free.

(E) Essentially all the protons present combine with neutrons to form helium nuclei, which mostly survive until the present time.

(F) Essentially all the protons present combine with neutrons to form deuterium nuclei, which mostly survive until the present time.

---

PROBLEM 2: THE EFFECT OF PRESSURE ON COSMOLOGICAL EVOLUTION

This problem was Problem 2 of Problem Set 6.

---

8.286 QUIZ 2, FALL 2007

PROBLEM 2: THE EFFECT OF PRESSURE ON COSMOLOGICAL EVOLUTION

---

8.286 QUIZ 2, FALL 2007
PROBLEM 4: THE STABILITY OF SCHWARZSCHILD ORBITS

This problem is an elaboration of Problem 1 of Problem Set 5, for which both statement and the solution are reproduced at the end of this quiz. This material is reproduced for your reference, but you should be aware that the solution to the problem is self-contained and is meant to replace the earlier homework problem. You are expected to present a logical, self-contained answer to the question.

The coordinate $\tau$ defined in Problem 1 is not the coordinate $t$ of any coordinate system. Since the determinant $h(\tau)$ is negative, you may allow your answer to contain $h'(\tau)$, its derivative with respect to $\tau$.

We will consider a body which undergoes small oscillations about a circular orbit. In the solution to that homework problem, it was stated that further analysis of the electron-positron pairs, where the energy of the electron-positron pairs is greater than the rest mass of the particle, will not be a constant in this solution, will work out the explicit form of the Goedel equation

$$\frac{\Delta}{\Delta Y} - \tau = (\Delta) Y$$

where

$$\Delta = \frac{\Delta Y}{\Delta X}$$

We are given that $5 \text{ MeV}$, at $10^6 \text{ MeV}$, is much lighter than the muon, which has a rest energy of $10^6 \text{ MeV}$, and it is much heavier than the electron, which has a rest energy of $0.511 \text{ MeV}$.

The first step, since $h = 0$, where $\theta d\phi = \sin^2 \theta d\phi$ and $r = \rho$, is a constant. The coordinate $\rho$, where $r$ is a constant, will not be a constant in this solution, will be $\tau$, but all the other coordinates will vary as the body changes its orbit.

The mass density at $t = 0$, where $\Phi = \theta d\phi$, and the charge $Q = 0$, where $Q = 0$, will be the value of $\Phi$, at $10^6 \text{ MeV}$, at $10^6 \text{ MeV}$, express your answer in the units of $\text{g/cm}^3$. Find the charge $Q$.

(a) (10 points) What would be the value of $\Phi$ if this was the Schwarzschild solution?

(b) (10 points) What would be the value of $\Phi$ if this was the Schwarzschild solution?

(c) (5 points) What would be the value of $\Phi$ if this was the Schwarzschild solution?

(d) (10 points) What would be the value of $\Phi$ if this was the Schwarzschild solution?

(e) (5 points) Express your answer in the units of $\text{g/cm}^3$. Find the charge $Q$.

(f) (5 points) Find the charge $Q$.

(g) (5 points) Find the charge $Q$.

(h) (5 points) Find the charge $Q$.

(i) (5 points) Find the charge $Q$.

(j) (5 points) Find the charge $Q$.

(k) (5 points) Find the charge $Q$.

(l) (5 points) Find the charge $Q$.

(m) (5 points) Find the charge $Q$.

(n) (5 points) Find the charge $Q$.

(o) (5 points) Find the charge $Q$.

(p) (5 points) Find the charge $Q$.

(q) (5 points) Find the charge $Q$.

(r) (5 points) Find the charge $Q$.

(s) (5 points) Find the charge $Q$.

(t) (5 points) Find the charge $Q$.

(u) (5 points) Find the charge $Q$.

(v) (5 points) Find the charge $Q$.

(w) (5 points) Find the charge $Q$.

(x) (5 points) Find the charge $Q$.

(y) (5 points) Find the charge $Q$.

(z) (5 points) Find the charge $Q$.
\[ \left( \frac{\mathcal{H}}{\mathcal{F}} \right) + \beta \left( \frac{e}{A} \right) \mathcal{F} = \mathcal{H} + \beta \left( \frac{e}{A} \right) \mathcal{F} = (\mathcal{H}) \left( \frac{e}{A} \right) \]

\[ \mathcal{H} \left( \frac{e^2}{d^3} + d \right) \mathcal{F} \mathcal{F} = \mathcal{H} \left( \frac{e}{A} \right) - d \mathcal{F} \mathcal{F} = \left( \frac{\mathcal{H}}{\mathcal{F}} \right) = \mathcal{H} \]

\[ \mathcal{H} \left( \frac{e^2}{d^3} + d \right) \mathcal{F} \mathcal{F} = \mathcal{H} \left( \frac{e}{A} \right) - d \mathcal{F} \mathcal{F} = \left( \frac{\mathcal{H}}{\mathcal{F}} \right) = \mathcal{H} \]

**Cosmological Evolution:**

\[ \mathcal{H} \left( \frac{e^2}{d^3} + d \right) \mathcal{F} \mathcal{F} = \mathcal{H} \left( \frac{e}{A} \right) - d \mathcal{F} \mathcal{F} = \left( \frac{\mathcal{H}}{\mathcal{F}} \right) = \mathcal{H} \]

\[ \mathcal{H} \left( \frac{e^2}{d^3} + d \right) \mathcal{F} \mathcal{F} = \mathcal{H} \left( \frac{e}{A} \right) - d \mathcal{F} \math{cal{F}} = \left( \frac{\mathcal{H}}{\mathcal{F}} \right) = \mathcal{H} \]

**Problem:**

- Carry out the algebra of inserting the explicit forms that you have found for \( \rho \) in terms of the unspecified functions \( f \).
- Use this equation to simplify the expression for the external force giving the orbiting body a small kick in the radial direction.
- Write an equation that determines the stability of the orbit. (That is, if some small perturbation about the circular orbit at \( E = \frac{1}{2} \rho \), and \( \beta \) is some function, how is the equation of motion for \( \phi \) modified?)

**Solution:**

1. Use the equation of motion for \( \phi \) modified to include the small perturbation.
2. Substitute Eq. (4) into Eq. (5) to determine the stability of the orbit.

**Useful Information:**

- **Lorentz-Fitzgerald Contraction:**
  - Length: \( \mathcal{L} \) (Special Relativity) = \( \mathcal{L} \)
  - Velocity: \( v/c \) (Lorentz-Fitzgerald) = \( v/c \)

- **Time Dilation:**
  - \( \frac{\mathcal{L}}{\mathcal{L}} = \mathcal{L} \)
  - \( v/c \) (Lorentz-Fitzgerald) = \( v/c \)

- **Cosmological Redshift:**
  - \( \hat{v}/z = \mathcal{L} \)
  - \( \mathcal{L} \) (Special Relativity) = \( \mathcal{L} \)
  - \( \mathcal{L} \) (Lorentz-Fitzgerald) = \( \mathcal{L} \)

- **Doppler Shift:**
  - For motion along a line:
  - \( \hat{v}/z = \mathcal{L} \)

**Questions:**

(a) Find an expression for the cosmological redshift of a distant galaxy that is moving away from us.

(b) Determine the stability of the orbit. (That is, if a small perturbation is applied to the orbit, how does the orbit respond?)
\[ \frac{\rho}{\rho_0} = \left( \frac{1 + \cos \theta}{2} \right)^{\frac{1}{2}} \]

where

\[ \frac{\rho}{\rho_0} \equiv \frac{\rho}{\rho_0} = \left( \frac{1 + \cos \theta}{2} \right)^{\frac{1}{2}} \]

(\text{Open}) \quad \begin{cases} \frac{\rho}{\rho_0} \equiv \frac{\rho}{\rho_0} & (0 > \gamma) \\ \frac{\rho}{\rho_0} \equiv \frac{\rho}{\rho_0} & (0 < \gamma) \end{cases}

(\text{Closed}) \quad \begin{cases} \frac{\rho}{\rho_0} \equiv \frac{\rho}{\rho_0} & (\theta > \sinh \theta) \quad \text{where} \\ \frac{\rho}{\rho_0} \equiv \frac{\rho}{\rho_0} & (\theta < \sinh \theta) \quad \text{where} \end{cases}

\text{EVOLUTION OF A MATTER-DOMINATED UNIVERSE:}

\[ \begin{align*}
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0} \\
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0} \\
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0}
\end{align*} \]

where

\[ \frac{\rho}{\rho_0} \equiv \left( \frac{\rho}{\rho_0} \right) \frac{\rho}{\rho_0} \]

(\text{Total}) \quad \begin{cases} \frac{\rho}{\rho_0} \equiv \frac{\rho}{\rho_0} & (0 = \gamma) \quad \text{Flat} \\ \frac{\rho}{\rho_0} \equiv \frac{\rho}{\rho_0} & (0 = \gamma) \quad \text{Flat} \end{cases}

\[ \begin{align*}
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0} \\
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0} \\
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0}
\end{align*} \]

(\text{Total}) \quad \begin{cases} \frac{\rho}{\rho_0} \equiv \frac{\rho}{\rho_0} & (0 = \gamma) \quad \text{Flat} \\ \frac{\rho}{\rho_0} \equiv \frac{\rho}{\rho_0} & (0 = \gamma) \quad \text{Flat} \end{cases}

\[ \begin{align*}
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0} \\
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0} \\
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0}
\end{align*} \]

\[ \begin{align*}
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0} \\
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0} \\
\frac{\rho}{\rho_0} & \equiv \frac{\rho}{\rho_0}
\end{align*} \]
EVOLUTION OF A FLAT RADIATION-DOMINATED UNIVERSE:

\[ \rho = \frac{3}{32} \pi G t^2 \]

\[ kT = \left( \frac{45 \bar{h}^3 c^5}{16 \pi g G} \right)^{1/4} \frac{1}{\sqrt{t}} \]

For mass \( m_{\mu} = 10^6 \text{ MeV} \gg kT \gg m_e = 0.511 \text{ MeV} \),

\[ g = 10^{0.75} \]

and then

\[ kT = 0.860 \text{ MeV} \sqrt{t} \text{ (in sec)} \]

After the freeze-out of electron-positron pairs,

\[ T_\nu T_\gamma = \left( \frac{4}{11} \right)^{1/3} \]

HORIZON DISTANCE:

\[ \ell_{\text{horizon}}(t) = \frac{R(t)}{c} = \begin{cases} 3ct & \text{flat, matter-dominated,} \\ 2ct & \text{flat, radiation-dominated.} \end{cases} \]

PHYSICAL CONSTANTS:

- \( G = 6.673 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2} \)
- \( k = \text{Boltzmann's constant} = 8.617 \times 10^{-16} \text{ eV/K} \)
- \( \bar{h} = \frac{h}{2\pi} = 1.055 \times 10^{-27} \text{ erg} \cdot \text{s} \)
- \( c = 2.998 \times 10^{10} \text{ cm/s} \)
- \( 1 \text{ year} = 3.156 \times 10^{7} \text{ s} \)
- \( 1 \text{ electron} = 1.602 \times 10^{-12} \text{ erg} \)
- \( 1 \text{ GeV} = 1.783 \times 10^{-24} \text{ gram} \)

\( \equiv c \) (where \( c \equiv 1 \)).