Problem 1: Did you do the reading?

(a) True or False? In the form of VACCHO, the ionization fraction of the universe is 1.

(b) True or False? MACHOs are thought to be a significant fraction of dark matter. This answer must be within 10% of the true number density of dark matter.

(c) True or False? The virial theorem can be applied to a cluster of galaxies to find its total mass, most of which is dark matter.

(d) True or False? The speed of the Local Group with respect to the CMB, expressed as a fraction of the speed of light, is 0.05. This question concerns some numbers related to the cosmic microwave background (CMB) that one should never forget. State the values of these numbers, to within an order of magnitude unless otherwise stated. In all cases, assume the CMB to be at rest at 2.73 K.

(i) The temperature of the CMB is 2.73 K.

(ii) The speed of the Local Group relative to the CMB is 0.05 x c. (The speed of the Local Group is found by measuring the dipole pattern of the CMB to determine the peculiar velocity of the observer.)

(iii) The intrinsic relative temperature fluctuations ∆T/T are about 1%. This question refers to the present value of these quantities.

(iv) Magnetic monopoles are thought to comprise a significant fraction of the energy density of dark matter. This answer must be within 10% of the true number density of dark matter.

(v) The angular size of the dipole anisotropy corresponding to the motion of the observer relative to the galaxy, and the galaxy relative to the center of mass of the Local Group.

(vi) Photons traveling toward us from the surface of last scattering appear redder because of the Doppler effect.

(vii) Photons from fluid which had a velocity toward us along the line of sight appear bluer because of the Doppler effect.

(viii) Photons from overdense regions at the surface of last scattering appear redder because of absorption in the intergalactic medium.

(ix) Photons which travel through a spatially varying potential acquire a redshift or blueshift depending on whether they are going up or down.

(x) Photons originating in the denser regions start bluer because they must climb out of the gravitational potential well.

(xi) Photons from overdense regions at the surface of last scattering appear bluer because of the Doppler effect.

(xii) Photons from fluid which had a velocity toward us along the line of sight appear bluer because of the Doppler effect.

(xiii) Photons from overdense regions at the surface of last scattering appear redder because of absorption in the intergalactic medium.

(xiv) Photons which travel through a spatially varying potential acquire a redshift or blueshift depending on whether they are going up or down.

(xv) Photons originating in the denser regions start bluer because of absorption in the intergalactic medium.

Explanation: Denser regions have a deeper (more negative) gravitational potential. Photons which travel through a spatially varying potential acquire a redshift or blueshift depending on whether they are going up or down. Photons originating in the denser regions start bluer because of absorption in the intergalactic medium.

Problem 2: Guess Date: December 3, 2009 Quiz Date: December 23, 2009

(a) True or False? The LIGO project, which is the Large Array, is planned to be run by LIGO.

(b) True or False? The LIGO project, which is the Large Array, is planned to be run by LIGO.

(c) True or False? The LIGO project, which is the Large Array, is planned to be run by LIGO.

(d) True or False? The LIGO project, which is the Large Array, is planned to be run by LIGO.

(e) True or False? The LIGO project, which is the Large Array, is planned to be run by LIGO.

(f) True or False? The LIGO project, which is the Large Array, is planned to be run by LIGO.

(g) True or False? The LIGO project, which is the Large Array, is planned to be run by LIGO.

(h) True or False? The LIGO project, which is the Large Array, is planned to be run by LIGO.

(i) True or False? The LIGO project, which is the Large Array, is planned to be run by LIGO.

Problem 3: Use the Early Universe

(a) True or False? The temperature of the universe is 10^10 K.

(b) True or False? The temperature of the universe is 10^10 K.

(c) True or False? The temperature of the universe is 10^10 K.

(d) True or False? The temperature of the universe is 10^10 K.

(e) True or False? The temperature of the universe is 10^10 K.

(f) True or False? The temperature of the universe is 10^10 K.

(g) True or False? The temperature of the universe is 10^10 K.

(h) True or False? The temperature of the universe is 10^10 K.

(i) True or False? The temperature of the universe is 10^10 K.
PROBLEM 2: PRESSURE, ENERGY DENSITY, AND COSMIC EVOLUTION WITH MYSTERIOUS STUFF

(a) If \( u \propto \frac{1}{\sqrt{V}} \), then one can write
\[
\frac{\rho}{\sqrt{V}} = \frac{\rho_0}{\sqrt{V + \Delta V}} = \frac{\rho_0}{\sqrt{V}}
\]

The above expression is proportional to \( \frac{1}{\sqrt{V + \Delta V}} \), and reduces to \( u = u_0 \) when \( \Delta V = 0 \).

The total energy is the energy density times the volume, so
\[
U(V + \Delta V) = \rho_0 \sqrt{V + \Delta V} = \rho_0 \sqrt{V} (V + \Delta V)
\]

Expanding to first order,
\[
U(V + \Delta V) = U_0 \sqrt{1 + \frac{\Delta V}{V}} \approx U_0 (1 + \frac{1}{2} \frac{\Delta V}{V})
\]

where \( U_0 = \rho_0 V \) is the original energy inside the piston.

Then
\[
\Delta U = U_0 \frac{\Delta V}{V}
\]

(b) The work done by the agent must be the negative of the work done by the gas, which is
\[
p \Delta V
\]

So
\[
\Delta W = -p \Delta V
\]

(c) The agent must supply the full change in energy, so
\[
\Delta W = \Delta U = \frac{1}{2} \frac{\Delta V}{V} U_0
\]

Combining this with the expression for \( \Delta W \) from part (b), one sees immediately
\[
\rho_c = \frac{-1}{2} \frac{U_0}{V} = \frac{-1}{2} u_0
\]

(d) The critical density \( \rho_c \) is defined as that density for which
\[
\frac{\rho}{\sqrt{V}} = \frac{\rho_0}{\sqrt{V + \Delta V}}
\]

where
\[
\rho_0 = \frac{3 H^2}{8 \pi G}
\]

The mass density today of any species \( X \) is related to \( \Omega_X \) by
\[
\rho_{X,0} = \rho_c \Omega_X
\]

The total mass density today is expressed in terms of its four components
\[
\rho_0 = \rho_{m,0} \Omega_m + \rho_{r,0} \Omega_r + \rho_{v,0} \Omega_v + \rho_{ms,0} \Omega_{ms}
\]

But we also know how each of these contributions to the mass density scales with \( x(t) \):
\[
\frac{\rho_m}{\rho_0} \propto \frac{1}{x^3}, \quad \frac{\rho_r}{\rho_0} \propto \frac{1}{x^4}, \quad \frac{\rho_v}{\rho_0} \propto 1, \quad \text{and} \quad \frac{\rho_{ms}}{\rho_0} \propto \frac{1}{\sqrt{x}} \propto \frac{1}{x^{3/2}}
\]

Inserting these factors,
\[
\rho(t) = \rho_0 \frac{H_0^2}{8 \pi G} \left[ \Omega_m x^3 + \Omega_r x^4 + \Omega_v + \Omega_{ms} x^{3/2} \right]
\]

(e) The Friedmann equation then becomes
\[
\left( \frac{\dot{x}}{x^2} \right)^2 = \frac{8 \pi G}{3} \rho_0 \left[ \Omega_m x^3 + \Omega_r x^4 + \Omega_v + \Omega_{ms} x^{3/2} \right] - \frac{k}{a^2}
\]

Defining
\[
\frac{\rho}{\rho_0} \lambda Y = \frac{\rho}{\rho_0} \lambda Y
\]

the Friedmann equation becomes
\[
\frac{\dot{\lambda}}{\lambda} = \frac{\rho}{\rho_0} \lambda Y
\]

where \( \lambda = 0 \) is the original energy inside the piston. Then
\[
\left( \frac{\lambda}{\lambda} \frac{\rho}{\rho_0} \lambda Y \right)^{\rho/\rho_0} \approx \frac{\lambda}{\lambda} \frac{\rho}{\rho_0} \lambda Y = \left( \frac{\lambda}{\lambda} \frac{\rho}{\rho_0} \lambda Y \right)
\]

Expanding to first order,
\[
\frac{\lambda}{\lambda} \frac{\rho}{\rho_0} \lambda Y = \left( \frac{\lambda}{\lambda} \frac{\rho}{\rho_0} \lambda Y \right)
\]

when the total energy is the energy density times the volume, so
\[
\rho_{0m} = (\lambda + \Delta \lambda) n
\]

where \( n = \rho_0 \lambda Y \) is proportional to \( \rho_0 \lambda Y \). And reduces to \( 0 = \rho_0 \lambda Y \) when \( \rho_0 \lambda Y = \rho_0 \lambda Y \). Finally, one can write
\[
\frac{\Delta \lambda}{\lambda} \rho_{0m} = (\lambda + \Delta \lambda) n
\]
\[
\frac{\rho \pi E}{V} = \rho H
\]

The mass density is then given by

\[
\rho = \frac{\pi E}{V} = \rho H
\]

The following conclusions can be drawn from the above equation:

1. In the standard model, the black-body radiation at \( E \) approximates \( 200 \text{ MeV} \).

PROBLEM 3: A NEW THEORY OF THE WEAK INTERACTIONS

You were not expected to evaluate this expression, but with a calculator you would find

\[
\frac{e^{\mu}}{\sqrt{3}} \left( \frac{\rho H}{V} \right) = \frac{\rho H}{V}
\]

So, the answer would be

\[
\frac{e^{\mu}}{\sqrt{3}} = \frac{\rho H}{V}
\]

The solution to this problem is

\[
\frac{e^{\mu}}{\sqrt{3}} \left( \frac{\rho H}{V} \right) = \frac{\rho H}{V}
\]

PROBLEM 3: A NEW THEORY OF THE WEAK INTERACTIONS

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So, the answer would be

\[
\frac{e^{\mu}}{\sqrt{3}} = \frac{\rho H}{V}
\]
The model, as long as the universe is in thermal equilibrium, is that entropy is conserved, where a common mistake was to leave out the conversion factor $1.602 \times 10^{-19} \text{ J/eV}$ (or 1.602 $\times 10^{-12} \text{ erg/eV}$), and instead to use $\bar{\hbar} = 6.582 \times 10^{-16} \text{ eV-s}$. But if one works out the units of this answer, they turn out to be $\text{eV-sec}^{2}/\text{m}^{5}$ (or $\text{eV-sec}^{2}/\text{cm}^{5}$), which is a peculiar set of units to measure mass density.

In the NTWI, we have in addition the contribution to the mass density from $+_{-} R +_{-} R$ pairs, which would act just like $e_{-} e_{-}$ pairs or $\mu_{-} \mu_{-}$ pairs, with $g = 3 \frac{1}{2}$. Thus $g_{\text{TOT}} = 20 \frac{3}{4}$, so

$$\rho_{\text{NTWI}} = 2.53 \times 10^{18} \frac{\text{kg}}{\text{m}^{3}} = 2.53 \times 10^{15} \frac{\text{g}}{\text{cm}^{3}}.$$

(b) As long as the universe is in thermal equilibrium, entropy is conserved. The entropy in a given volume of the comoving coordinate system is

$$S = a^3 t s_{\text{coord}},$$

where $s$ is the entropy density and $a^3 V_{\text{coord}}$ is the physical volume. So

$$S(t) = a^3 s_{\nu} \text{ and } a^3 s_{\text{other}}$$

are separately conserved, where $s_{\nu}$ is the entropy of everything except neutrinos. After the neutrinos decouple, $s_{\nu}$ and $s_{\text{other}}$ are separately conserved, where $s_{\text{other}}$ is the entropy of everything except neutrinos.

Note that $s_{\text{other}}$ can be written as

$$s_{\text{other}} = g_{\text{AT}}^{3} a^3 T^3$$

where $A$ is a constant. Before the disappearance of the $e_{-} e_{-}$, $\mu_{-} \mu_{-}$, and $R +_{-} R$ particles from the thermal equilibrium radiation,

$$s_{\nu} = \left( \frac{51}{4} \right) a^3 T^3$$

and

$$s_{\text{other}} = \left( \frac{151}{2} \right) a^3 T^3.$$

So

$$s_{\nu}/s_{\text{other}} = 5^{1} \frac{1}{4} 15^{1} \frac{1}{2} = \frac{9}{11}.$$

If $a^3 s_{\nu}$ and $a^3 s_{\text{other}}$ are conserved, then so is $s_{\nu}/s_{\text{other}}$. But today, the entropy in neutrinos is still

$$s_{\nu} = \left( \frac{51}{4} \right) a^3 T^3$$

and $s_{\text{other}}$ is still

$$s_{\text{other}} = \left( \frac{151}{2} \right) a^3 T^3.$$

Since $s_{\nu}/s_{\text{other}}$ is constant we know that

$$s_{\nu}/s_{\text{other}} = \left( \frac{51}{4} \right) T^3 / \left( 151/2 \right) T^3 = \frac{9}{11}.$$

(c) One can write

$$n = g^{*} B T^3,$$

where $B$ is a constant. Here $g^{*} \gamma = 2$, and $g^{*} \nu = 6 \times 3 \frac{4}{11} = 4 \frac{1}{2}$. In the standard model, one has today

$$n_{\nu} n_{\gamma} = g^{*} \nu T^3 / g^{*} \gamma T^3 = \left( \frac{4 \frac{1}{2}}{\frac{9}{11}} \right) = \frac{9}{11}. $$

Note that in the NTWI, we have in addition the contribution to the mass density from $+_{-} R +_{-} R$ pairs, which would act just like $e_{-} e_{-}$ pairs or $\mu_{-} \mu_{-}$ pairs, with $g = 3 \frac{1}{2}$. Thus $g_{\text{TOT}} = 20 \frac{3}{4}$, so

$$\rho_{\text{NTWI}} = 2.53 \times 10^{18} \frac{\text{kg}}{\text{m}^{3}} = 2.53 \times 10^{15} \frac{\text{g}}{\text{cm}^{3}}.$$
For a thermal equilibrium calculation gives $n_e/T = n_n/T = \frac{1}{2}$.

This corresponds to a thermal equilibrium of neutrons and protons, with $n_n = n_p = n_e/T$. The temperature and density of neutrons and protons in the standard model are

$$\frac{n_n}{T} = \frac{1}{2} \Rightarrow n_n = \frac{T}{2} \Rightarrow n_p = \frac{T}{2}$$

In the NTWI, neutron-neutron collisions would change this to

$$\frac{n_n}{T} = \frac{1}{2} \Rightarrow n_n = \frac{T}{2} \Rightarrow n_p = \frac{T}{4}$$

To calculate how much less, note that Ryden estimates the start of nucleosynthesis as $t_{\text{start}} \approx 10^6$ s, so in the NTWI it would be longer by the factor

$$t_{\text{NTWI}}/t_{\text{SM}} = 2.$$
Note that Ryden gives $t_{\text{nuc}} \approx 200$ s, while Weinberg places it at $3.3$ minutes $\approx 225$ s, which is close enough.

To follow the effect of this free decay, it is easiest to do it by considering the ratio neutrons to baryon number, $n_B$, since $n_B$ does not change during this period. At freeze-out, when $kT \approx 200$ MeV, $A \approx 1$. Since all the surviving neutrons are bound into He, the corrected value of $A$ is simply decreased by multiplying by $\exp\left( \frac{\text{mass}}{\text{ entropy}} \right)$.

If free decay is ignored, we found $Y = 1$. Since all the surviving neutrons are bound into He, the corrected value of $A$ is simply decreased by multiplying by $\exp\left( \frac{\text{mass}}{\text{ entropy}} \right)$.

Thus, the prediction of Ryden is $A \approx 0.76$. The fraction of neutrons that do not undergo decay, $\frac{n_B}{A}$, is given above.

When evaluated numerically, the ratio of neutrons to baryon number, $n_B$, is given above.

Note that Ryden gives $t_{\text{nuc}} \approx 200$ s, while Weinberg places it at $3.3$ minutes $\approx 225$ s, which is close enough.