MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — October 6, 2009
Prof. Alan Guth

QUIZ 1

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A SUMMARY OF USEFUL INFORMATION IS AT THE END OF THE EXAM

Your Name

Problem	Maximum	Score
1	30	
2	15	
3	55	
TOTAL	100	

PROBLEM 1: QUESTIONS BASED ON READING AND SHORT CALCULATIONS *(30 points)*

Please answer part (a) directly next to each part, but answer parts (b) and (c) on the blank page at the right.

(a) *(7 points)* The following quantities all have a power law dependence on the cosmological scale factor, $a(t)$. State the dependences (in the form of $\propto a^n$):

(i) The number density of baryons

(ii) The number density of photons

(iii) The energy density of baryons (protons and neutrons)

(iv) The energy density of photons

(v) The pressure of photons

(vi) The wavelength of photons

(vii) The temperature of a blackbody distribution of photons

(b) *(5 points)* On some problems you have been assigned, you were asked to assume that time dependence of the scale factor was that of a flat matter-dominated universe, $a(t) \propto t^{2/3}$. (As usual, "matter-dominated" is shorthand for "dominated by the mass density of nonrelativistic matter.") Argue from one or more of the above dependences why this assumption might not always be valid.

(c) *(6 points)* What is the temperature of the background radiation (i) today and (ii) when the universe first became optically transparent? Combine these numbers with your answer from (a)-(vii) and the relationship between a and z to estimate the redshift when the universe became transparent.

— Problem 1 continues on the following page —

Problem 1, Continued.

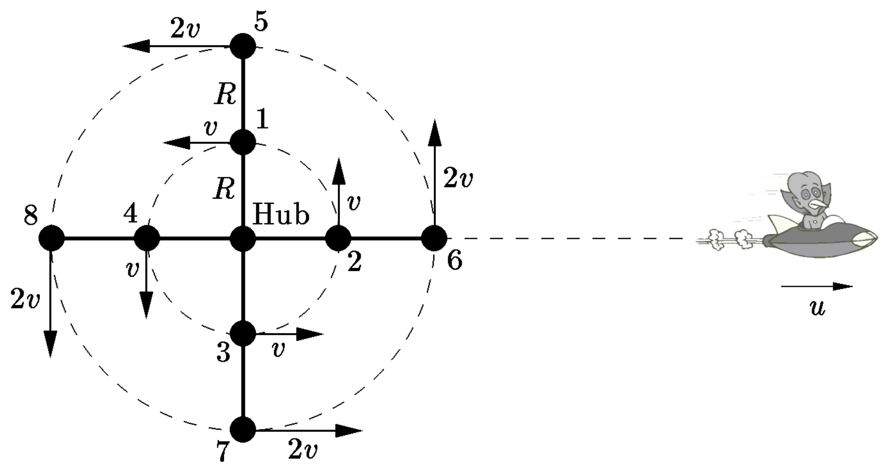
(d) *(6 points)* Assume for the moment (contrary to (b)) that the universe was always matter dominated, and take $H_0 = 72$ km-s^{-1}-Mpc^{-1}. Using the matter-dominated form of $a(t)$, what is the age of the universe (i) today and (ii) when it became transparent? (If you did not remember the temperature at which the universe became transparent, you will get full credit here if you answer the question assuming 10,000 K. You may find it useful to know that $1/(10^{10}$ yr$) = 97.8$ km-s^{-1}-Mpc^{-1}. If you are unclear about how to find the age of the universe in terms of H_0, note that you can first find H in terms of t, and then invert the formula.)

(e) *(6 points)* Write down a formula for the horizon distance $d_h(t)$ in terms of an arbitrary $a(t)$, and then evaluate it for the particular case of a matter-dominated universe. Taking the ages from (d), find the horizon distance (i) today and (ii) when the universe became transparent. (The horizon distance is defined as the present distance of objects that are so far away that light has just barely had time to reach us.)

PROBLEM 2: A TWO-LEVEL HIGH-SPEED MERRY-GO-ROUND *(15 points)*

This was Problem 20 on the Review Problems for Quiz 1, and it originated as Problem 3 of Quiz 1, 2007.

Consider a high-speed merry-go-round which is similar to the one discussed in Problem 3 of Problem Set 1, but which has two levels. That is, there are four evenly spaced cars which travel around a central hub at speed v at a distance R from a central hub, and also another four cars that are attached to extensions of the four radial arms, each moving at a speed $2v$ at a distance $2R$ from the center. In this problem we will consider only light waves, not sound waves, and we will assume that v is not negligible compared to c, but that $2v < c$.



We learned in Problem Set 1 that there is no redshift when light from one car at radius R is received by an observer on another car at radius R.

(a) *(5 points)* Suppose that cars 5–8 are all emitting light waves in all directions. If an observer in car 1 receives light waves from each of these cars, what redshift z does she observe for each of the four signals?

(b) *(10 points)* Suppose that a spaceship is receding to the right at a relativistic speed u along a line through the hub, as shown in the diagram. Suppose that an observer in car 6 receives a radio signal from the spaceship, at the time when the car is in the position shown in the diagram. What redshift z is observed?

PROBLEM 3: SIGNAL PROPAGATION IN A FLAT MATTER-DOMINATED UNIVERSE *(55 points)*

Consider a flat, matter-dominated universe, with scale factor

$$a(t) = bt^{2/3} ,$$

where b is an arbitrary constant. For the following questions, the answer to any part may contain symbols representing the answers to previous parts, whether or not the previous part was answered correctly.

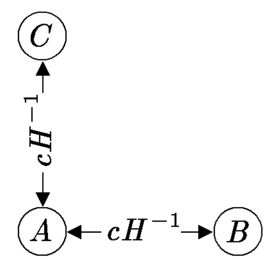
(a) *(10 points)* At time $t = t_1$, a light signal is sent from galaxy A. Let $\ell_{p,sA}(t)$ denote the physical distance of the signal from A at time t. (Note that $t = 0$ corresponds to the origin of the universe, not to the emission of the signal.) (i) Find the speed of separation of the light signal from A, defined as $\mathrm{d}\ell_{p,sA}/\mathrm{d}t$. What is the value of this speed (ii) at the time of emission, t_1, and (iii) what is its limiting value at arbitrarily late times?

(b) *(5 points)* Suppose that there is a second galaxy, galaxy B, that is located at a physical distance cH^{-1} from A at time t_1, where $H(t)$ denotes the Hubble expansion rate and c is the speed of light. (cH^{-1} is called the Hubble length.) Suppose that the light signal described above, which is emitted from galaxy A at time t_1, is directed toward galaxy B. At what time t_2 does it arrive at galaxy B?

(c) *(10 points)* Let $\ell_{p,sB}(t)$ denote the physical distance of the light signal from galaxy B at time t. (i) Find the speed of approach of the light signal towards B, defined as $-\mathrm{d}\ell_{p,sB}/\mathrm{d}t$. What is the value of this speed (ii) at the time of emission, t_1, and (iii) at the time of reception, t_2?

(d) *(10 points)* If an astronomer on galaxy A observes the light arriving from galaxy B at time t_1, what is its redshift z_{BA}?

(e) *(10 points)* Suppose that there is another galaxy, galaxy C, also located at a physical distance cH^{-1} from A at time t_1, but in a direction orthogonal to that of B. If galaxy B is observed from galaxy C at time t_1, what is the observed redshift z_{BC}? Recall that this universe is flat, so Euclidean geometry applies.



(f) *(10 points)* Suppose that galaxy A, at time t_1, emits electromagnetic radiation spherically symmetrically, with power output P. (P might be measured, for example, in watts, where 1 watt = 1 joule/second.) What is the radiation energy flux J that is received by galaxy B at time t_2, when the radiation reaches galaxy B? (J might be measured, for example, in watts per meter^2. Units are mentioned here only to help clarify the meaning of these quantities — your answer should have no explicit units, but should be expressed in terms of any or all of the given quantities t_1, P, and c, plus perhaps symbols representing the answers to previous parts.)

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QUIZ 1 FORMULA SHEET

DOPPLER SHIFT (For motion along a line):

$$z = v/u \quad \text{(nonrelativistic, source moving)}$$

$$z = \frac{v/u}{1 - v/u} \quad \text{(nonrelativistic, observer moving)}$$

$$z = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \quad \text{(special relativity, with } \beta = v/c\text{)}$$

COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{a(t_{\text{observed}})}{a(t_{\text{emitted}})}$$

SPECIAL RELATIVITY:

Time Dilation Factor:

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \ , \qquad \beta \equiv v/c$$

Lorentz-Fitzgerald Contraction Factor: γ

Relativity of Simultaneity:
Trailing clock reads later by an amount $\beta \ell_0 / c$.

EVOLUTION OF A MATTER-DOMINATED UNIVERSE:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{a^2} \ , \quad \ddot{a} = -\frac{4\pi}{3} G\rho a \ ,$$

$$\rho(t) = \frac{a^3(t_i)}{a^3(t)}\, \rho(t_i)$$

$$\Omega \equiv \rho/\rho_c \ , \quad \text{where} \quad \rho_c = \frac{3H^2}{8\pi G} \ .$$

$$\text{Flat } (k = 0): \qquad a(t) \propto t^{2/3} \ ,$$

$$\Omega = 1 \ .$$